#Goals – Lesson 9.2

Mallory's New Year Resolution is to become a better runner, but she quickly forgets about her goal. Once school lets out for the summer, she decides to start running in earnest. On June 1st, she starts by running 15 minutes and then adds 5 minutes to her run every day.

1. Mallory tries to record how much she ran every day, but she notices that she forgot to track her running on several days. Complete her running log for the indicated days.

	Date	Time spent running
a_1	June 1st	15 minutes
a_2	June 2nd	20 min.
a_5	June 5th	35 min.
a_{11}	June 11th	
a_{29}	June 29th	
a_{30}	June 30th	160 minutes

3 days, 3.5 min (+15 min.), +5

Recursive

$\begin{cases} a_1 = 15 \\ a_n = a_{n-1} + 5 \end{cases}$

"previous term"

Explicit Formula

2. Write a rule that gives the amount of time that Mallory runs on any given day in June. Prove that your rule works.

? $a_n = 15 + 5n$?

Test it: $a_2 = 15 + 5 \cdot 2 = 25$ ⇒ since the 1st is already established, add $(n-1)$ groups of 5 ⇒ $a_n = 15 + 5(n-1)$

$a_2 = 15 + 5 \cdot 1 = 20$ ✓

3. a. What was Mallory's average run length during the month of June? How do you know?

$$\text{avg} = \frac{\text{low} + \text{high}}{2} = \frac{15 + 160}{2} = \frac{175}{2} = 87.5 \text{ min}$$

b. Was there a day when she ran this amount exactly? Why or why not?

No: none have half minutes

Sum

4. How many minutes total did Mallory run in the entire month of June?

$$\text{avg} \cdot 30 = (87.5)(30) = 2625 \text{ min.}$$

5. Can you come up with a rule for how many miles Mallory has run in total by the *nth* day of June?

avg 1st & last days, then mult. by n: $S_n = \frac{15 + a_n}{2} \cdot n$

6. Mallory's brother, Dale, is inspired by his sister and also begins running regularly. His first run is on June 5th and he runs 10 minutes. Every day after he adds 6 minutes to his time. On June 22nd, Mallory and Dale head out on a run together but they don't end at the same time. Who ran longer? How do you know?

Lesson 9.2—Arithmetic Sequences and Series

Arithmetic sequences have a constant difference, d. (Think of it like a linear equation with constant rate of change, or slope.)

$y = mx + b$

Explicit Formulas

$a_n = a_1 + d(n-1)$ or $a_n = a_0 + d \cdot n$

(a_1: first term; a_0: "zero" term)

Recursive Formula

$\begin{cases} a_1 = \square \\ a_n = a_{n-1} + d \end{cases}$

(a_{n-1}: previous term)

Partial Sums: $S_n = \frac{a_1 + a_n}{2} \cdot n$

$S_n = \frac{a_1 + (a_0 + dn)}{2} \cdot n$

average of 1st & last
- with a_n as last
- must find a_n

1. Find the 7th, 8th, and 14th term in the arithmetic sequence 2, 6, 10, 14, … Then write an explicit rule for the sequence.

+4 +4 +4 $\quad d = 4$

Recursive way: 2, 6, 10, 14, 18, 22, 26 …

7th Term	8th Term	14th Term	Explicit Rule: Two forms
$a_7 = 26$ (+4)	$a_8 = 30$	Explicit way: $2 + 4(13) = 54$; $a_1 + d(n-1)$	$a_n = 2 + 4(n-1) = 2 + 4n - 4$ OR $a_n = -2 + 4n$ (zero term)

2. Write a scenario that could be modeled with an arithmetic sequence. Be sure to explain what n and a_n represent in your context.

Start with \$20 (a_1) and save \$5 each week ($d = 15$)

n = week #

a_n = money saved

3. If a_n represents an arithmetic sequence with $a_4 = 5$ and $a_7 = -13$, find a_0.

Set up: a_0 a_1 a_2 a_3 a_4 a_5 a_6 a_7

29 … 5 … −13

+6 +6 +6 +6 −6 −6 −6

Step 1 (//)

Step 2 ⇒ GO BACKWARDS! drops 18 in 3 hops: $d = -6$

4. There are 26 seats in the first row of the auditorium, 29 seats in the second row, 32 seats in the third row, and so on. (a_1)

Stage

+3 → 26; +3 → 29; +3 → 32

Partial Sum

a. How many total students can be seated in the first 6 rows?

$26 + 29 + 32 + 35 + 38 + 41 = 201$ students

b. ~~How many rows of seats do we need in order to seat 2500 students?~~

b) How many TOTAL can be seated in the first 20 rows?

Step 1: $S_n = \frac{a_1 + a_n}{2} \cdot n$

$S_{20} = \frac{26 + \square}{2} \cdot 20$

Step 2: need 20th row

$\begin{cases} a_{20} = 26 + 3(19) = 83 \\ \text{OR } = 23 + 3 \cdot 20 = 83 \end{cases}$

Step 3: $S_{20} = \frac{26 + 83}{2} \cdot 20$

$= 1090$ students