**Algebra 2 Chapter 4 Calendar**

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**M-F 10/7/19-10/11/19**

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* Be prepared for daily quizzes.
* Every student is expected to do every assignment for the entire unit.
* Try [www.khanacademy.org](http://www.khanacademy.org) if you need help outside of school hours.
* Students who complete 100% of their homework for the semester will receive a 2% bonus!

### 4.1: Graphing in \((h,k)\) form or vertex form

The Parent Function of the Quadratic:

\[ y = x^2 \]

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y = x^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>((-2)^2 = 4)</td>
</tr>
<tr>
<td>-1</td>
<td>((-1)^2 = 1)</td>
</tr>
<tr>
<td>0</td>
<td>(0^2 = 0)</td>
</tr>
<tr>
<td>1</td>
<td>(1^2 = 1)</td>
</tr>
<tr>
<td>2</td>
<td>(2^2 = 4)</td>
</tr>
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</table>

\[ y = a(x - h)^2 + k \]

This is called **vertex form** or **\((h, k)\) form** or **graphing form**!
Examples:
1. \( y = (x + 2)^2 - 5 \)
   Vertex: \((-2, -5)\)  
   y-intercept: \((0, -1)\)  
   Transformations?

\[
\begin{array}{c}
\text{Max/Min:} -5 \quad \text{lowest y-value (from vertex)} \\
\text{Domain:} \mathbb{R} \quad \text{Range:} \{y | y \geq -5\} \\
\text{Axis of symmetry:} \ x = -2
\end{array}
\]

2. \( y = -(x - 3)^2 + 6 \)
   Vertex: \((3, 6)\)  
   y-intercept: \((0, -3)\)  
   Transformations?

\[
\begin{array}{c}
\text{Max/Min:} 6 \\
\text{Domain:} \mathbb{R} \quad \text{Range:} \ y \leq 6 \\
\text{Axis of symmetry:} \ x = 3
\end{array}
\]

3. \( y = (x + 2)^2 \)
   Vertex: \((-2, 0)\)  
   y-intercept: \((0, 4)\)  
   Transformations?

\[
\begin{array}{c}
\text{Max/Min:} 0 \\
\text{Domain:} \mathbb{R} \quad \text{Range:} \ y \geq 0 \\
\text{Axis of symmetry:} \ x = -2
\end{array}
\]

4. \( y = -x^2 + 6 \)
   Vertex: \((0, 6)\)  
   y-intercept: \((0, 6)\)  
   Transformations?

\[
\begin{array}{c}
\text{Max/Min:} 6 \\
\text{Domain:} \mathbb{R} \quad \text{Range:} \ y \leq 6 \\
\text{Axis of symmetry:} \ x = 0
\end{array}
\]
You try!!

a) \( y = (x + 4)^2 - 4 \)
   \( y = x^2 - 4 \)
   Vertex: \((-4, -4)\)  
   y-intercept: \((0, 12)\)
   Transformations?
   \( \downarrow 4 \)  \( \downarrow 4 \)
   Max/Min: \(-4\)
   Domain: \(\mathbb{R}\)  Range: \(y \geq -4\)
   Axis of symmetry: \(x = -4\)

b) \( y = -(x - 2)^2 + 1 \)
   \( y = -x^2 + 4x - 3 \)
   Vertex: \((2, 1)\)  
   y-intercept: \((0, -3)\)
   Transformations?
   \( \rightarrow 2 \) \( \Rightarrow \) \( \text{Reflect} \)
   Max/Min: \(1\)
   Domain: \(\mathbb{R}\)  Range: \(y \leq 1\)
   Axis of symmetry: \(x = 2\)

c) \( y = -x^2 - 2 \)
   Vertex: \((0, -2)\)  
   y-intercept: \((0, -2)\)
   Transformations?
   \( \downarrow 2 \) \( \Rightarrow \) \( \text{Reflect} \)
   Max/Min: \(-2\)
   Domain: \(\mathbb{R}\)  Range: \(y \leq -2\)
   Axis of symmetry: \(x = 0\)

d) \( y = (x + 5)^2 \)
   Vertex: \((-5, 0)\)  
   y-intercept: \((0, 25)\)
   Transformations?
   \( \leftarrow 5 \)
   Max/Min: 0
   Domain: \(\mathbb{R}\)  Range: \(y \geq 0\)
   Axis of symmetry: \(x = -5\)
When $a = 1$ or $a = -1$, there's no stretch or compression...

When $a = 2$...

$y = 2x^2$

When $a = 4$...

$y = 4x^2$

When $a = \frac{1}{2}$...

$y = \frac{1}{2}x^2$

When $|a| > 1$, $a$ is said to be **STRETCHED**.

When $|a| < 1$, $a$ is said to be **COMPRESSED**.
Examples:
1. \( y = 3(x + 2)^2 - 5 \)
   
   Vertex: \((-2, -5)\)  
   y-intercept: \((0, 7)\)  
   
   Transformations?  
   \(-2 \uparrow \downarrow \ 5 \quad \text{stretch 3} \)
   
   Max/Min: \(-5\)  
   
   Domain: \(\mathbb{R}\)  
   Range: \(y \leq -5\)  
   
   Axis of symmetry: \(x = -2\)

2. \( y = -4(x - 3)^2 + 6 \)
   
   Vertex: \((3, 6)\)  
   y-intercept: \((0, 30)\)  
   
   Transformations?  
   \(-3 \uparrow \downarrow \text{reflect} \quad \text{stretch 4} \)
   
   Max/Min: \(6\)  
   
   Domain: \(\mathbb{R}\)  
   Range: \(y \leq 6\)  
   
   Axis of symmetry: \(x = 3\)

3. \( y = \frac{1}{2}(x + 4)^2 - 1 \)
   
   Vertex: \((-4, -1)\)  
   y-intercept: \((0, 7)\)  
   
   Transformations?  
   \(-4 \uparrow \downarrow \ 1\)
   
   Max/Min: \(1\)  
   
   Domain: \(\mathbb{R}\)  
   Range: \(y \geq 1\)  
   
   Axis of symmetry: \(x = -4\)

4. \( y = -\frac{1}{3}x^2 \)
   
   Vertex: \((0, 0)\)  
   y-intercept: \((0, 0)\)  
   
   Transformations?  
   \(- \text{reflect} \quad \text{compress} \ \frac{1}{3}\)
   
   Max/Min: \(0\)  
   
   Domain: \(\mathbb{R}\)  
   Range: \(y \leq 0\)  
   
   Axis of symmetry: \(x = 0\)
5. Describe the transformations on the functions shown below from the parent function \( f(x) = x^2 \) and write the equation for each parabola.

\[
\begin{align*}
\text{Vertex: } &(-4, 5) \\
\text{Left } &4 \to \text{Right } 5 \\
\text{Reflect} &
\end{align*}
\]
\[
y = -(x+4)^2 - 5
\]

\[
\begin{align*}
\text{Vertex: } & (0, -6) \\
\downarrow & \text{Stretch 3} \\
\text{Reflect} & \text{Stretch 2}
\end{align*}
\]
\[
y = 3(x-0)^2 - 6 \quad \text{or} \quad y = 3x^2 - 6
\]

\[
\begin{align*}
\text{Vertex: } & (3, 0) \\
\rightarrow & 3 \text{ Reflect} \\
\text{Stretch} & 2
\end{align*}
\]
\[
y = 2(x-3)^2
\]

4.2: Rewriting Quadratic equations into vertex \((h, k)\) form

ESQ: Can you rewrite a quadratic equation in vertex form?

**Vertex Form:**

\[
y = a(x - h)^2 + k
\]

Vertex: \((h, k)\)

**Standard Form:**

\[
y = ax^2 + bx + c
\]

Vertex: \((-\frac{b}{2a}, f\left(-\frac{b}{2a}\right))\)

\[6 \text{ Page}\]
Example 3: Rewrite the equation in vertex form.

\[ y = 4x^2 - 24x + 31 \]
\[ a = 4 \]
\[ b = -24 \]
\[ c = 31 \]
\[ x = \frac{-b}{2a} \]
\[ x = \frac{-(-24)}{2(4)} \]
\[ x = \frac{24}{8} \]
\[ x = 3 \]
\[ y = 4(3)^2 - 24(3) + 31 \]
\[ = 36 - 72 + 31 \]
\[ = -5 \]
\[ y = 4(x - 3)^2 - 5 \]

Steps to writing an equation in vertex form:

1) Put the equation in **Standard** form.
\[ y = ax^2 + bx + c \]

2) Identify a, b, and c.

3) Find the x-coordinate of the vertex using the equation
\[ h = -\frac{b}{2a} \]

4) Find the y-coordinate of the vertex by evaluating
\[ k = f\left(-\frac{b}{2a}\right) \]

5) Substitute a, h, and k into the equation.
\[ y = a(x - h)^2 + k \]

Example 4: Write the following equations in (h, k) form, then write the vertex and draw a sketch.

a.) \[ y = x^2 - 6x - 2 \]
\[ x = \frac{-(-6)}{2(1)} = \frac{6}{2} = 3 \]
\[ y = (3)^2 - 6(3) - 2 \]
\[ = 9 - 18 - 2 \]
\[ = -11 \]

\[ y - \text{int} = c \]

<table>
<thead>
<tr>
<th>Vertex:</th>
<th>((3, -11))</th>
</tr>
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<tbody>
<tr>
<td>y-intercept:</td>
<td>(-2)</td>
</tr>
<tr>
<td>Domain:</td>
<td>((\infty, \infty))</td>
</tr>
<tr>
<td>Range:</td>
<td>([-11, \infty))</td>
</tr>
<tr>
<td>Max/Min:</td>
<td>(-11)</td>
</tr>
<tr>
<td>Transformations:</td>
<td>(\rightarrow 3 \downarrow 1)</td>
</tr>
</tbody>
</table>
b.) \( y = x^2 - 8x + 25 \)
\[
\begin{align*}
x &= \frac{-(-8)}{2(1)} = 4 \\
y &= (4)^2 - 8(4) + 25 \\
&= 16 - 32 + 25 \\
&= 9
\end{align*}
\]
vertex: \((4, 9)\)

What is the range in example b? \([9, \infty)\)

What are the transformations in example b? \(\rightarrow 4 \uparrow 9\)

You try!!!

d.) \( y = x^2 + 8x + 12 \)
\[
\begin{align*}
x &= -\frac{8}{2} = -4 \\
y &= (-4)^2 + 8(-4) + 12 \\
&= 16 - 32 + 12 \\
&= -4
\end{align*}
\]
vertex: \((-4, -4)\)

Example 5: Write the following equations in \((h, k)\) form, then write the vertex and draw a sketch.

a.) \( y = x^2 + 18x + 4 \)
\[
\begin{align*}
x &= -\frac{18}{2} = -9 \\
y &= (-9)^2 + 18(-9) + 4 \\
&= -77
\end{align*}
\]
vertex: \((-9, -77)\)

b.) \( y = 2x^2 + 20x + 6 \)
\[
\begin{align*}
x &= -\frac{20}{2(2)} = -\frac{20}{4} = -5 \\
y &= 2(-5)^2 + 20(-5) + 6 \\
&= 2 \cdot 25 - 100 + 6 \\
&= -44
\end{align*}
\]
vertex: \((-5, -44)\)
c) \( y = -2x^2 - 8x + 5 \quad a = -2 \)
\[
\begin{align*}
  x &= \frac{-(-8)}{2(-2)} = \frac{8}{-4} = -2 \\
  y &= -2(-2)^2 - 8(-2) + 5 \\
    &= 3
\end{align*}
\]
Vertex: \((-2, 3)\)
\[
y = -2(x+2)^2 + 3
\]

You try!!!

b) \( y = 2x^2 + 8x + 12 \quad a = 2 \)
\[
\begin{align*}
  x &= \frac{-8}{4} = -2 \\
  y &= 2(-2)^2 + 8(-2) + 12 \\
    &= 4
\end{align*}
\]
\[
y = 2(x+2)^2 + 4
\]

Example 6:
What are the transformations on the function \( y = 2x^2 + 12x + 19 \)?
\[
\begin{align*}
  x &= \frac{-12}{2(2)} = -\frac{12}{4} = -3 \\
  a &= 2 \\
  y &= 2(-3)^2 + 12(-3) + 19 = 1
\end{align*}
\]
Vertex: \((-3, 1)\)
\[
y = 2(x+3)^2 + 1
\]
\[
\leq 3 \quad \uparrow 1 \\
stretch 2
\]
4.3: Solving Quadratics Using Square Roots

*Can you solve an equation by completing the square? Can you graph a function including the vertex AND the x-intercepts?*

Work with a partner to solve the following quadratics by Square Rooting. If needed, write answers in terms of \(i\), and simplify radical answers.

1) \(x^2 + 25 = 0\)
\[
\sqrt{x^2} = \sqrt{-25}
\]
\[
x = \pm 5i
\]

2) \(-3x^2 - 30 = 6\)
\[
\frac{-3x^2}{-3} = \frac{36}{-3}
\]
\[
\sqrt{x^2} = \sqrt{-12}
\]
\[
x = \pm 2i \sqrt{3}
\]

3) \((x - 2)^2 - 9 = 0\)
\[
\sqrt{(x-2)^2} = \sqrt{9}
\]
\[
x - 2 = \pm 3
\]
\[
x = 2 \pm 3 \rightarrow -1
\]

4) \(\frac{1}{4}(y - 6)^2 = 8\)
\[
\frac{1}{4}(y-6)^2 = \sqrt{32}
\]
\[
y - 6 = \pm 4 \sqrt{2}
\]
\[
y = 6 \pm 4 \sqrt{2}
\]

---

**Solving Quadratics by Square Rooting**

- We can have \(0\) real solutions, \(1\) real solution \((0)\), or \(2\) real solutions.

---
Now that we can change a quadratic function from standard form to vertex form \(t\), we can easily find the vertex. In order to find the solutions (roots, \(x\)-intercepts, zeros), all we need to do is solve by using square roots.

**Example 1)** Solve \(y = x^2 - 8x - 33\) by rewriting in vertex form and graph completely.

- \(x = \frac{8}{2} = 4\)
- \(y = (4)^2 - 8(4) - 33 = -49\)
- \(y = (x - 4)^2 - 49\)

Set \(y = 0\)

- \(0 = (x - 4)^2 - 49\)
- \(49 = (x - 4)^2\)
- \(\pm 7 = x - 4\)
- \(4 \pm 7 \rightarrow -3\)

**Example 2)** Solve \(y = -x^2 - 2x + 3\) by rewriting in vertex form and graph completely.

- \(a = -1\)
- \(x = \frac{-(-2)}{2(-1)} = -1\)
- \(y = -(1)^2 - 2(-1) + 3 = 4\)
- \(y = -(x + 1)^2 + 4\)
- \(d = -(x + 1)^2 + 4\)
- \(\pm 2 = x + 1\)
- \(-1 \pm 2\)
- **Vertex:** \((-1, 4)\)
- **Domain:** \(\mathbb{R}\)
- **Max/Min:** 4

**Steps to solving by completing the square:**

1) Rewrite the equation in vertex form.

2) Graph the vertex

3) Set \(y = 0\) and solve by using square roots.

4) Graph \(x\)-intercepts (solutions)

\(\text{Solutions: } 1 \text{ \& } -3\)

\(\text{Range: } y \leq 4\)

\(\text{y-intercept: } (0, 3)\)
Example 3) Solve \( y = x^2 + 6x + 9 \) by rewriting in vertex form and graph completely.

\[
 x = \frac{-b}{2a} = -3 \\
 y = (-3)^2 + 6(-3) + 9 = 0 \\
 \text{Vertex: } (-3, 0) \\
 y = (x+3)^2
\]

\[
 x - \text{int} \\
 0 = (x+3)^2 \\
 0 = x + 3 \\
 x = -3
\]

Vertex: \((-3, 0)\)  
Domain: \(\mathbb{R}\)  
Max/Min: \(\circ\)  
Solutions: \(-3\)  
Range: \(y \geq 0\)  
y-intercept: \((0, 9)\)

**You try!!!**

a) \( y = x^2 - 4x - 21 \)

\[
 x = \frac{-b}{2a} = 2 \\
 y = 2^2 - 4(2) - 21 = -25 \\
 y = (x - 2)^2 - 25 \\
 0 = (x - 2)^2 - 25 \\
 \pm 5 = x - 2 \\
 \pm 5 + 2 = x \\
 7 = x \\
 5 = x
\]

Vertex: \((2, -25)\)  
Domain: \(\mathbb{R}\)  
Max/Min: \(\circ\)  
Solutions: \(7 \neq -3\)  
Range: \(y \leq -25\)  
y-intercept: \((0, -21)\)
Example 4) A football is kicked in the air, and its path can be modeled by the equation \( f(x) = -16(x - 5)^2 + 21 \), where \( x \) is the horizontal distance (in feet) and \( f(x) \) is the height. What is the maximum height of the football?

\[
\text{vertex:} (5, 21)
\]

\[
\text{max height}
\]

Example 5) A rocket is launched off a platform with an initial velocity of 19.6 meters per second. The path of the rocket can be modeled by the equation \( h = -4.9(t - 2)^2 + 78.4 \) where \( h \) is the height of the rocket, and \( t \) is the time in seconds.

What is the maximum height the rocket reaches?

\[
\text{vertex:} (2, 78.4)
\]

\[
\text{max height}
\]

After how many seconds will the rocket hit the ground? \( \text{height} = 0 \) \( \text{find} \) \( X \)-int

\[
-4.9(t - 2)^2 + 78.4 = 0
\]

\[
-4.9(t - 2)^2 = -78.4
\]

\[
-4.9
\]

\[
(t - 2)^2 = \frac{-78.4}{-4.9}
\]

\[
(t - 2) = \pm \frac{\sqrt{16}}{2}
\]

\[
(\pm 4)
\]

6 seconds

Example 6) Given the function \( f(x) = x^2 + 2x + 7 \), state whether the parabola opens up or down and the maximum or minimum. What do you need to find the maximum or minimum?

\[
\text{opens up}
\]

\[
\text{min} @ 0
\]

\[
x = \frac{-2}{2} = -1
\]

\[
\text{vertex}
\]

\[
y = (-1)^2 + 2(-1) + 7 = 6
\]

\[
y = (x + 1)^2 + 6
\]
4.4: More Graphing - Key Features of Quadratics

Can you identify key features of a quadratic and analyze....

With a partner, fill in the tables...

**Graph 1**
- **Vertex:** (3, 4)
- **y-intercept:** (0, -5)
- **Domain:** (-∞, ∞)
- **Range:** (-∞, 4]
- **Max/Min:** 4
- **Transformations:** → 3 ↑ 4 reflect
- **Roots:** 1 & 5

**Graph 2**
- **Vertex:** (-1, -12)
- **y-intercept:** (0, -9)
- **Domain:** (-∞, ∞)
- **Range:** [-12, ∞)
- **Max/Min:** -12
- **Transformations:** ← 1 ↓ 12 stretch 3
- **Roots:** -3 & 1

1.) If \((x + 3)(x - 1) = (x - h)^2 + k\), then what is the value of k?

**FOIL**

\[
x^2 + 2x - 3
\]

\[
x = \frac{-2}{2} = -1
\]

\[
y = (-1)^2 + 2(-1) - 3 = -4
\]
2.) Find the vertex from #1

\((-1, -4)\)

3.) What is the vertex of the function \(y = 3.2(x + 4)^2 - 5.1\)?

\((-4, -5.1)\)

4.) What is the y-intercept of the function \(f(x) = -2(x - 3)^2 + 10?\)

\[\begin{align*}
y &= -2(0 - 3)^2 + 10 \\
&= -18 + 10 \\
&= -8
\end{align*}\]

5.) What are the zeroes of the function \(h(x) = -3(x + 4)^2 + 3\)?

\[\begin{align*}
0 &= -3(x + 4)^2 + 3 \\
-3 &= -3(x + 4)^2 \\
1 &= (x + 4)^2 \\
\pm \sqrt{1} &= x + 4 \\
\pm 1 &= x + 4 \\
&\Rightarrow x = -4 \pm 1 \\
&\Rightarrow x = -3, -5
\end{align*}\]

6.) What are the x-intercepts of the function \(y = x^2 + 6x - 27\)?

\[\begin{align*}
0 &= x^2 + 6x - 27 \\
0 &= (x + 9)(x - 3) \\
&\Rightarrow x = -9, 3
\end{align*}\]

7.) A parabola has a vertex of \((-1, -5)\) and passes through the point \((3, -37)\). In the \(y = a(x - h)^2 + k\) form of the parabola, what is the value of \(a\)?

\[\begin{align*}
y &= a(x + 1)^2 - 5 \\
-37 &= a(3 + 1)^2 - 5 \\
-32 &= a(4)^2 \\
-32 &= 16a \\
-2 &= a
\end{align*}\]
8.) A parabola has a vertex of (5, 2) and passes through the point (6, 3). In the $y = a(x - h)^2 + k$ form of the parabola, what is the value of $a$?

$$y = a(x - 5)^2 + 2.$$ 

$$3 = a(6 - 5)^2 + 2.$$ 

$$3 = a + 2.$$ 

You try!

9.) A parabola has a vertex of $(-4, 3)$ and passes through the point $(-2, 19)$. In the $y = a(x - h)^2 + k$ form of the parabola, what is the value of $a$?

$$19 = a(-2 + 4)^2 + 3.$$ 

$$19 = 4a + 3.$$ 

$$16 = 4a.$$ 

$$a = 4.$$ 

10.) If $g(x) = -x^2 + 10x - 21 = a(x - h)^2 + k$, then what is the value of $k$?

$$a = -1.$$ 

$$x = \frac{10}{2(-1)} = 5.$$ 

$$y = -1(5)^2 + 10(5) - 21 = 4.$$ 

11.)Which of the following functions have the range $[4, \infty)$?

(A) $y = 2(x + 3)^2 + 4$ 

(B) $y = (x + 2)^2$ 

(C) $y = (x + 2)^2$ 

(D) $f(x) = 2x^2 + 6x + 4$ 

$$2(x + \frac{3}{2})^2 - 1/2.$$ 

12.) The graph $f(x) = x^2$ has a vertical stretch by a factor of 5 and is reflected vertically. What is the equation of the function after the transformation?

$$C(x) = -5x^2.$$
13.) Describe in words how the graph of \( g(x) = \frac{1}{2}(x + 4)^2 - 3 \) would be transformed from the parent function \( f(x) = x^2 \).

\[ \quad \downarrow 4 \quad \downarrow 3 \quad \text{compress} \quad \downarrow \frac{1}{2} \]

14.) The graph \( f(x) = x^2 \) is compressed by a factor of \( \frac{2}{3} \), shifted 3 units to the left, and 6 units up. What is the equation of the function after the transformation?

\[ f(x) = \frac{2}{3} (x + 3)^2 + 6 \]

15.) When evaluating the function \( f(x) = -2(x + 1)^2 + 3 \) for any real number \( x \), what must be true about the value of \( f(x) \)?

\[ (-1, 3) \quad \text{or} \quad \text{max @ 3} \]

\( f(x) \) must be in the range \( \mathbb{R} \mid y \leq 3 \}

\(-\infty, 3\]\n
Example 16: The storage building shown can be modeled by the graph of the function \( y = -x^2 + 24x - 44 \) where \( x \) is the horizontal distance and \( y \) is the height (in cm). What is the maximum height of the building?

\[ a = 1 \]

\[ x = \frac{-24}{2(-1)} = 12 \]

\[ y = -(12)^2 + 24(12) - 44 = 100 \]

What is the width of the building at the base?

find \( x - 1 \) in

\[ -1(x - 12)^2 + 100 = 0 \]

\[ (x - 12)^2 = 100 \]

\[ x - 12 = \pm 10 \]

\[ x = 12 \pm 10 \]

\[ width = 22 - 2 \]

\[ 20 \text{ cm} \]

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17.) Compare the two functions represented below. Determine which of the following statements is true.

<table>
<thead>
<tr>
<th>Function $f(x)$</th>
<th>Function $g(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Graph" /></td>
<td>$\text{Roots: } (x-1)^2 + 6 = 0$</td>
</tr>
<tr>
<td></td>
<td>$x^2 = -6$</td>
</tr>
<tr>
<td></td>
<td>$y = \text{Int.}: g(x) = (x-1)^2 + 6 = 7$</td>
</tr>
<tr>
<td></td>
<td>$\text{Vertex: } (1, 0)$</td>
</tr>
<tr>
<td></td>
<td>$x = 1$ Axis of Sym</td>
</tr>
</tbody>
</table>

I. $f(x)$ and $g(x)$ have the same $y$-intercept.
II. $f(x)$ and $g(x)$ have the same roots.
III. $f(x)$ and $g(x)$ have the same axis of symmetry. $x = 1$
IV. $f(x)$ and $g(x)$ have the same range.

18.) Find the key features of the function $y = -0.5x^2 - 3x + 4$

| Vertex: $(-3, 8.5)$ |
| y-intercept: 4 |
| Domain: $(-\infty, \infty)$ |
| Range: $(-\infty, 8.5]$ |
| Max/Min: 8.5 |
| Transformations: $\leq 3 \rightarrow 8.5$, reflect, compress |
| Roots: $-7.12, 4.12$ |

19.) Find the key features of the function $y = 2.5(x + 10.2)^2 - 5$

| Vertex: $(-10.2, -5)$ |
| y-intercept: $255.1$ |
| Domain: $(-\infty, \infty)$ |
| Range: $[-5, \infty)$ |
| Max/Min: -5 |
| Transformations: $\leq 10.2 \rightarrow 5$, stretch $2.5$ |
| Roots: $-11.6, -8.8$ |