**CONSTRUCT ARGUMENTS** Determine whether each quadrilateral is a parallelogram. Justify your answer.
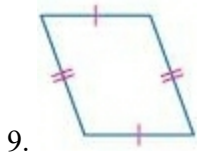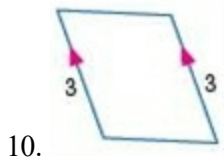


9.

*SOLUTION:*

The quadrilateral is a parallelogram because both pairs of opposite sides are congruent.

*ANSWER:*

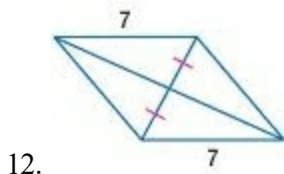Yes; both pairs of opp. sides are ≅.



10.

*SOLUTION:*

The quadrilateral is a parallelogram because one pair of opposite sides are parallel and congruent. By Theorem 6.12, this quadrilateral is a parallelogram.

*ANSWER:*

Yes; one pair of opp. sides is ∥ and ≅.



12.

*SOLUTION:*

The quadrilateral is not a parallelogram because none of the tests for parallelograms are fulfilled.

*ANSWER:*

No; none of the tests for ▱ are fulfilled.



13.

*SOLUTION:*

The quadrilateral is a parallelogram since the diagonals bisect each other.

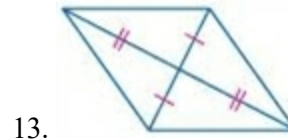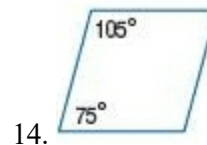*ANSWER:*
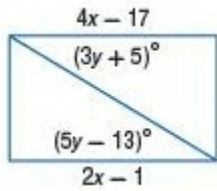
Yes; the diagonals bisect each other.



14.

*SOLUTION:*

The quadrilateral is not a parallelogram since none of the tests for parallelograms are fulfilled.

*ANSWER:*

No; none of the tests for ▱ are fulfilled.

**ALGEBRA** Find $x$ and $y$ so that the quadrilateral is a parallelogram.

19.



*SOLUTION:*

Opposite sides of a parallelogram are congruent.

Solve for $x$.

$4x - 17 = 2x - 1$

$2x - 17 = -1$

$2x = 16$

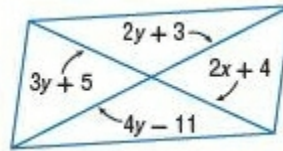$x = 8$

Solve for $y$.

$5y - 13 = 3y + 5$

$2y - 13 = 5$

$2y = 18$

$y = 9$

*ANSWER:*

$x = 8,\ y = 9$

**ALGEBRA** Find $x$ and $y$ so that the quadrilateral is a parallelogram.

21.



*SOLUTION:*

Diagonals of a parallelogram bisect each other.

So, $3y + 5 = 2x + 4$ and $4y - 11 = 2y + 3$.

Solve for $y$.

$4y - 11 = 2y + 3$

$2y = 14$

$y = 7$

Substitute $y = 7$ in $3y + 5 = 2x + 4$.

$3(7) + 5 = 2x + 4$

$21 + 5 = 2x + 4$

$26 = 2x + 4$

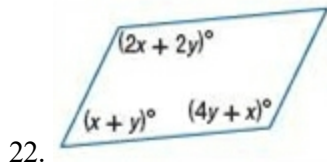$-2x = -22$

$x = 11$

*ANSWER:*

$x = 11,\ y = 7$

22.



*SOLUTION:*

Opposite angles of a parallelogram are congruent.
So, $2x + 2y = 4y + x$. We know that consecutive angles in a parallelogram are supplementary.
So, $x + y + 4y + x = 180$.
Solve for $x$.

$$2x + 2y = 4y + x$$
$$x + 2y = 4y$$
$$x = 2y$$

Substitute $x = 2y$ in $x + y + 4y + x = 180$.

$$x + y + 4y + x = 180$$
$$2y + y + 4y + 2y = 180$$
$$9y = 180$$
$$y = 20$$

So, $x = 2(20)$ or 40.

*ANSWER:*

$x = 40, y = 20$

**COORDINATE GEOMETRY  Graph each quadrilateral with the given vertices. Determine whether the figure is a parallelogram. Justify your answer with the method indicated.**

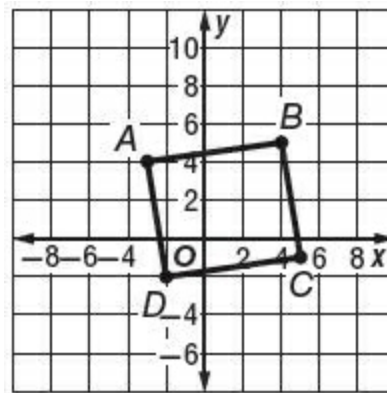24. $A(-3, 4)$, $B(4, 5)$, $C(5, -1)$, $D(-2, -2)$; Slope Formula

*SOLUTION:*

$$\text{Slope of } \overline{AB} = \frac{5-4}{4+3}$$
$$= \frac{1}{7}$$

$$\text{Slope of } \overline{BC} = \frac{-1-5}{5-4}$$
$$= -6$$

$$\text{Slope of } \overline{CD} = \frac{-2-(-1)}{-2-5}$$
$$= \frac{1}{7}$$

$$\text{Slope of } \overline{AD} = \frac{-2-4}{-2-(-3)}$$
$$= -6$$

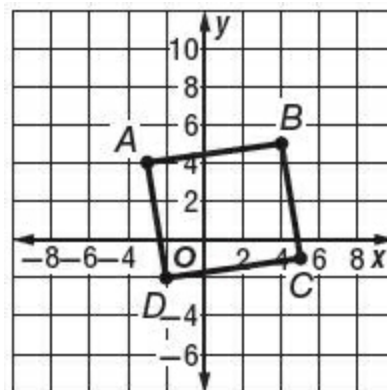Since both pairs of opposite sides are parallel, *ABCD* is a parallelogram.



*ANSWER:*

Yes; slope of $\overline{AB} = \dfrac{1}{7} = $ slope of $\overline{CD}$. So, $\overline{AB} \parallel \overline{CD}$.
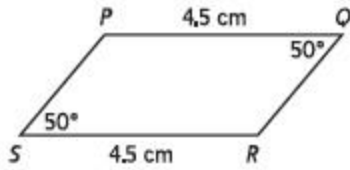
Slope of $\overline{BC} = -6 = $ slope of $\overline{AD}$. So, $\overline{BC} \parallel \overline{AD}$.
Because both pairs of opposite sides are parallel, *ABCD* is a parallelogram.

**46.** Amir is using software to design a geometric pattern for a quilt. He starts by making quadrilateral *PQRS*, as shown. He wants to know if the quadrilateral he made is a parallelogram.



Which of the following best describes quadrilateral *PQRS*?

**A** It is a parallelogram, because a pair of opposite sides are congruent.

**B** It is a parallelogram, because a pair of opposite angles are congruent.

**C** It is a parallelogram, because a pair of opposite sides are both parallel and congruent.

**D** There is not enough information to conclude that the quadrilateral is a parallelogram.

*SOLUTION:*

Choice A is not correct because it states that a pair of opposite sides are congruent whereas Theorem 6.9 indicates that BOTH pairs of opposite sides need to be congruent.

Choice B is not correct because it states that a pair of opposite angles are congruent whereas Theorem 6.10 indicates that BOTH pairs of opposite angles need to be congruent.

Choice C is not correct because it states that a pair of opposite sides are parallel and congruent. Although there is a pair of congruent opposite sides, there is not enough information available to prove that this same pair of opposite sides is also parallel.

Therefore, Choice D is the correct answer.

*ANSWER:*

D

**48.** In quadrilateral *WXYZ*, $WX = 4x - 15$, $XY = 4x + 20$, $YZ = 3x + 5$, and $ZW = 6x - 20$. For what value of $x$ is quadrilateral *WXYZ* a parallelogram?

*SOLUTION:*

Set $WX = YZ$ and solve for $x$. Then, do the same for $XY$ and $ZW$.

$$
\begin{aligned}
WX &= ZY \\
4x - 15 &= 3x + 5 \\
x - 15 &= 5 \\
x &= 20 \\
XY &= ZW \\
4x + 20 &= 6x - 20 \\
-2x &= -40 \\
x &= 20
\end{aligned}
$$

If $x = 20$, then both pairs of opposite sides are congruent and *WXYZ* is a parallelogram. The correct answer is 20.

*ANSWER:*

20