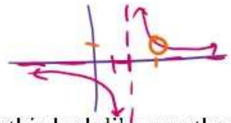
6.3 Notes: Graphing Rational Functions with Holes

Exploration: Consider the function $g(x) = \frac{x-3}{x^2-5x+6} = \frac{\cancel{x-3}}{(\cancel{x-3})(x-2)} = \frac{1}{x-2}$

Based on what you know so far about rational functions, what do you anticipate $g(x)$ would look like? Consider asymptotes as part of your decision.

VA: $x = 2$

HA: $y = 0$



Use a graphing calculator to sketch $g(x)$. Does this look like you thought it would? What is surprising about this graph?

yes

What's happening @ $x = ?$

Find $g(3)$ by evaluating $g(x)$ at 3. What do you notice? However, what does $g(3)$ appear to be on the graph? What is happening?

$g(3) = \frac{3-3}{9-15+6} = \frac{0}{0} = ?$

$(3, ?)$ ← hole in the graph

Equivalent Expressions: Some rational expressions can be written as simpler equivalent expressions. This happens when a factor is repeated on the numerator and the denominator. Consider $f(x) = \frac{x^2-16}{x+4}$.
Factor fully, and *reduce out* any factors that repeat on the numerator and denominator.

$f(x) = \frac{(\cancel{x+4})(x-4)}{(\cancel{x+4})} = \underbrace{x-4}_{g(x)}$

at $x = -4$

An expression equivalent to $f(x) = \frac{x^2-16}{x+4}$ is $g(x) =$ $x-4$. Compare the graph of each by using your graphing calculator.

*Find $f(4)$ and $g(4)$. What do you notice?

$f(-4) = \frac{(-4)^2-16}{-4+4} = \frac{0}{0} \to ?$

$(-4, ?)$ hole

$g(-4) = -4-4 = -8$

$(-4, -8)$

Holes of a rational function: A single point that is undefined for the graph.

- ✓ Plot as an Open circle.
- ✓ Is a repeated factor on the numerator and denominator.
- ✓ Write as an ordered pair.

Examples: Find the *coordinates* of any holes in the rational functions below.

1) $y = \frac{x^2-4x}{x^2-6x+8} = \frac{x\cancel{(x-4)}}{\cancel{(x-4)}(x-2)} = \frac{x}{x-2}$

hole VA

$\boxed{(4, 2)}$

x	y
4	$\frac{4}{2}=2$

must use the reduced equiv. expression

2) $y = \frac{x^2-3x}{5x-2x^2} = \frac{\cancel{x}(x-3)}{\cancel{x}(5-2x)} = \frac{x-3}{5-2x}$

hole VA

$\boxed{(0, \frac{-3}{5})}$

x	y
0	$\frac{-3}{5}$

3) $y = \frac{x+1}{x^2+7x+6} = \frac{\cancel{(x+1)}}{\cancel{(x+1)}(x+6)} = \frac{1}{x+6}$

hole VA

$\boxed{(-1, \frac{1}{5})}$

x	y
-1	$\frac{1}{5}$

Graphing Rational Functions with Holes

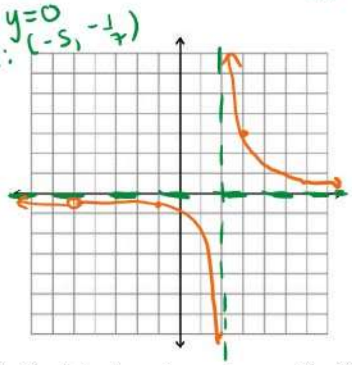
1. factor
2. Plot HA & VA first
3. used reduced equiv. express find the hole
4. test points

Examples: Graph each rational function. Include any asymptotes and holes.

4) $y = \frac{x+5}{x^2+3x-10} = \frac{\cancel{x+5}}{\cancel{(x+5)}(x-2)} = \frac{1}{x-2}$

hole VA

VA: $x=2$
HA: $y=0$
hole: $(-5, -\frac{1}{7})$



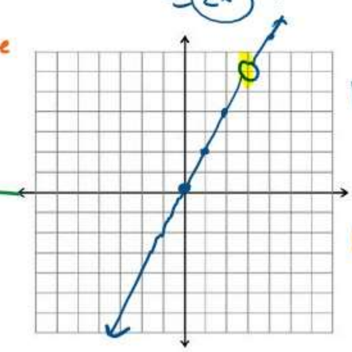
x	y
-5	$\frac{1}{-7}$ hole
-1	$\frac{1}{-3}$
3	$\frac{3}{1}$

5) $y = \frac{2x^2-6x}{x-3} = \frac{2x\cancel{(x-3)}}{\cancel{x-3}}$

hole

$= 2x$

no VA
no HA



	x	y
hole	3	$2(3)=6$

D R
(3, 6)

$y = 2x + 0$
$y = mx + b$

6) Find the domain and range for #5 above.

D: $\{x \mid x \neq 3\}$
R: $\{y \mid y \neq 6\}$

Domain: All defined values of the input (x) of a function. The domain is restricted at the following values:

Domain: All defined values of the input (x) of a function. The domain is restricted at the following values: VA + x-coord of any holes

Range: All defined values of the output (y) of a function. The range is restricted at the following values:
HA + y-coord of any holes

Example 7: Graph the given rational function. Include all asymptotes, holes, and intercepts. Find the domain, range, and end behavior.

$$y=\frac{x^2+9x+20}{x^2+7x+12}=\frac{\overbrace{(x+5)}^{x\text{-int}}\cancel{\overbrace{(x+4)}^{\text{hole}}}}{\cancel{(x+4)}(x+3)}=\frac{x+5}{x+3}$$

HA @ $y=1$

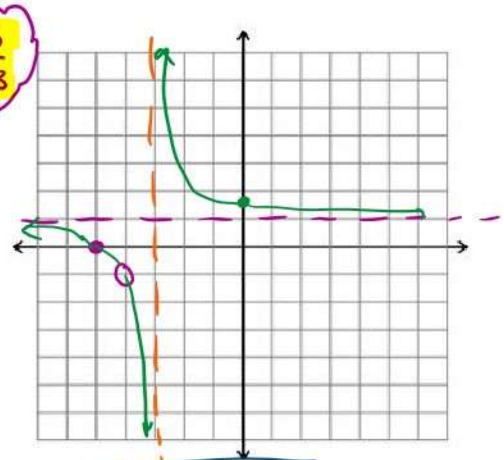
hole $(-4,-1)$

VA @ $x=-3$

x-int $(-5,0)$

x	y
-4	$\frac{1}{-1}=-1$

y-int $(0,\frac{5}{3})$ $\quad \frac{0+5}{0+3}\rightarrow$



D: $\{x \mid x \neq -3,-4\}$

R: $\{y \mid y \neq 1,-1\}$

as $x\to\infty$, $y\to 1$

as $x\to-\infty$, $y\to 1$

Summary for Graphing Rational Functions that are not in graphing form:

1. Factor fully.
2. Factors that reduce out identify the x-coordinate of any holes in the graph. Find the y value(s) by evaluating the reduced expression at the given x-value. Holes *must* be written as ordered pairs.
3. Factors of the denominator that are *not* repeated on the numerator give the values of vertical asymptotes. Write as x = constant.
4. Compare the degree of the numerator and denominator to identify any horizontal asymptotes. Write as y = constant.
5. Factors of the numerator that are not reduced out are the values of x-intercepts.
6. To find the y-intercept, evaluate the expression at $x = 0$. As needed, use the reduced equivalent expression.
7. As needed, test points on either side of any vertical asymptotes in order to find points on the graph. Fit the curve to the asymptotes.

Alg 2H Unit 6 Part 2 Notes

Monday, October 4, 2021 2:19 PM

Alg 2H Unit 6 Part 2...

6.4: Simplifying, Multiplying, and Dividing Rational Expressions

Rational Function – A function of the form $f(x) = \frac{p(x)}{q(x)}$

Ex: $y = \frac{x-2}{x^2+5x-6}$ or $y = \frac{5}{x-10}$

Domains of fractions are undefined when the denominator = 0.

Find the domain:

1) $y = \frac{x-2}{3x}$ → $3x=0$ D: $\{x \mid x \neq 0\}$

2) $f(x) = \frac{7}{(x+2)(3x-5)}$ D: $\{x \mid x \neq -2, \frac{5}{3}\}$

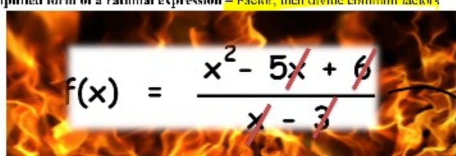
3) $g(x) = \frac{5x}{4x^3-9x} = \frac{5x}{x(4x^2-9)} = \frac{\cancel{5x}}{\cancel{x}(2x+3)(2x-3)} = \frac{5}{(2x+3)(2x-3)}$ D: $\{x \mid x \neq 0, -\frac{2}{3}, \frac{2}{3}\}$ VA

Reminder: What are these domain restrictions representing on the graph of the rational function?

VA @ $x=0$ VA @ $x=-2$; $x=\frac{5}{3}$ hole $(0, -\frac{5}{9})$ VA $x=-\frac{3}{2}, x=\frac{3}{2}$

Simplified form of a rational expression – Factor, then divide common factors

No!



WRONG!

$\frac{2 + 3}{3} = \frac{5}{3}$

$\frac{(x-2)\cancel{(x-3)}}{\cancel{(x-3)}} = x-2$

WRONG!

Examples: Simplify completely. Identify any restrictions on the domain.

1) $\frac{x^2+7x+10}{x^2-4} = \frac{(x+5)\cancel{(x+2)}}{\cancel{(x+2)}(x-2)} = \boxed{\frac{x+5}{x-2}}$ $\boxed{x \neq \pm 2}$ Restrictions

2) $\frac{x^2+5x+4}{x^2+x-12} = \frac{(x+4)(x+1)}{(x+4)(x-3)} = \boxed{\frac{x+1}{x-3}}$ $\boxed{x \neq -4, 3}$

Multiplying and Dividing Rational Expressions

1. Factor, if possible.
2. For division, multiply by the reciprocal of the fraction after the ÷ sign. Skip Flip multiply
3. Reduce repeated factors.

mult and any num w/ any den

$\frac{1\ 5}{2\ 8} \cdot \frac{6\ 2}{3\ 1} \cdot \frac{9}{25\ 5} \cdot \frac{4\ 1}{2\ 1} = \frac{9}{10}$

Examples: Simplify completely. Identify any restrictions on the domain.

1) $\frac{5}{4x^2} \cdot \frac{2x^3}{18} \cdot \frac{6}{15x^4} = \frac{1x^3}{12x^6}$ $\boxed{\frac{1}{12x^3}}$ $x \neq 0$

2) $\frac{x^2+4x-12}{x^2+11x+30} \cdot \frac{x^2-2x-35}{x+4}$

$\frac{\cancel{(x+6)}(x-2)}{\cancel{(x+6)}\cancel{(x+5)}} \cdot \frac{(x-7)\cancel{(x+5)}}{(x+4)} = \boxed{\frac{(x-2)(x-7)}{x+4}}$ better

$\boxed{x \neq -6, -5, -4}$

3) $\frac{21a^3}{12} \div \frac{14a}{16}$

$\frac{21a^3}{12} \cdot \frac{16}{14a} = \frac{2a^3}{a}$

$\frac{2aa\cancel{a}}{\cancel{a}} = \boxed{2a^2}$

$\boxed{a \neq 0}$

4) You Try! $\frac{x^2-25}{x^2+2x-3} \div \frac{x+5}{x^2-3x-18}$

$\frac{\cancel{(x+5)}(x-5)}{(x+3)(x-1)} \cdot \frac{(x-6)\cancel{(x+3)}}{\cancel{(x+5)}}$ before after

$= \boxed{\frac{(x-5)(x-6)}{x-1}}$

$\boxed{x \neq -3, 1, 6, -5}$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

5) $\frac{-2x^2}{x^3-27} \cdot (x^2+3x+9)$

$x^3 - 3^3$

$\frac{-2x^2}{(x-3)\cancel{(x^2+3x+9)}} \cdot \frac{\cancel{(x^2+3x+9)}}{1}$

no solution

$\boxed{\frac{-2x^2}{x-3}}$

$\boxed{x \neq 3}$

6) $\frac{3x^2+1x-2}{x^3+1x} \div \frac{9x^2-4}{5x}$

$x(x^2+1)$

$\frac{\cancel{(3x-2)}\cancel{(x+1)}}{\cancel{x}(x+1)(x^2+x+1)} \cdot \frac{5\cancel{x}}{(3x+2)\cancel{(3x-2)}}$

no solution

$= \boxed{\frac{5}{(x^2+x+1)(3x+2)}}$

$\boxed{x \neq 0, -1, \pm\frac{2}{3}}$

$$\boxed{\frac{-2x^2}{x-3}}$$

$\boxed{x \neq 3}$

$$= \boxed{\frac{5}{(x^2+x+1)(3x+2)}}$$

$\boxed{x \neq 0, -1, \pm\frac{2}{3}}$

Simplifying Complex Fractions

Examples: Simplify each complex fraction.

7) $$\frac{\frac{b^2-4}{b^2-2b+1}}{\frac{b+2}{b-1}}$$

$$\frac{b^2-4}{b^2-2b+1} \div \frac{b+2}{b-1}$$

$$\frac{\cancel{(b+2)}(b-2)}{(b-1)^{\cancel{2}}} \cdot \frac{\cancel{b-1}}{\cancel{b+2}}$$

$$\boxed{\frac{b-2}{b-1}}$$

$\boxed{b \neq 1, -2}$

8) $$\frac{x-5}{2x^2-5x-3} \div \left(\frac{x^2-4x-5}{x^2-9} \cdot \frac{x^2+2x+1}{x^2+5x+6}\right)$$

$$\frac{x-5}{(2x+1)(x-3)} \div \left(\frac{(x-5)\cancel{(x+1)}}{\cancel{(x+3)}(x-3)} \cdot \frac{(x+2)\cancel{(x+3)}}{\cancel{(x+1)}(x+1)}\right)$$

$$\frac{x-5}{(2x+1)(x-3)} \div \frac{(x-5)(x+2)}{(x-3)(x+1)}$$

$$\frac{\cancel{(x-5)}}{(2x+1)\cancel{(x-3)}} \cdot \frac{\cancel{(x-3)}(x+1)}{\cancel{(x-5)}(x+2)} = \boxed{\frac{x+1}{(2x+1)(x+2)}}$$

$\boxed{x \neq -\frac{1}{2}, \pm 3, -2, -1, 5}$

HW: 6.4 Practice

6.5: Adding and Subtracting Rational Expressions

To add or subtract rational expressions
1. Get a common denominator (factor denominator if possible). Keep!
2. Add or subtract.
3. Factor if possible.
4. Reduce (holes).

Examples: Simplify fully. Identify any restrictions on the domain.

1) $$\frac{9}{x+7} + \frac{2}{x+7} = \frac{11}{x+7}$$

$\boxed{x \neq -7}$

2) $$\frac{7x}{x-4} - \frac{4x+12}{x-4} = \frac{7x-4x-12}{x-4} = \frac{3x-12}{x-4}$$

$$= \frac{3\cancel{(x-4)}}{\cancel{(x-4)}} = \boxed{3}$$

$\boxed{x \neq 4}$

3) $$\frac{x}{x+6} + \frac{-72}{x^2-36}$$

$$\frac{x}{x+6} \cdot \frac{(x-6)}{(x-6)} + \frac{-72}{(x+6)(x-6)}$$

$$\frac{x^2-6x}{(x+6)(x-6)} + \frac{-72}{(x+6)(x-6)}$$

$$\frac{x^2-6x-72}{(x+6)(x-6)}$$

$$\frac{(x-12)\cancel{(x+6)}}{\cancel{(x+6)}(x-6)}$$

$$\boxed{\frac{x-12}{x-6}}$$

$\boxed{x \neq \pm 6}$

4) $$\frac{x}{3x-3} - \frac{x}{x^2-1}$$

CD: $3(x-1)(x+1)$

$$\frac{x(x+1)}{3(x-1)(x+1)} - \frac{3x}{3(x+1)(x-1)}$$

$$\frac{x^2+x-3x}{3(x-1)(x+1)} = \boxed{\frac{x^2-2x}{3(x-1)(x+1)}}$$

$$= \boxed{\frac{x(x-2)}{3(x-1)(x+1)}}$$

$x \neq \pm 1$

$$\frac{5 \cdot 7}{3 \cdot 7} + \frac{2 \cdot 3}{7 \cdot 3}$$

$$\frac{35}{21} + \frac{6}{21} = \frac{41}{21}$$

5) You Try! $\frac{5x}{x^2-2x-15} - \frac{3}{x^2+4x+3}$ CD: $(x-5)(x+3)(x+1)$

$\frac{5x\,(x+1)}{(x-5)(x+3)(x+1)} - \frac{3\,(x-5)}{(x+3)(x+1)(x-5)}$

$\frac{5x^2+5x-3x+15}{(x-5)(x+3)(x+1)} = \boxed{\frac{5x^2+2x+15}{(x-5)(x+3)(x+1)}}$

$\boxed{x \neq 5, -3, -1}$

6) $\dfrac{\frac{2x}{x+1}}{\frac{1}{3x} + \frac{2}{x+1}} = \dfrac{\frac{2x}{x+1}}{\frac{x+1+6x}{3x(x+1)}}$

Complex fraction
① simp num to one fract
② simp denom to one fract
③ Divide (mult by reciprocal)

$\frac{2x}{(\cancel{x+1})} \cdot \frac{3x(\cancel{x+1})}{7x+1} = \boxed{\frac{6x^2}{7x+1}}$

$\boxed{x \neq -1, 0, -\frac{1}{7}}$

7) $\dfrac{2}{3+\frac{5}{a}} - \dfrac{2}{3-\frac{5}{a}}$

$\frac{2}{\left(\frac{3a+5}{a}\right)} - \frac{2}{\left(\frac{3a-5}{a}\right)}$ $\boxed{a \neq 0, \pm\frac{5}{3}}$

$\frac{2}{1} \cdot \frac{a}{3a+5} - \frac{2}{1} \cdot \frac{a}{3a-5}$

$\frac{2a\,(3a-5)}{(3a+5)(3a-5)} - \frac{2a\,(3a+5)}{(3a-5)(3a+5)}$

$\frac{6a^2-10a-6a^2-10a}{(3a+5)(3a-5)} = \boxed{\frac{-20a}{(3a+5)(3a-5)}}$

8) $\dfrac{x^{-1}+y^{-1}}{1-x^{-1}}$ $\dfrac{\frac{1\cdot y}{x\cdot y} + \frac{1\cdot x}{y\cdot x}}{\frac{1\cdot x}{1\cdot x} - \frac{1}{x}} = \dfrac{\frac{x+y}{xy}}{\frac{x-1}{x}}$

$\frac{(x+y)}{\cancel{x}y} \cdot \frac{\cancel{x}}{(x-1)} = \boxed{\frac{x+y}{y(x-1)}}$

$\boxed{x \neq 0, 1 \quad y \neq 0}$

HW: 6.5 Practice

6.6: Solving Rational Equations

Steps for Solving Rational Equations:
① mult each term by common denom (clearing out the fractions) → factor
② Solve for x.
③ Check your answers → reject any answers that are domain restrictions! (domain → denom)

$\frac{x}{5} = \frac{3}{2}$ CD: 10

$\cancel{10}^{2} \cdot \frac{x}{5} = \frac{3}{2} \cdot \cancel{10}^{5}$

$2x = 15$

Cross-Multiplying: Why does this work? Solve example 1 in two ways (by cross-multiplying, and by clearing out the denominator.)

fract = fract

Examples: Solve each equation for the variable. Include the restrictions on the domain.

1) $\frac{x}{x+3} = \frac{6}{x-1}$ $\boxed{x \neq -3, 1}$

$x(x-1) = 6(x+3)$

$x^2 - x = 6x + 18$

$x^2 - 7x - 18 = 0$

$(x-9)(x+2) = 0$

$\boxed{x = 9, -2}$

CD: $(x+3)(x-1)$

$(\cancel{x+3})(x-1) \cdot \frac{x}{\cancel{x+3}} = \frac{6}{(\cancel{x-1})}(x+3)(\cancel{x-1})$ $\boxed{x \neq -3, 1}$

$x^2 - x = 6x + 18$

$x^2 - 7x - 18 = 0$

$(x-9)(x+2)$

$\boxed{x = 9, -2}$

2) $\frac{x}{x+2} - \frac{8}{x^2-4} = \frac{2}{x-2}$ CD: $(x+2)(x-2)$

$(\cancel{x+2})(x-2)\frac{x}{\cancel{x+2}} - \frac{8\,(\cancel{x+2})(\cancel{x-2})}{(\cancel{x+2})(\cancel{x-2})} = \frac{2\,(x+2)(\cancel{x-2})}{\cancel{x-2}}$

$x^2 - 2x - 8 = 2x + 4$

$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

$\boxed{x = 6}$ $\cancel{-2}$

$\boxed{x \neq \pm 2}$

3) **You Try!** $\dfrac{x}{x-3} - \dfrac{2x}{x+3} = \dfrac{18}{x^2-9}$

$\dfrac{x\,(x+3)\cancel{(x-3)}}{\cancel{(x-3)}} - \dfrac{2x\,\cancel{(x+3)}(x-3)}{\cancel{(x+3)}} = \dfrac{18\,\cancel{(x+3)(x-3)}}{\cancel{(x+3)(x-3)}}$

$1x^2 + 3x - 2x^2 + 6x = 18$ $\boxed{x \neq \pm 3}$

$-x^2 + 9x = 18$ (1, 18; 2, 9; 3, 6)

$0 = x^2 - 9x + 18$

$0 = (x-3)(x-6)$

$\cancel{x=3}$ $\boxed{x=6}$

4) $\dfrac{3x-5}{x+3} = \dfrac{2}{1} + \dfrac{8}{(x+3)(x-4)}$ CD: $(x+3)(x-4)$

$(x-4)(3x-5) = 2(x+3)(x-4) + 8$

$2(x^2-x-12)$

$3x^2 - 17x + 20 = 2x^2 - 2x \underbrace{-24+8}_{-16}$ (3, 12)

$x^2 - 15x + 36 = 0$

$(x-3)(x-12) = 0$

$\boxed{x=3 \mid x=12}$ $\boxed{x \neq -3, 4}$

5) $\dfrac{6x^2}{(x+4)(x-4)} - \dfrac{3x}{x+4} = \dfrac{4}{x-4}$

$6x^2 - 3x^2 + 12x = 4x + 16$ $\boxed{x \neq \pm 4}$

$3x^2 + 12x = 4x + 16$

$3x^2 + 8x - 16 = 0$

$(3x - 4)(x + 4) = 0$

$\boxed{x = \frac{4}{3}}$, $\cancel{-4}$

Review of Graphing Rational Functions

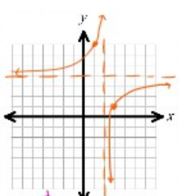
Graphing Form: $y = \frac{a}{x-h} + k$ VA $x=h$; HA $y=k$

6) Graph $y = \frac{-3}{x-2} + 4$

VA: $x=2$

HA: $y=4$

x	y
3	$\frac{-3}{1}+4=1$
1	$\frac{-3}{-1}+4=7$



7) Graph: $y = \frac{3x-9}{x+1}$, and find the requested information.

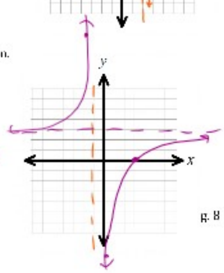
VA

$y = \frac{3(x-3)}{x+1}$

VA: $x=-1$

HA: $y=3$

x	y
-2	$\frac{3(-5)}{-1}=15$



Systems of Linear and Rational Equations

Examples: Solve the following systems by using algebra and by graphing. Then check using your graphing calculator.

$g(x) = f(x)$; find x

8) $f(x) = \frac{x+2}{x-1}$ *and* $g(x) = 2x$

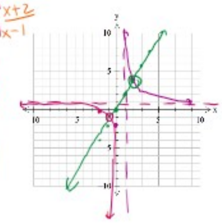
$f(x) = \frac{1x+2}{1x-1}$

$x \neq 1$

$\frac{x+2}{x-1} = \frac{2x}{1}$

$y = 2x + 0$

x	y
0	-2
2	$\frac{4}{1}$



$x + 2 = 2x(x-1)$

$x + 2 = 2x^2 - 2x$

$0 = 2x^2 - 3x - 2$

$0 = (2x + 1)(x - 2)$

$x = -\frac{1}{2}$ $\quad x = 2$

Given $h(x) = j(x)$

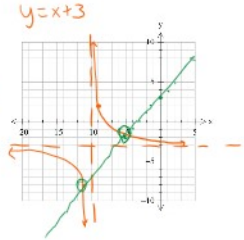
9) $h(x) = \frac{5}{x+10} - 3$ and $j(x) = x + 3$

$y = x + 3$

$\frac{5}{x+10} - 3 = x + 3$

x	y
-9	$\frac{5}{1} - 3$
-11	$\frac{5}{-1} - 3$
0	$\frac{1}{2} - 3$

$\frac{5}{x+10} = \frac{x+6}{1}$ $\quad x \neq -10$



$5 = (x+10)(x+6)$

$5 = x^2 + 16x + 60$ $\quad$ 1, 55; 5, 11

$0 = x^2 + 16x + 55$

$0 = (x+5)(x+11)$

$\boxed{x = -5, -11}$

not right

6.7 Notes: Slant (Oblique) Asymptotes

HA: none $\qquad \frac{\text{higher}}{\text{lower}}$

When the degree of the numerator is exactly one more than the degree of the denominator, the graph of the rational function will have a slant asymptote. *A rational function will never have more than one slant asymptote. It will also never have a horizontal asymptote and a slant asymptote at the same time.*

To find the equation of a slant asymptote, perform long division (synthetic if the denominator is a binomial of degree 1) by dividing the denominator into the numerator. As x gets very large (this is the far left or far right ends of the graph), the remainder portion becomes very small, almost zero. So, to find the equation of the slant asymptote, perform the division and discard the remainder.

$y = mx + b$

Example: Find the equation of the slant asymptotes for $y = \frac{x^2-6x+5}{x-4}$, if any. Since the degree of the numerator (2) is exactly one more than the degree of the denominator (1), a slant asymptote exists.

$\frac{(x-5)(x-1)}{x-4}$

VA @ $x = 4$

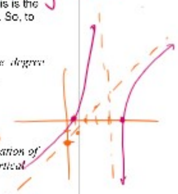
x-int @ (5,0) and (1,0)

$$\begin{array}{r|rrr} 4 & 1 & -6 & 5 \\ & \downarrow & 4 & -8 \\ \hline & 1x & -2 & \cancel{-3} \end{array}$$

mult ignore

$\boxed{y = x - 2}$

Disregarding the remainder, the quotient is $x - 2$, so the equation of the slant asymptote is $y = x - 2$. Note that there is also a vertical asymptote at $x = 4$.



Slant Asymptotes: Written in the form $y = mx + b$ (ignore remainder)

- ✓ There is a slant asymptote when the degree of the numerator is one degree higher than the denom degree
- ✓ Use synthetic division to find the equation (or long division)
  - Ignore the remainder
- ✓ There may be a slant asymptote when there is no HA.

Example 1: Find the value of any slant asymptote for $y = \frac{x^2-3x-4}{x+2}$. Then graph the function and its oblique asymptote. Find the listed key features of the graph.

$y = \frac{(x-4)(x+1)}{x+2}$

x	y
-3	$\frac{(-7)(-2)}{-1} = -14$

VA: $x = -2$

HA: none

Slant Asymptote: $y = x - 5$

D: $\{x \mid x \neq -2\}$

R: $\{y \mid y \leq -11.9$ and $y \geq -2.1\}$

End Behavior:

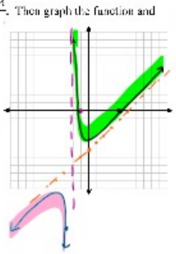
as $x \to \infty$, $y \to \infty$

as $x \to -\infty$, $y \to -\infty$

x-int: (4,0) (-1,0)

$$\begin{array}{r|rrr} -2 & 1 & -3 & -4 \\ & \downarrow & -2 & 10 \\ \hline & 1 & -5 & \cancel{6} \end{array}$$

$y = 1x - 5$



Your turn: **Example 2:** Find the value of any slant asymptote for $y = \frac{2x^2-5}{x-1}$. Then graph the function and its oblique asymptote.

$2x^2 + 0x - 5$

$$\begin{array}{r|rrr} 1 & 2 & 0 & -5 \\ & \downarrow & 2 & 2 \\ \hline & 2x + & 2 & \cancel{-3} \end{array}$$

VA: $x = 1$

HA: none

Slant Asymptote: → $y = 2x + 2$

D: $\{x \mid x \neq 1\}$

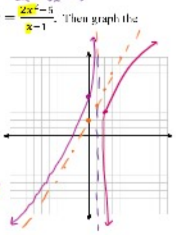
R: $\{y \mid y \neq 2x+2\}$ or $\mathbb{R}$

End Behavior:

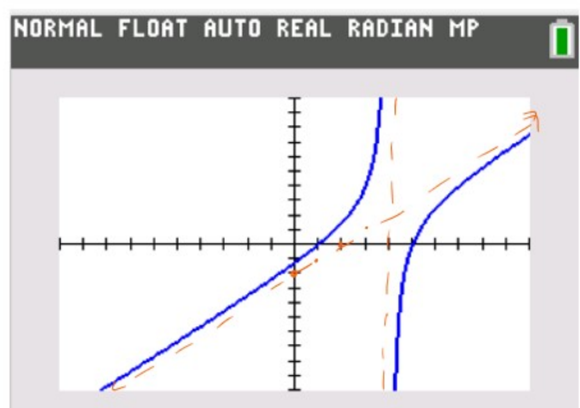
as $x \to \infty$, $y \to \infty$

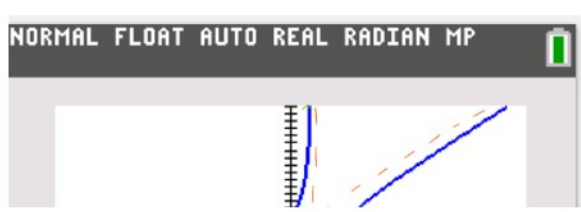
as $x \to \infty$, $y \to -\infty$

x	y
0	5
2	$\frac{2(2)^2-5}{2-1} = 3$



x-int (-5,0)





1=3

Example 3: Graph $g(x) = \dfrac{x^2+9x+20}{x^2+7x+12} = \dfrac{\cancel{(x+4)}(x+5)}{(x+3)\cancel{(x+4)}}$

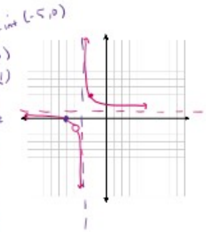
x-int (-5,0)

VA · hole

VA: $x = -3$
HA: $y = 1$
Slant Asymptote: none
Hole: $(-4, -1)$
D: $\{x \mid x \neq -3, -4\}$
R: $\{y \mid y \neq 1, -1\}$
End Behavior:
as $x \to \infty$, $g(x) \to 1$
as $x \to -\infty$, $g(x) \to 1$

$g(x) = \dfrac{x+5}{x+3}$

x	y
-4	$\frac{1}{-1} = -1$
-2	$\frac{3}{1}$
0	$\frac{5}{3}$

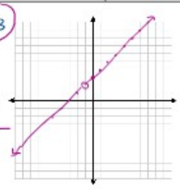


Example 4: Graph $y = \dfrac{x^2+4x+3}{x+1} = \dfrac{(x+3)\cancel{(x+1)}}{\cancel{x+1}} = x+3$

hole

VA: none
HA: none
Slant Asymptote: none
Hole: $(-1, 2)$
D: $\{x \mid x \neq -1\}$
R: $\{y \mid y \neq 2\}$
End Behavior:
as $x \to \infty$, $y \to \infty$
as $x \to -\infty$, $y \to -\infty$

x	y
-1	2



$y = x+3$

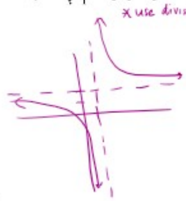
Example 5: What could you do to write the function, $y = \dfrac{2x+3}{x-1}$, in graphing form?

* use division (long or synthetic)

$$\begin{array}{r|rr} 1 & 2 & 3 \\ & \downarrow & 2 \\ \hline & 2 & 5 \end{array}$$

$2 + \dfrac{5}{x-1}$

$\boxed{y = \dfrac{5}{x-1} + 2}$



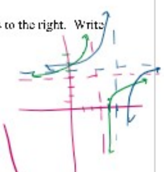
Example 6: Translate the graph of $f(x) = \dfrac{4x+9}{x+3}$ one unit up and two units to the right. Write the function in graphing form.

$$\begin{array}{r|rr} -3 & 4 & 9 \\ & \downarrow & -12 \\ \hline & 4 & -3 \end{array}$$

$4 + \dfrac{-3}{x+3}$

$y = \dfrac{-3}{x-3-2} + 4 + 1$

$\boxed{y = \dfrac{-3}{x-5} + 5}$



HW: 6.7 Practice