9.4: ANNUITIES, STOCKS, AND BONDS

Objectives

1. Determine the value of an annuity.
2. Determine regular annuity payments needed to achieve a financial goal.
3. Understand stocks and bonds as investments.
4. Read stock tables.

often used to save for retirement

■ **Annuities**: An ***annuity*** is a sequence of equal payments made at equal time periods.

The ***value of an annuity*** is the sum of all payments plus all interest paid.

■ Annuity Interest Compounded Once a Year

If P is the deposit made at the end of each year for an annuity that pays an annual interest rate r (in decimal form) compounded once a year, the value, A, of the annuity after t years is

$$A = \frac{P\left[(1+r)^t - 1\right]}{r}.$$

P =

Example 1: To save for retirement, you decide to deposit $1000 into an IRA at the end of each year for the next 30 years. If you can count on an interest rate of 10% per year compounded annually, n = 1, r = .10

a. How much will you have from the IRA after 30 years?

$$A = \frac{1000\left[(1+.10)^{30} - 1\right]}{.10}$$

$$A = 164{,}492.02$$

b. Find the interest.

accumulated value − all deposits

$164{,}492.02 - 30(1000)$

$= 164{,}492.02 - 30{,}000$

$= \boxed{134{,}492.02}$ Wow!

What if you start earlier and invest for 40 years?

$$A = \frac{1000\left[(1+.10)^{40} - 1\right]}{.10}$$

$= 442{,}592.56$

Interest:

$442{,}592.56 - 40(1000)$

$= 442{,}592.56 - 40{,}000$

$= \boxed{402{,}592.56}$

4X as much!!!

■ Annuity Interest Compounded N Times Per Year

If P is the deposit made at the end of each compounding period for an annuity that pays an annual interest rate r (in decimal form) compounded n times per year, the value, A, of the annuity after t years is

$$A = \frac{P\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}{\left(\frac{r}{n}\right)}$$

Example 3 At age 25 to save for retirement, you decide to deposit $200 into an IRA at the end of each month at an interest rate of 7.5% per year compounded monthly.

a. How much will you have from the IRA when you retire at age 65? $t = 40$ years (age 65 − age 25)

$r = .075$ $P = 200$ $n = 12$

$$A = \frac{200\left[\left(1+\frac{.075}{12}\right)^{12\cdot 40} - 1\right]}{\left(\frac{.075}{12}\right)} = \$604{,}764.43$$

b. Find the interest.

accumulated value − all deposits

$604{,}764.43 - 200(12)(40)$
$= 604{,}764.43 - 96{,}000$
$= \$508{,}764.43$ interest

each year: $200/mo \times 12\ mo = 2400/yr$

all: $2400/yr \times 40\ years = 96000$

The interest is more than 5 times the amount of your contributions to the IRA.

→ divide to compare

$\frac{\text{interest}}{\text{\$ invested}} = \frac{508764}{96000} = 5.3$ (or) $\approx \frac{500{,}000}{100{,}000} \approx 5$

■Planning for the Future with an Annuity : How much to invest?

P, the deposit that must be made at the end of each compounding period into an annuity that pays an annual interest rate r (in decimal form) compounded n times per year in order to achieve a value of A dollars after t years is

$$P = \frac{A\left(\frac{r}{n}\right)}{\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}.$$

Example 4: You would like to have \$20,000 to use as a down payment for a home in five years by making regular, end-of-the-month deposits in an annuity that pays 8% compounded monthly.

GOAL: $A = 20{,}000$ $\quad t = 5$ $\quad r = .08$ $\quad n = 12$

a. How much should you deposit each month?

Find P

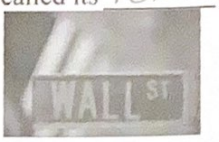
$$P = \frac{20{,}000\left(\frac{.08}{12}\right)}{\left(\left(1+\frac{.08}{12}\right)^{12\cdot 5}-1\right)} = \$272.19$$

b. How much of the \$20,000 down payment comes from deposits and how much comes from interest?

$(272.19)(12)(5) = 16{,}331.40$ $\quad 20{,}000 - 16{,}331.40 = 3{,}668.60$

Investments $$$

- When depositing money into a bank account, you are making an investment.
- The account's interest rate guarantees a certain percent increase in your investment, called its return
- Other kind of investments that are riskier are called ***stocks*** and ***bonds***.



Stocks

- Investors purchase ***stock***, shares of ownership in a company.

For example, if a company has issued a total of 1 million shares and an investor owns 20,000 of these shares, that investor owns 2% of the company.

"out of" → $\frac{20{,}000}{1{,}000{,}000} = 0.02 = 2\%$

- Any investor who owns some percentage of the company is called a shareholder.
- Buying or selling stocks is referred to as trading.
- Stocks are traded on a stock exchange.

There are two ways to make money by investing in stock:

- You sell shares for more money than you paid for them, in which case you have a capital gain on the sale of stock.
- While you own the stock, the company distributes all or part of its profits to shareholders as dividends.

Reading Stock Tables

Daily newspapers and online services give current stock prices and other information about stocks.

- ***52-week high*** refers to the highest price at which a company traded during the past 52 weeks (1 year)
- ***52-week low*** refers to the lowest price at which a company traded during the past 52 weeks.
- ***Stock*** refers to the company name.
- ***SYM*** refers to the symbol the company uses for trading.
- ***Div*** refers to dividends paid per share to stockholders last year.
- ***Yld%*** stands for percent yield. = annual return
- ***Vol100s*** stands for sales volume in 100's for yesterday. (mult × 100)
- ***Hi*** stands for the highest price at which the company's stock traded yesterday.
- ***Lo*** stands for the lowest price at which the company's stock traded yesterday.
- ***Close*** stands for the price at which shares traded when the stock exchange closed yesterday.
- ***Net Chg*** stands for net change (from the close two days ago to the close yesterday).
- ***PE*** stands for the price-to-earnings ratio.

$$\text{PE ratio} = \frac{\text{Yesterday's closing price per share}}{\text{Annual earnings per share}}$$

- Annual earnings per share = $\frac{\text{yesterday's closing price per share}}{\text{PE ratio}}$

Example 5: Use the stock table for Disney to answer the following questions:

← Year ; Yesterday →

52-week High	Low	Stock	SYM	Div per share	Yld % annual return	PE	Vol 100s x 100	Hi	Lo	Close price	Net Chg change in price
42.38	22.50	Disney	DIS	.21	.6	43	115900	32.50	31.25	32.50	...

a. What were the high and low prices for the past 52 weeks?

↳ $42.38 ↳ $22.50 per share

b. If you owned 3000 shares of Disney stock last year, what dividend did you receive?

.21 per share

↳ $3000 \times (.21) = \boxed{\$630}$

c. What is the annual return for dividends alone? How does this compare to a bank account offering a 3.5% interest rate?

.6% = .006

Dividend is lower ($.006 < .035$ or $.6\% < 3.5\%$)

BUT can also make $ from stocks by buying low & selling high ... (COULD be a better investment)

d. How many shares of Disney were traded yesterday?

Vol 100's, volume of sales

$115900 \times 100 = \boxed{11{,}590{,}000 \text{ shares}}$

e. What were the high and low prices for Disney shares yesterday?

↳ $32.50 per share ↳ $31.25 per share

f. What was the price at which Disney shares traded when the stock exchange closed yesterday?

$32.50

g. What does "..." in the net change column mean?

No change

h. Compute Disney's annual earnings per share using

$$\text{Annual earnings per share} = \frac{\text{Yesterday's closing price per share}}{\text{PE ratio}} = \frac{32.50}{43} = .7558$$

$\approx \boxed{\$.76}$