6NP1 filled out

Wednesday, September 29, 2021 9:30 AM

Alg 2H Unit 6 Notes

6.1 Notes: Graphing Rational Functions with Transformations

Rational Function: $y = \frac{a}{x-h} + k$ undef $= \frac{\#}{0}$ domain restriction

"fraction"

pass the vertical line test

$y = \frac{\text{polynomial}}{\text{polynomial}}$

Asymptote: • Horizontal Asymptotes (HA) • Vertical Asymptotes (VA) ← never be crossed!

lines that a function approaches but [usually] does not cross.

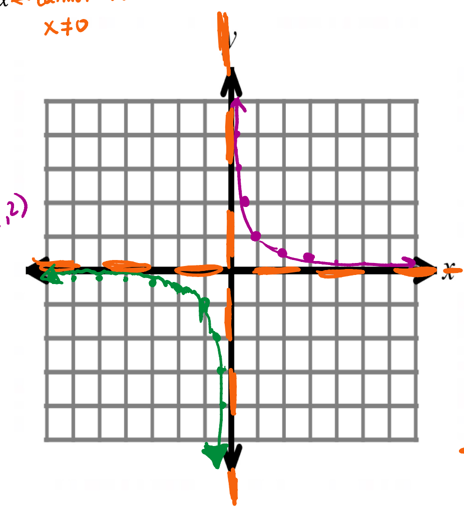
dotted line

Note: horizontal asymptotes may be crossed (although rarely are); vertical asymptotes are never crossed.

The parent function: $y = \frac{1}{x}$ ← cannot have a 0 on denom

$x \neq 0$

x	y
3	$\frac{1}{3}$
2	$\frac{1}{2}$
1	$\frac{1}{1} = 1$ (1, 1)
$\frac{1}{2}$	$\frac{1}{\frac{1}{2}} = 1 \cdot 2 = 2$ ($\frac{1}{2}$, 2)
$\frac{1}{3}$	$\frac{1}{\frac{1}{3}} = 1 \cdot 3 = 3$
$\frac{1}{4}$	$\frac{1}{\frac{1}{4}} = 1 \cdot 4 = 4$
0	$\frac{1}{0}$ undef
$-\frac{1}{4}$	$\frac{1}{-\frac{1}{4}} = 1 \cdot -4 = -4$
$-\frac{1}{3}$	$\frac{1}{-\frac{1}{3}} = 1 \cdot -3 = -3$
$-\frac{1}{2}$	$\frac{1}{-\frac{1}{2}} = 1 \cdot -2 = -2$
-1	$\frac{1}{-1} = -1$
-2	$\frac{1}{-2} = -\frac{1}{2}$
-3	$\frac{1}{-3} = -\frac{1}{3}$



Vertical Asymptote:

$x = 0$

Horizontal Asymptote:

$y = 0$

Domain:

$(-\infty, 0) \cup (0, \infty)$ interval notation

$\{x \mid x \neq 0\}$ set notation

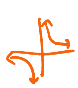
Range:

$(-\infty, 0) \cup (0, \infty)$ interval notation

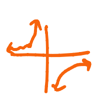
$\{y \mid y \neq 0\}$ set notation

Graphing Form of a Rational Function: $y = \frac{a}{x-h} + k$

$a > 0$



$a < 0$ vert. reflection



VA @ $x = h$ (sign change)

HA @ $y = k$

$|a| > 1$ stretch

$|a| < 1$ compression

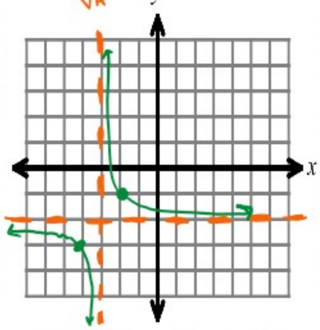
Horizontal Shifts: move VA ⟷ (depending on h) ↑ sign change

Vertical Shifts: move HA ↕ depending on k ↑ no sign change

Examples: Draw a sketch of each rational function by using shifts of the parent function. Then write the equations for any asymptotes, and find the domain and range. SET NOTATION!

1) $y = \frac{1}{x+3} - 2$ k HA @ $y = -2$

$x = -3$ VA



VA: $x = -3$

HA: $y = -2$

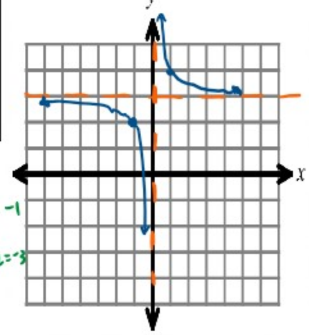
Domain: $\{x \mid x \neq -3\}$

Range: $\{y \mid y \neq -2\}$

x	y
-2	$\frac{1}{-2+3} - 2 = 1 - 2 = -1$
-4	$\frac{1}{-4+3} - 2 = -1 - 2 = -3$

2) $y = \frac{1}{x} + 3$ $y = 3$

$x = 0$



VA: $x = 0$

HA: $y = 3$

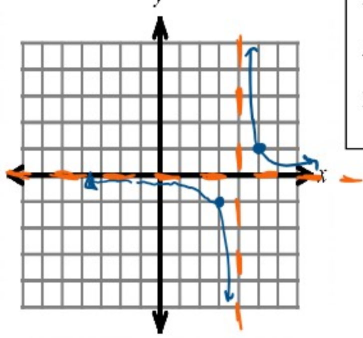
Domain: $\{x \mid x \neq 0\}$

Range: $\{y \mid y \neq 3\}$

x	y
1	$\frac{1}{1} + 3 = 4$
-1	$\frac{1}{-1} + 3 = 2$

3) $y = \frac{1}{x-4} + 0$

$x = 4$



VA: $x = 4$

HA: $y = 0$

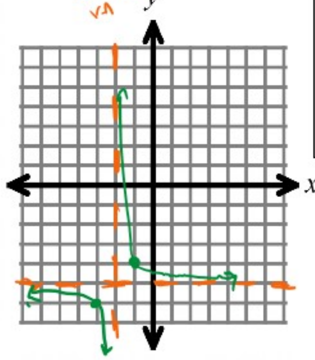
Domain: $\{x \mid x \neq 4\}$

Range: $\{y \mid y \neq 0\}$

x	y
5	$\frac{1}{5-4} = 1$
3	$\frac{1}{3-4} = -1$

4) $y = \frac{1}{x+2} - 5$ HA

$x = -2$ VA



VA: $x = -2$

HA: $y = -5$

Domain: $\{x \mid x \neq -2\}$

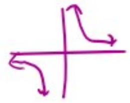
Range: $\{y \mid y \neq -5\}$

x	y
-1	$\frac{1}{-1+2} - 5 = 1 - 5 = -4$
-3	$\frac{1}{-3+2} - 5 = -1 - 5 = -6$

Graphing Form of a Rational Function: $y = \frac{a}{x-h} + k$

Vertical Reflection:

$a > 0$



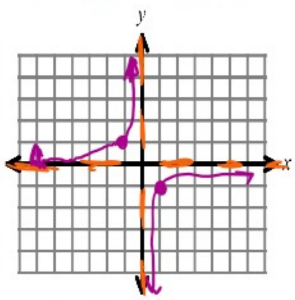
$a < 0$ vert reflection

Vertical Stretch:

$|a| > 1 \rightarrow$ stretch

Examples: Draw a sketch of each rational function by using transformations of the parent function. Then write the equations for all asymptotes, and find the domain and range. SET NOTATION

5) $y = \frac{-1}{x-0} + 0$



VA: $x = 0$

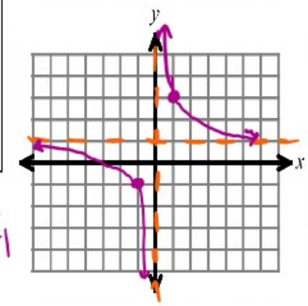
HA: $y = 0$

Domain: $\{x \mid x \neq 0\}$

Range: $\{y \mid y \neq 0\}$

x	y
1	$-\frac{1}{1} = -1$
-1	$-\frac{1}{-1} = 1$

6) $y = \frac{2}{x-0} + 1$



VA: $x = 0$

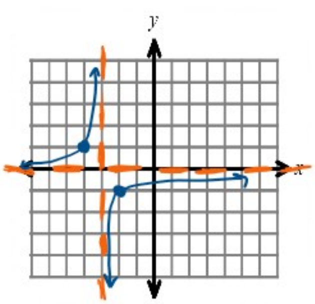
HA: $y = 1$

Domain: $\{x \mid x \neq 0\}$

Range: $\{y \mid y \neq 0\}$

x	y
1	$\frac{2}{1} + 1 = 3$
-1	$\frac{2}{-1} + 1 = -1$

7) $y = \frac{-1}{x+3} + 0$



VA: $x = -3$

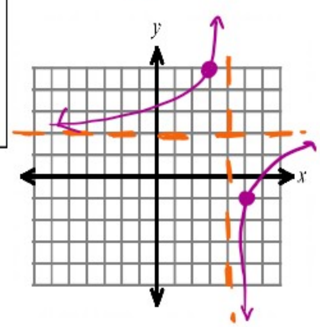
HA: $y = 0$

Domain: $\{x \mid x \neq -3\}$

Range: $\{y \mid y \neq 0\}$

x	y
-2	$\frac{-1}{-2+3} = -1$
-4	$\frac{-1}{-4+3} = 1$

8) $y = \frac{-3}{x-4} + 2$ $\rightarrow 4$ $\uparrow$



VA: $x = 4$

HA: $y = 2$

Domain: $\{x \mid x \neq 4\}$

Range: $\{y \mid y \neq 2\}$

x	y
5	$\frac{-3}{5-4} + 2 = -3 + 2 = -1$
3	$\frac{-3}{3-4} + 2 = 3 + 2 = 5$

End Behavior of a Rational Function: looking at far right & far left of graph

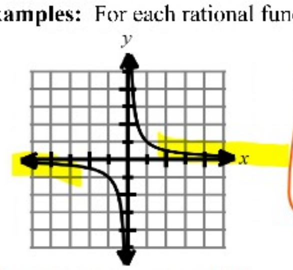
Form: as $x \to \infty$, $y \to$ $\boxed{\text{height}}$ (right)

as $x \to -\infty$, $y \to$ $\boxed{\text{height}}$ (left)

HA (K)

Examples: For each rational function, describe its end behavior.

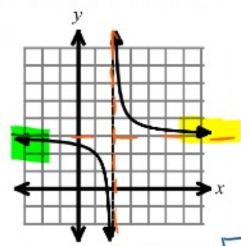
9)



as $x \to \infty$, $y \to 0$

as $x \to -\infty$, $y \to 0$

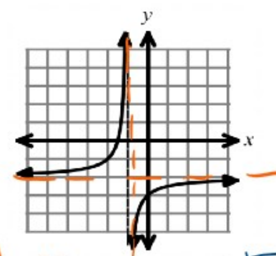
10)



as $x \to \infty$, $y \to \boxed{3}$

as $x \to -\infty$, $y \to \boxed{3}$

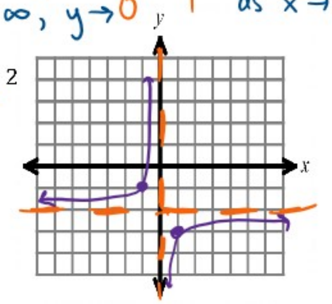
11)



as $x \to \infty$, $y \to \boxed{-2}$

as $x \to -\infty$, $y \to \boxed{-2}$

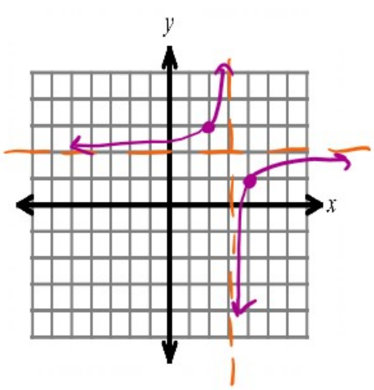
12) $y = -\frac{1}{x} - 2$



as $x \to \infty$, $y \to \boxed{-2}$

as $x \to -\infty$, $y \to \boxed{-2}$

13) Write an equation, in graphing form, of a rational function with a vertical asymptote at $x = 3$, a horizontal asymptote at $y = 2$, and a vertical reflection of the parent function. Then sketch the function below.



reflect/stretch

$$y = \frac{a}{x-h} + K$$

HA

VA

$$y = \frac{-1}{x-3} + 2$$

6.2 Notes: Graphing Rational Functions in General Form

General Form of a Rational Function:

$y = \frac{p(x)}{q(x)}$ or $y = \frac{\text{constant}}{p(x)}$

Vertical Asymptotes: A vertical line that a graph approaches but never touches or crosses.

- ✓ Comes from the undefined values on the denominator.
- ✓ Cannot be a factor of the numerator.
- ✓ Factor the denominator to find the vertical asymptotes.
- ✓ Draw as a dotted line.

Horizontal Asymptote: A horizontal line that a graph approaches but can touch or cross. (Note: occasionally a horizontal asymptote can be crossed by a rational function. This is rare, but can happen close to vertical asymptotes.)

- ✓ Compare the degree of the numerator and the degree of the denominator.
- ✓ Which is growing faster, as x values approach $\pm\infty$?

Case 1: If degree of the numerator is larger, then there is NO horizontal asymptote.

$y = \frac{5x^3 - 10}{x^2 + 2}$ large / smaller

Case 2: If the degree of the denom is larger, then the horizontal asymptote is $y = 0$.

$y = \frac{x+2}{5x^3 - 10}$ small / larger

Case 3: If the degree of the numerator is the SAME as the degree of the denominator, then use the leading coefficients $y = \frac{P}{Q}$.

$y = \frac{3 - 4x^2}{5x^2 + 1}$ $y = \frac{-4}{5}$

Examples: For each rational function below, write the equation of any vertical and horizontal asymptotes.

1) $y = \frac{1}{x-2}$

VA: $x = 2$
HA: $y = 0$

2) $y = \frac{x-1}{x+3}$

VA: $x = -3$
HA: $y = 1$

3) $y = \frac{x^2+5}{x}$

VA: $x = 0$
HA: none

4) $y = \frac{2x+5}{x^2-9} = \frac{2x+5}{(x+3)(x-3)}$

VA: $x = -3$ and $x = 3$
HA: $y = 0$

5) $\frac{7x+2x^2}{x+1} = \frac{x(7+2x)}{x+1}$

VA: $x = -1$
HA: none

6) $\frac{4-3x+7x^3}{x^3+5x^2+4x} = \frac{4-3x+7x^3}{x(x+4)(x+1)}$

$x(x^2+5x+4)$

VA: $x = 0$; $x = -4$; $x = -1$
HA: $y = 7$

Graphing with horizontal and vertical asymptotes and testing points:

Steps:

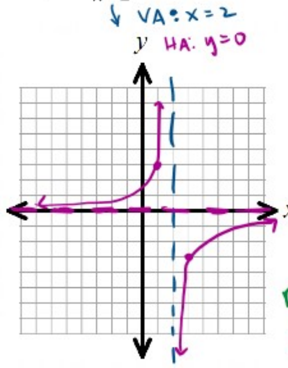
1. Factor the expression fully.
2. Zeros of the denominator identify any VA (as long as that factor is not repeated on the numerator)
3. Compare the degree of the numerator and denominator to identify any HA.
4. Sketch the asymptotes.
5. Choose an input value on *both sides* of any VA. Find the output, and sketch at least one point on each side of each VA. Fit the curve to the asymptotes.

Examples: Sketch each rational function. Include all asymptotes, and identify the domain and range.

1) $y = \frac{-3}{x-2}$

VA: $x=2$

HA: $y=0$



x	y
3	$\frac{-3}{1}=3$
1	$\frac{-3}{-1}=3$

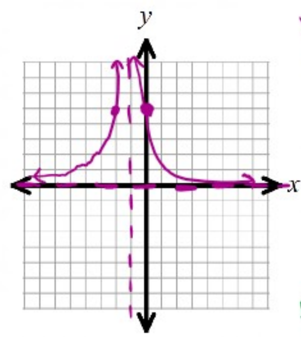
D: $\{x \mid x \neq 2\}$

R: $\{y \mid y \neq 0\}$

2) $f(x) = \frac{5}{x^2+2x+1} = \frac{5}{(x+1)^2}$ $\frac{+}{+}$

VA: $x=-1$

HA: $y=0$



x	y
0	$\frac{5}{1}$
-2	$\frac{5}{1}$

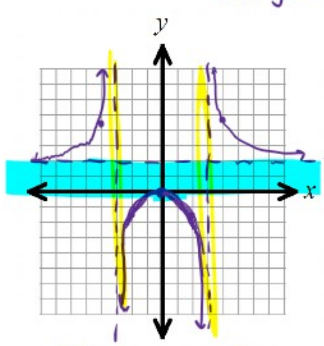
D: $\{x \mid x \neq -1\}$

R: $\{y \mid y > 0\}$

3) $h(x) = \frac{2x^2}{x^2-9} = \frac{2x^2}{(x+3)(x-3)}$

VA: @ $x=-3$ and $x=3$

HA: $y=2$



x	y
2	$\frac{8}{5 \cdot -1} = \frac{-8}{5}$
4	$\frac{32}{7 \cdot 1} = \frac{32}{7}$
-2	$\frac{8}{1 \cdot -5} = \frac{-8}{5}$
-4	$\frac{32}{-1 \cdot -7} = \frac{32}{7}$
0	$\frac{0}{-9} = 0$

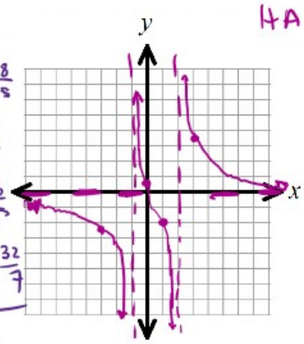
D: $\{x \mid x \neq -2 \text{ and } 2\}$

R: $\{y \mid y > 2 \text{ and } y \leq 0\}$

4) $y = \frac{5x-1}{(x^2-x-2)} = \frac{5x-1}{(x-2)(x+1)}$

VA @ $x=2$ + $x=-1$

HA @ $x=0$



x	y
3	$\frac{14}{1 \cdot 4} = \frac{7}{2}$
1	$\frac{4}{-1 \cdot 2} = -2$
0	$\frac{-1}{-2} = \frac{1}{2}$
-2	$\frac{-11}{-4 \cdot -1} = -\frac{11}{4}$

Other helpful points when graphing rational functions:

- y-intercept
  - plug in a 0 for x
- x-intercepts
  - zeros of numerator (if they are not repeated on denom)

Examples: Graph each rational function. Include the following: all asymptotes, all intercepts, domain, range, and end behavior.

5) $y = \dfrac{x^2+5x-6}{x^2-4} = \dfrac{(x+6)(x-1)}{(x+2)(x-2)}$

VA: $x = -2$ and $x = 2$

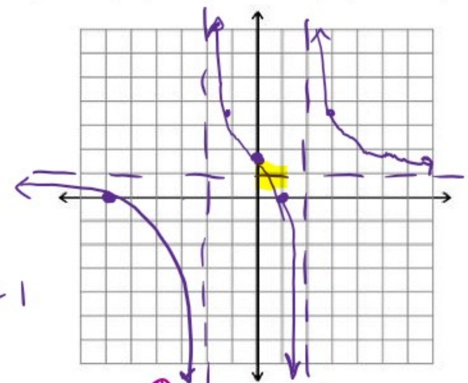
HA: $y = 1$

x-int: @ $x = -6$ and $x = 1$

y-int: $\frac{-6}{-4} = \frac{3}{2}$ $(0, \frac{3}{2})$

D: $\{x \mid x \neq -2 \text{ and } 2\}$

R: $\{y \mid \text{all real \#s}\}$



x	y
3	$\frac{9 \cdot 2}{5 \cdot 1} = \frac{18}{5}$
-1	$\frac{5 \cdot -2}{1 \cdot -3} = \frac{10}{3}$

end behavior:
as $x \to \infty$, $y \to 1$
as $x \to -\infty$, $y \to 1$

6) $y = \dfrac{4x-8}{x+3} = \dfrac{4(x-2)}{x+3}$

VA @ $x = -3$

HA @ $y = 4$

x-int @ $x = 2 \to (2, 0)$

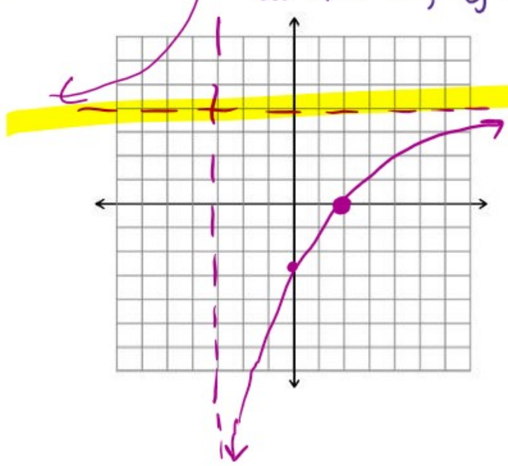
y-int $= \frac{-8}{3}$ $(0, -\frac{8}{3})$

D: $\{x \mid x \neq -3\}$

R: $\{y \mid y \neq 4\}$

as $x \to \infty$, $y \to 4$

as $x \to -\infty$, $y \to 4$



x	y
-4	$\frac{4(-6)}{-1}$