6-1 Angles of Polygons

Find the sum of the measures of the interior angles of each convex polygon.

12. dodecagon

SOLUTION:

A dodecagon has twelve sides. Use the Polygon Interior Angles Sum Theorem to find the sum of its interior angle measures.

Substitute $n = 12$ in $(n-2)180$.

$$\begin{aligned}(n-2)180 &= (12-2)180\\ &= 10 \cdot 180\\ &= 1800\end{aligned}$$

ANSWER:

1800

14. 29-gon

SOLUTION:

A 29-gon has twenty nine sides. Use the Polygon Interior Angles Sum Theorem to find the sum of its interior angle measures.

Substitute $n = 29$ in $(n-2)180$.

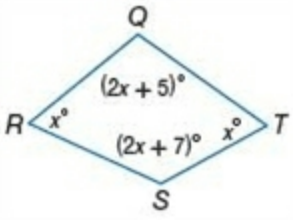
$$\begin{aligned}(n-2)180 &= (29-2)180\\ &= 27 \cdot 180\\ &= 4860\end{aligned}$$

ANSWER:

4860

Find the measure of each interior angle.

16.



SOLUTION:

The sum of the interior angle measures is $(4-2)180$ or 360.

$$\begin{aligned}m\angle Q + m\angle R + m\angle S + m\angle T &= 360\\ (2x+5) + x + (2x+7) + x &= 360\\ 6x + 12 &= 360\\ 6x &= 348\\ x &= 58\end{aligned}$$

Use the value of x to find the measure of each angle.

$$\begin{aligned}m\angle Q &= 2x+5\\ &= 2(58)+5\\ &= 116+5\\ &= 121\end{aligned}$$

$$\begin{aligned}m\angle R &= x\\ &= 58\end{aligned}$$

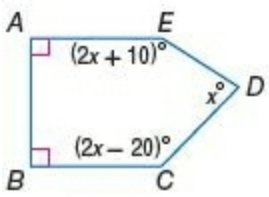
$$\begin{aligned}m\angle S &= 2x+7\\ &= 2(58)+7\\ &= 116+7\\ &= 123\end{aligned}$$

$$\begin{aligned}m\angle T &= x\\ &= 58\end{aligned}$$

ANSWER:

$m\angle Q = 121, m\angle R = 58, m\angle S = 123, m\angle T = 58$

6-1 Angles of Polygons

18. 

SOLUTION:

The sum of the interior angle measures is $(5-2)180$ or 540.

$$m\angle A + m\angle B + m\angle C + m\angle D + m\angle E = 540$$
$$90 + 90 + (2x - 20) + x + (2x + 10) = 540$$
$$5x + 170 = 540$$
$$5x = 370$$
$$x = 74$$

Use the value of x to find the measure of each angle.

$$m\angle A = 90$$
$$m\angle B = 90$$
$$m\angle C = 2x - 20$$
$$= 2(74) - 20$$
$$= 148 - 20$$
$$= 128$$
$$m\angle D = x$$
$$= 74$$
$$m\angle E = 2x + 10$$
$$= 2(74) + 10$$
$$= 148 + 10$$
$$= 158$$

ANSWER:

$m\angle A = 90, m\angle B = 90, m\angle C = 128, m\angle D = 74, m\angle E = 158$

20. **BASEBALL** In baseball, home plate is a pentagon. The dimensions of home plate are shown. What is the sum of the measures of the interior angles of home plate?

SOLUTION:

A pentagon has five sides. Use the Polygon Interior Angles Sum Theorem to find the sum of its interior angle measures.

Substitute $n = 5$ in $(n-2)180$.

$$(n-2)180 = (5-2)180$$
$$= 3 \cdot 180$$
$$= 540$$

ANSWER:

540

Find the measure of each interior angle of each regular polygon.

22. pentagon

SOLUTION:

Let n be the number of sides in the polygon and x be the measure of each interior angle of a regular polygon with 5 sides. Since all angles of a regular pentagon are congruent, the sum of the interior angle measures is $5x$. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$$5x = (n-2)180$$
$$5x = (5-2)180$$
$$5x = (3)180$$
$$5x = 540$$
$$x = 108$$

The measure of each interior angle of a regular pentagon is 108.

ANSWER:

108

24. nonagon

SOLUTION:

Let n be the number of sides in the polygon and x be the measure of each interior angle of a regular polygon with 9 sides. Since all angles of a regular nonagon are congruent, the sum of the interior angle measures is $9x$. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$$9x = (9-2)180$$
$$9x = (9-2)180$$
$$9x = (7)180$$
$$9x = 1260$$
$$x = 140$$

The measure of each interior angle of a regular nonagon is 140.

ANSWER:

140

The measure of an interior angle of a regular polygon is given. Find the number of sides in the polygon.

26. 60

SOLUTION:

Let n be the number of sides. Since all angles of a regular polygon are congruent, the sum of the interior angle measures is $60n$. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

$$60n = (n-2)180$$
$$60n = 180n - 360$$
$$-120n = -360$$
$$n = 3$$

ANSWER:

3

28. 120

SOLUTION:

Let n be the number of sides. Since all angles of a regular polygon are congruent, the sum of the interior angle measures is $120n$. By the Polygon Interior Angles Sum Theorem, the sum of the interior angle measures can also be expressed as $(n-2)180$.

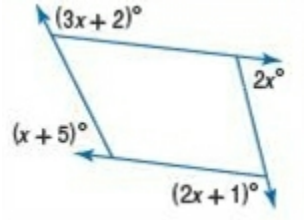
$$120n = (n-2)180$$
$$120n = 180n - 360$$
$$-60n = -360$$
$$n = 6$$

ANSWER:

6

6-1 Angles of Polygons

Find the value of x in each diagram.

32. 

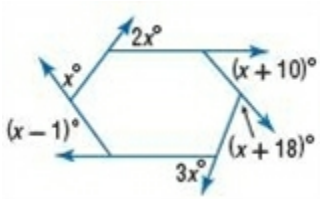
SOLUTION:

Use the Polygon Exterior Angles Sum Theorem to write an equation. Then solve for x.

$$(3x+2)+2x+(2x+1)+(x+5)=360$$
$$3x+2+2x+2x+1+x+5=360$$
$$8x+8=360$$
$$8x=352$$
$$x=44$$

ANSWER:

44

33. 

SOLUTION:

Use the Polygon Exterior Angles Sum Theorem to write an equation. Then solve for x.

$$x+2x+(x+10)+(x+18)+3x+(x-1)=360$$
$$x+2x+x+10+x+18+3x+x-1=360$$
$$9x+27=360$$
$$9x=333$$
$$x=37$$

ANSWER:

37

Find the measure of each exterior angle of each regular polygon.

34. decagon

SOLUTION:

A regular decagon has 10 congruent sides and 10 congruent interior angles. The exterior angles are also congruent, since angles supplementary to congruent angles are congruent. Let n be the measure of each exterior angle.

Use the Polygon Exterior Angles Sum Theorem to write an equation.

$10n = 360$

Solve for n.

$n = 36$

The measure of each exterior angle of a regular decagon is 36.

ANSWER:

36

35. pentagon

SOLUTION:

A regular pentagon has 5 congruent sides and 5 congruent interior angles. The exterior angles are also congruent, since angles supplementary to congruent angles are congruent. Let n be the measure of each exterior angle.

Use the Polygon Exterior Angles Sum Theorem to write an equation.

$5n = 360$

Solve for n.

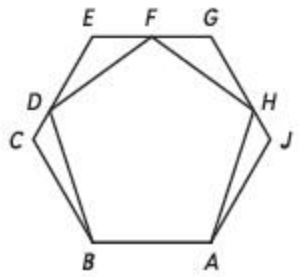
$n = 72$

The measure of each exterior angle of a regular pentagon is 72.

ANSWER:

72

55. Polygons *ABCEGJ* and *ABDFH* are both regular polygons.



What is the measure of angle *CBD*?

A 12
B 18
C 108
D 120

SOLUTION:
First, find the interior angle of the hexagon *ABCEGJ*.

$$\begin{aligned}\frac{180(n-2)}{n} &= \frac{180(6-2)}{6}\\ &= \frac{180(4)}{6}\\ &= \frac{720}{6}\\ &= 120\end{aligned}$$

Each interior angle of the regular hexagon is 120°.

Second, find the interior angle of the pentagon *ABDFH*.

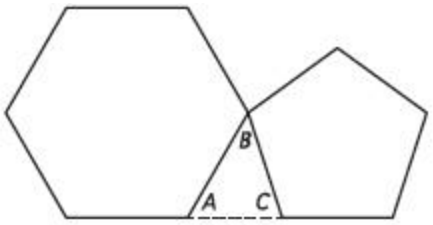
$$\begin{aligned}\frac{180(n-2)}{n} &= \frac{180(5-2)}{5}\\ &= \frac{180(5)}{5}\\ &= \frac{900}{5}\\ &= 108\end{aligned}$$

Each interior angle of the regular pentagon is 108°.

The measure of $\angle CBD$ is the difference between the measure of an interior angle of the hexagon and the pentagon. Therefore, the measure of $\angle CBD$ is 120 – 108 = 12. So, the correct answer is choice A.

ANSWER:
A

57. The figure below shows a regular hexagon and a regular pentagon.



Find the measure of angle *ABC*.

A 12
B 48
C 84
D 132

SOLUTION:
Use the Polygon Exterior Angles Sum Theorem to find the measures of angles *A* and *C*.

$$\begin{aligned}6n &= 360\\ &= 60\end{aligned}$$

The measure of each exterior angle *A* of the hexagon is 60°.

$$\begin{aligned}5n &= 360\\ &= 72\end{aligned}$$

The measure of each exterior angle *C* of the pentagon is 72°.

Use the Triangle Angles Sum Theorem to find the measure of the missing angle.

$$\begin{aligned}m\angle A + m\angle B + m\angle C &= 180\\ 60 + m\angle B + 72 &= 180\\ m\angle B + 132 &= 180\\ m\angle B &= 48\end{aligned}$$

The measure of angle *ABC* is 48. So, the correct answer is choice B.

ANSWER:
B