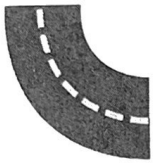
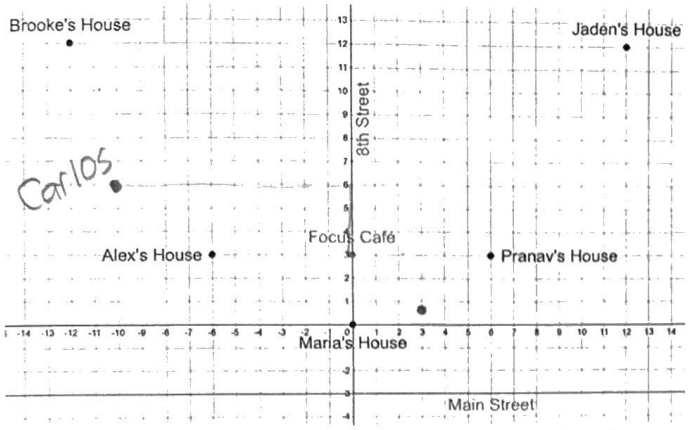
on 8.2: Welcome to Conics Neighborhood (Parabolas)



In Bolapara City, the most sought after residential area is on Conicopia Drive, located just north of Main Street and close to the local café. In fact, this is exactly what the city developer had in mind when he zoned out this winding stretch of road. Brooke, Alex, Maria, Pranav, and Jaden own 5 of the lots in this coveted new neighborhood.



Let each square on the graph represent 1 block.

1. Which of the five residents lives closest to Main Street?

   Maria

   Which of the five residents lives closest to the Café?

   Maria

2. Fill in the table below with distances of each person to the café and Main Street.

Person	Distance to Café	Distance to Main Street
Maria	3	3
Alex	6	6
Pranav	6	6
Brooke	15	15
Jaden	15	15

pythag. thm

3. What do you notice about each person living on Conicopia Drive?

   They live the same distance from the cafe and main st.

4. Carlos lives 10 blocks west and 6 blocks north of Maria's house. Does he live on Conicopia Drive? How do you know?

   NO

   distance to main = 9 blocks

   distance to cafe = $\sqrt{10^2+3^2} = \sqrt{109} \neq 9$

5. Write an equation for the curve going through Conicopia Drive. (Hint: what shape does it make?)

   $y = ax^2 \rightarrow 3 = a(6)^2 \rightarrow a = \frac{1}{12} \rightarrow y = \frac{1}{12}x^2$

6. How many houses are necessary to determine the equation for Conicopia Drive? Explain.

   3 houses (2 only makes a line)

7. Phuong also lives on Conicopia Drive, 3 blocks east of 8th Street. How far away from Main Street does she live? Plot Phuong's location on the map.

   $y = \frac{1}{12}(3)^2 = \frac{9}{12} = \frac{3}{4}$ (3, 3/4)

   3.75 blocks from main st.

8. Let's look more closely at Alex, Pranav, and Maria's house. Describe each of their locations with respect to the café.

Maria is 3 blocks away

Pranav and Alex are 6 blocks away.

9. Way back in Unit 2, we learned several forms for writing parabolas, but today we'll look at one more. It's $y = \frac{1}{4p}x^2$. What is the "p" value for the parabola that passes through Conicopia Drive?

$4p = 12 \rightarrow \boxed{p = 3}$

10. Find each of the following distances in terms of p.

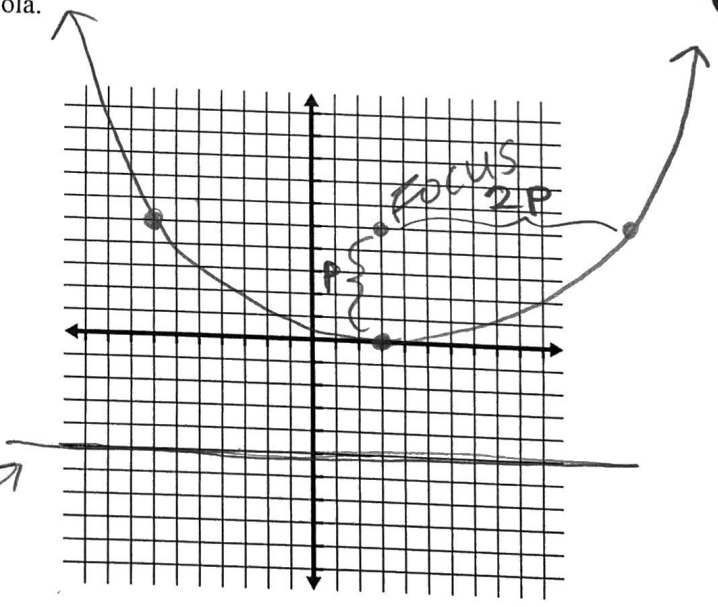
	Distance (# of blocks)	Distance in terms of p
Maria to Café	3	p
Maria to Main Street	3	p
Pranav to Café	6	2p
Alex to Café	6	2p
Alex to Pranav	12	4p

11. Now let's consider the parabola given by $y = \frac{1}{20}(x-3)^2$. Using what you learned above, find the vertex and two other points on this parabola. Then graph the parabola.

Vertex: (3, 0) $p = 5$

Two additional points:

(-7, 5) (13, 5)

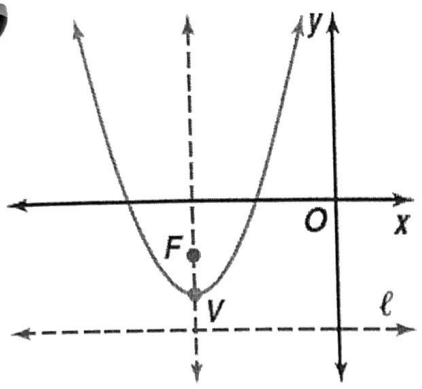


A **Parabola** is the set of points equidistant from its directrix and focus.

P is called the **Focal Length** and is the distance from the vertex to the directrix and focus.

Vertical Parabolas

$$y = \frac{1}{4p}(x-h)^2 + k \quad \text{OR} \quad (x-h)^2 = 4p(y-k)$$



Vertex: (h, k)

(F) **Focus:** $(h, k+p)$ OR $(h, k-p)$

Axis of Symmetry: $x = h$

(d) **Directrix:** $y = k+p$ or $y = k-p$

Opens Up: $p > 0$

Opens Down: $p < 0$

Domain: $(-\infty, \infty)$

Range: $[k, \infty)$ or $(-\infty, k]$ $y \geq k$

Examples:

1. Given $y = \frac{1}{8}(x-2)^2 - 5$, identify the following:

   a. Vertex: $(2, -5)$

   b. Focus: $(2, -3)$

   c. Directrix: $y = -7$

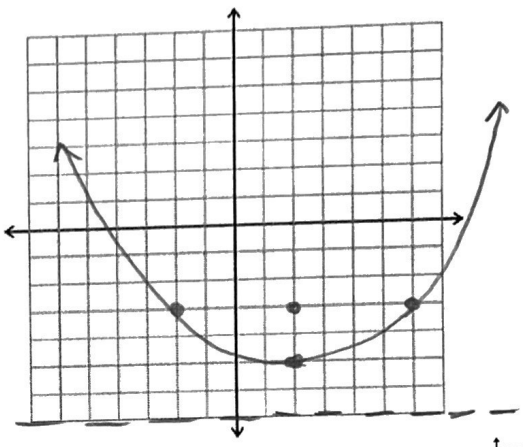
   d. Domain: $\mathbb{R}$

   e. Range: $y \geq -5$

   f. Graph the parabola.

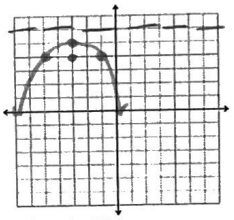
$8 = 4p$

$p = 2$



2. Write the equation of a parabola with a focus at $(-3, 4)$ and a directrix at $y = 6$.

$y = -\frac{1}{4}(x+3)^2 + 5$ $\quad p = 1$



3. Consider the function $(x-2)^2 = 4(y+1)$. Find the focus and directrix (Hint: what is "p"?)

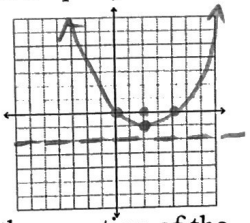
$4 = 4p$

$\boxed{p = 1}$

vertex: $(2, -1)$

Focus: $(2, 0)$

Directrix: $y = -2$



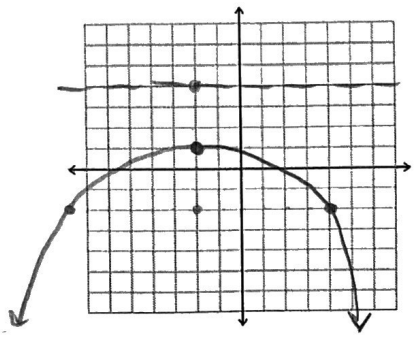
4. Write the parabola in standard form, then graph. State the vertex, focal length, and the equation of the directrix. $x^2 + 4x + 12y = +8$

$x^2 + 4x + \underline{4} + 12y = 8 + \underline{4}$

$(x+2)^2 + 12y = 12$

$12y = -(x+2)^2 + 12$

$y = -\frac{1}{12}(x+2)^2 + 1$

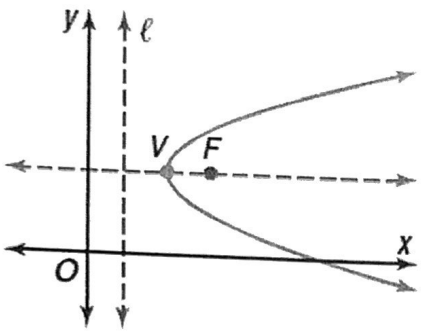


Vertex: $(-2, 1)$

Focal Length (p): 3

Directrix: $y = 4$

Horizontal Parabolas $x = \frac{1}{4p}(y-k)^2 + h$ OR $(y-k)^2 = 4p(x-h)$



Vertex: (h, k)

Focus: $(h+p, k)$

Axis of Symmetry: $y = k$

Directrix: $x = h - p$

Opens Right: $p > 0$

Opens Left: $p < 0$

Domain: $[h, \infty)$ or $(-\infty, h]$

Range: $(-\infty, \infty)$

Examples:

5. Given $x = \frac{1}{12}(y-3)^2 - 1$, identify the following:

   a. Vertex: $(-1, 3)$ $\quad 12 = 4p$

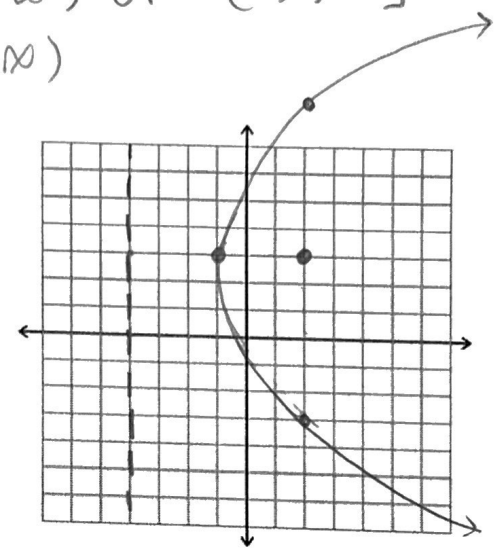
   b. Focus: $(2, 3)$ $\quad p = 3$

   c. Directrix: $x = -4$

   d. Domain: $[-1, \infty)$ or $x \geq -1$

   e. Range: $(-\infty, \infty)$ or $\mathbb{R}$

   f. Graph



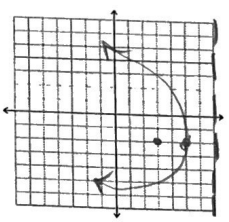
6. Write the equation of a parabola with a focus at $(3, -2)$ and a directrix at $x = 7$.

$p = 2$

$V: (5, -2)$

$x = \frac{1}{4p}(y-k)^2 + h$

$x = \frac{1}{8}(y+2)^2 + 5$



7. Write the equation in standard form and find the vertex, focus, and directrix: $y^2 - 8y + 20x - 24 = 0$.

$y^2 - 8y + \underline{16} + 20x - 24 = 0 + \underline{16}$

$\frac{-8}{2} = (-4)^2$

$(y-4)^2 + 20x - 24 = 16$

$\frac{20x}{20} = \frac{40}{20} - \frac{(y-4)^2}{20}$

$x = -\frac{1}{20}(y-4)^2 + 2$

8. Which of the following parabolas is not equivalent to the rest?

   a. $x = \frac{1}{2}(y-3)^2 + 4$

   b. $y^2 - 6y - 2x + 17 = 0$

   c. $2x + 4 = (y-3)^2$

   d. $2x - 8 = (y-3)^2$