Little Red's Crumby Day - Lesson 9.3 Day 1

There are 16 tile squares on the path from Little Red Riding Hood's house to her grandmother's house. On the first tile, Little Red drops 2 crumbs of cake, on the second tile, Little Red drops 6 crumbs of cake, on the third tile she drops 18 crumbs, on the 4th tile 54, and so on.

1. Make a table that gives the number of crumbs dropped on each of the first six tiles. Write each term using ONLY 3's and 2's.

Tile	Number of crumbs	
1	2	2
2	$6 = 2\cdot3$	$2(3)$
3	$18 = 2\cdot3\cdot3$	$2(3)^2$
4	$54 = 2\cdot3\cdot3\cdot3$	$2(3)^3$
5	$162 = 2\cdot3\cdot3\cdot3\cdot3$	$2(3)^4$
6	$486 = 2\cdot3\cdot3\cdot3\cdot3\cdot3$	$2(3)^5$

2. Now can you write each term using only **one** 3 and **one** 2 (not counting exponents)? Make a new table above, to the right of your original table.

3. How many crumbs will she drop on the 8th tile? $2(3)^7 = 4374$

4. How many crumbs will she drop on the *n*th tile?

$$a_n = 2(3)^{n-1}$$

5. How many *total* crumbs has she dropped by the time she reaches the 3rd tile? The 5th tile? The 16th tile?

3rd	5th
26	242

6. What if we wanted to find a shortcut for the 16th term? We'll start with an easier problem and see if we can notice some patterns. Pause. Put your pencil down. Watch Ms. Stecher create a shortcut.

$$S_5 = 2 + \cancel{2\cdot3} + \cancel{2\cdot3^2} + \cancel{2\cdot3^3} + \cancel{2\cdot3^4}$$

$$3\cdot S_5 = \cancel{2\cdot3} + \cancel{2\cdot3^2} + \cancel{2\cdot3^3} + \cancel{2\cdot3^4} + 2\cdot3^5$$

$$S_5 - 3S_5 = 2 - 2\cdot3^5 \rightarrow S_5(1-3) = 2(1-3^5)$$

$$S_5 = \frac{2(1-3^5)}{(1-3)}$$

7. Find the total number of crumbs Little Red has dropped by the time she reaches the 16th tile.

$$S_n = \frac{2(1-r^n)}{1-r} = S_{16} = \frac{2(1-3^{16})}{1-3} = \boxed{43046720 \text{ crumbs}}$$

Lesson 9.3 Day 1—Geometric Sequences and Finite Series

Geometric sequences have a common multiplier, or a constant ratio. (Think of it like an exponential equation.)

Explicit Rules:

$a_n = a_1(r^{n-1})$ or $a_n = a_0(r^n)$

the term before a_1

r = multiplier

Recursive:

$\begin{cases} a_1 = \\ a_n = r \cdot a_{n-1} \end{cases}$

Partial sums:

$S_n = \frac{a_1(1-r^n)}{1-r}$ (works only when n is a finite number)

Examples:

1. Write a formula for the sequence given by 2, 10, 50, 250…

   $a_1 = 2$
   $r = 5$

   $a_n = 2 \cdot 5^{n-1}$ OR $a_n = \frac{2}{5}(5)^n$

2. If a_n is a geometric sequence with $a_4 = 48$ and $a_7 = 384$, find the common ratio, r, and a_1.

   $\underline{6}$ $\underline{12}$ $\underline{24}$ $\underline{48}$ ___ ___ $\underline{384}$

   a_1 ÷2 ÷2 ÷2 a_4 a_7

   $\boxed{a_1 = 6}$

   $48 \cdot r \cdot r \cdot r = 384$
   $r^3 = 8$
   $\boxed{r = 2} \rightarrow a_n = a_1(2)^{n-1}$

3. Use the formula for *nth* partial sums to find the sum of the sequence given by 3, 9, 27, 81, 243…up to the 15th term.

   $r = 3$ $n = 15$
   $a_1 = 3$

   $S_{15} = \frac{3(1-3^{15})}{1-3} = 21,523,359$

4. The first week of July was the opening week of the Deluxe Hotel. It had 200 guests that week. In the weeks immediately following this, the bookings decreased by 5% each week.

   200, 190, …

   a. How many guests did the hotel have in the 11th week?

   $a_n = 200(.95)^{n-1}$ $a_{11} = 119.75 \approx 120$ people $r = 0.95$

   b. How many total guests did the hotel have during the 13 weeks of July, August, and September?

   $S_{13} = \frac{200(1-.95^{13})}{1-.95} = 1946.63 \approx 1947$ guests

   ~~c.~~ In which week did the hotel have their 3000th guest?