Unit 9 Introduction

Name Key

Recursive Rule

Figure out next term from the last

+5 +7 +9

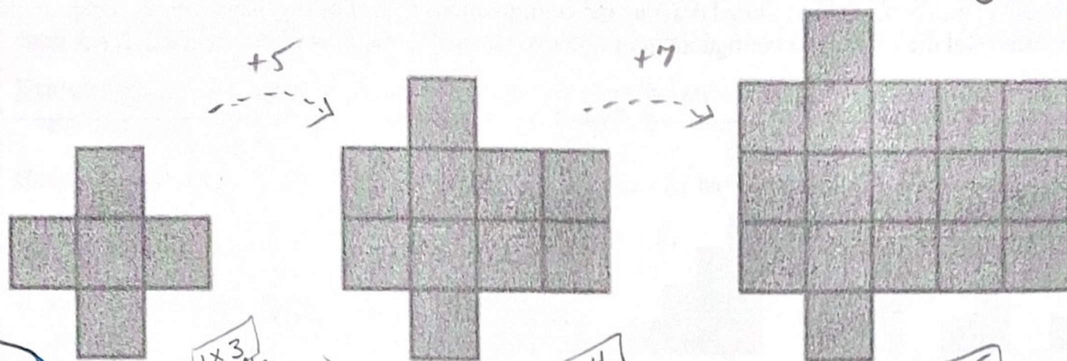


Figure 1 → $\boxed{1\times3}$ middle +2 (top/bottom) = 5 squares

Figure 2 → $\boxed{2\times4}$ middle +2 = 10 squares

Figure 3 → $\boxed{3\times5}$ middle +2 = 17 squares

Fig. 4 → $\boxed{4\times6}$ middle +2 = 26 squares

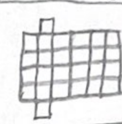
Explicit Rule

Can figure out any term!

1. How do you see the pattern growing? Use colors to show where you see the new squares being added.

The middle section is growing each time.

Fig. 4



2. Draw Figure 4. How many small squares would be in Figure 4? In Figure 5?

Fig. 4 → $\boxed{4\times6}$ middle section + 2 (top/bottom) = 24+2 = $\boxed{26}$

Fig. 5 → $\boxed{5\times7}$ middle + 2 (top/bottom) = 35+2 = $\boxed{37}$

3. How many small squares would be in Figure 43? Describe what it would look like.

Fig. 43 → $\boxed{43\times45}$ middle +2 (top/bottom) = 1935 + 2 = $\boxed{1937}$

★ Note: The Recursive Rule of add 5, then add 7, then add 9 would be really hard to apply here!

4. How many small squares would be in Figure 0? What would it look like?

→ $\boxed{0\times2}$ middle section + 2 = 0 + 2 = $\boxed{2}$

No middle!

5. Can you come up with a rule for how many small squares would be in Figure n?

→ $n\times(n+2)+2$

(Explicit Rule (can find any term)) → $\boxed{n(n+2)+2}$

Lesson 9.1—Using Sequences and Series to Describe Patterns

a_n gives a **sequence** where n is the term number and the output of a_n gives the term value.

Explicit rules do NOT depend on a previous term. The rule is in terms of n.

Ex: $\boxed{a_n = n^2 + 2}$ $\longrightarrow$ $a_1 = 1^2+2$, $a_1 = 3$ $\quad a_2 = 2^2+2$, $a_2 = 6$ $\quad a_3 = 3^2+2$, $a_3 = 11$ $\quad a_4 = 4^2+2$, $a_4 = 18$

Recursive rules DO depend on a previous term. The rule always has two parts.

Ex: $\begin{cases} a_1 = 3 \\ a_n = \underbrace{a_{n-1}}_{} + 2 \end{cases}$ "previous term"

$a_1 = 3$ $\quad a_2 = a_1 + 2$, $a_2 = 3+2$, $a_2 = 5$ $\quad a_3 = a_2 + 2$, $a_3 = 5+2$, $a_3 = 7$ $\quad a_4 = a_3 + 2$, $a_4 = 7+2$, $a_4 = 9$

A **series** is the sum of the terms of a sequence

$$S_n = \sum_{i=1}^{5} a_i = a_1 + a_2 + a_3 + a_4 + a_5$$

to → 5, from → $i=1$

Find all terms for a_1 to a_5 & ADD

Ex. $\sum_{i=1}^{3} 2i + 5 = 2\cdot1+5 + 2\cdot2+5 + 2\cdot3+5$
$= 7 + 9 + 11$
$= \boxed{27}$

1. Write the first 4 terms of each sequence, given the following explicit rules:

   a. $a_n = n^2 + 2$

   see above

   b. $a_n = \frac{(-1)^n}{n}$

   $a_1 = \frac{(-1)^1}{1}$ $\quad a_2 = \frac{(-1)^2}{2}$ $\quad a_3 = \frac{(-1)^3}{3}$ $\quad a_4 = \frac{(-1)^4}{4}$

   $a_1 = \boxed{-1}$ $\quad a_2 = \boxed{\frac{1}{2}}$ $\quad a_3 = \boxed{\frac{-1}{3}}$ $\quad a_4 = \boxed{\frac{1}{4}}$

2. Find a_7. Then write an explicit rule for a_n.

   a. 3, 6, 9, 12, 15, 18, … $\boxed{21}$

   Multiples of 3!

   $\boxed{a_n = 3n}$

   b. $\frac{5}{2}, \frac{5}{3}, \frac{5}{4}, 1, \frac{5}{6}, \ldots \frac{5}{7}, \boxed{\frac{5}{8}}$

   $\frac{5}{5}$

   $\boxed{a_n = \frac{5}{n+1}}$

3. Write the first 4 terms of the sequence, given this recursive rule:

   $\begin{cases} a_1 = 4 \\ a_n = \underbrace{a_{n-1}}_{\text{previous term}} + 10 \end{cases}$

   $a_1 = \boxed{4}$ $\quad a_2 = 4+10$, $a_2 = \boxed{14}$ $\quad a_3 = 14+10$, $a_3 = \boxed{24}$ $\quad a_4 = 24+10$, $a_4 = \boxed{34}$

4. Write a recursive rule for the a_n term:

   25, 18, 11, 4, -3…

   -7 -7

   $\begin{cases} a_1 = 25 \\ a_n = a_{n-1} - 7 \end{cases}$

5. Evaluate.

   to → 5, from → $i=1$

   $$\sum_{i=1}^{5} 3i - 1 = 3\cdot1-1 + 3\cdot2-1 + 3\cdot3-1 + 3\cdot4-1 + 3\cdot5-1$$
   $$= 2 + 5 + 8 + 11 + 14$$
   $$= \boxed{40}$$

   sum

6. How many small squares will be in figure 43?

2 blocks of 2x2 $\quad$ 2 blocks of 3x3

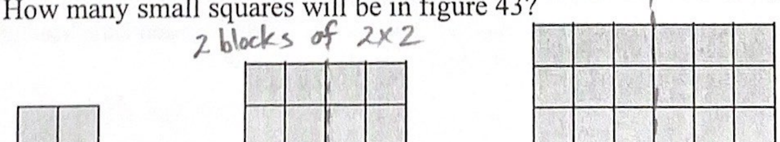


Figure 1 $\quad$ Figure 2 $\quad$ Figure 3

1×2 $\leftarrow 1\cdot2$ $\quad$ 2×4 $\leftarrow 2\cdot2$ $\quad$ 3×6 $\leftarrow 3\cdot2$

$=\boxed{2}$ $\quad$ $=\boxed{8}$ $\quad$ $=\boxed{18}$

Rule:

$a_n = n \times 2n$

$\boxed{a_n = 2n^2}$

$a_{43} = 2\cdot43^2$

$a_{43} = 3698$