Assignments for Prob/Stat/Discrete
Unit 5 Chapter 4: Discrete Probability Distributions

Day	Date	Assignment (Due the next class meeting)
		5.1 **Worksheet**
		5.2 **Worksheet**
		Unit 5 Practice Test
		Unit 5 Test

NOTE: You should be prepared for daily quizzes.

HW reminders:

- If you cannot solve a problem, get help **before** the assignment is due.
- Help is available before school, during lunch, or during IC.
- For extra practice, visit www.interactmath.com
- Don't forget that you can get 24-hour math help from www.smarthinking.com!

5.1 Probability Distributions

Section 5.1 Objectives:

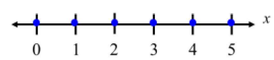
- Distinguish between discrete random variables and continuous random variables
- Construct a discrete probability distribution and its graph
- Determine if a distribution is a probability distribution
- Find the mean, variance, and standard deviation of a discrete probability distribution
- Find the expected value of a discrete probability distribution

Random Variables:

- Represents a ___________________ __________ associated with each outcome of a probability distribution.
- Denoted by ***x***
- Examples
  - *x* = Number of sales calls a salesperson makes in one day.
  - *x* = Hours spent on sales calls in one day.

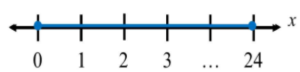
Discrete Random Variable

- Has a ______________ or _______________ number of possible outcomes that can be listed.
- Example
  - *x* = Number of sales calls a salesperson makes in one day.



Continuous Random Variable

- Has an _____________________ number of possible outcomes, represented by an interval on the number line.
- Example
  - *x* = Hours spent on sales calls in one day.



Example: Random Variables

Decide whether the random variable *x* is discrete or continuous.

1. *x* = The number of stocks in the Dow Jones Industrial Average that have share price increases on a given day.

2. *x* = The volume of water in a 32-ounce container.

Discrete probability distribution:

- Lists each possible value the random variable can assume, together with its probability.

- Must satisfy the following conditions:
  - The probability of each value of the discrete random variable is between ____ and ____, inclusive.
    $0 \leq P(x) \leq 1$
  - The sum of all the probabilities is _____.
    $\Sigma P(x) = 1$

Constructing a Discrete Probability Distribution

Let x be a discrete random variable with possible outcomes $x_1, x_2, \ldots, x_n$.

1. Make a ______________ distribution for the possible outcomes.
2. Find the __________ of the frequencies.
3. Find the probability of each possible outcome by dividing its frequency by the sum of the frequencies.
4. Check that each probability is between 0 and 1 and that the sum is 1.

Example: Constructing a Discrete Probability Distribution

An industrial psychologist administered a personality inventory test for passive-aggressive traits to 150 employees. Individuals were given a score from 1 to 5, where 1 was extremely passive and 5 extremely aggressive. A score of 3 indicated neither trait. Construct a probability distribution for the random variable x. Then graph the distribution using a histogram.

Score, x	Frequency, f
1	24
2	33
3	42
4	30
5	21

Solution: Constructing a Discrete Probability Distribution

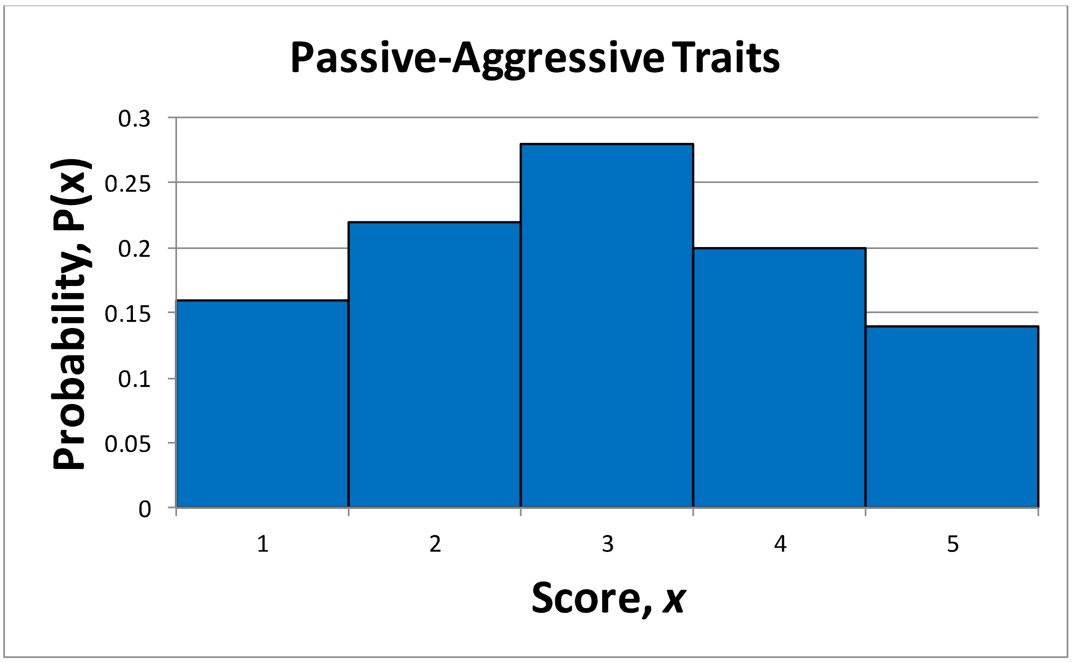
- Divide the frequency of each score by the total number of individuals in the study to find the probability for each value of the random variable.

- **Discrete probability distribution:**

x	1	2	3	4	5
xP(x)	**0.16**	**0.22**	**0.28**	**0.20**	**0.14**

This is a valid discrete probability distribution since

1. Each probability is between 0 and 1, inclusive, $0 \leq P(x) \leq 1$.
2. The sum of the probabilities equals 1, $\Sigma P(x) = 0.16 + 0.22 + 0.28 + 0.20 + 0.14 = 1$.



Because the width of each bar is one, the area of each bar is equal to the probability of a particular outcome.

Mean
Mean of a discrete probability distribution

$\mu = \Sigma xP(x)$

- **Each value of *x* is ________________ by its corresponding probability and the products are added.**

Example: Finding the Mean
The probability distribution for the personality inventory test for passive-aggressive traits is given. Find the mean.

Solution:

x	*P(x)*	*xP(x)*
1	0.16	1(0.16) =
2	0.22	2(0.22) =
3	0.28	3(0.28) =
4	0.20	4(0.20) =
5	0.14	5(0.14) =

Variance and Standard Deviation:
Variance of a discrete probability distribution

- $\sigma^2 = \Sigma(x - \mu)^2 P(x)$

Standard deviation of a discrete probability distribution

- $\sigma = \sqrt{\sigma^2} = \sqrt{\Sigma(x-\mu)^2 P(x)}$

Example*: *Finding the Variance and Standard Deviation:
The probability distribution for the personality inventory test for passive-aggressive traits is given. Find the variance and standard deviation. (μ = 2.94)

x	*P(x)*	*x – μ*	*(x – μ)²*	*(x – μ)²P(x)*
1	0.16			
2	0.22			
3	0.28			
4	0.20			
5	0.14			

Variance: $\sigma^2 = \Sigma(x - \mu)^2 P(x) =$

Standard Deviation:

Expected Value:
Expected value of a discrete random variable

- Equal to the ___________ of the random variable.
- **E(*x*) = μ = Σ*xP(x)***

Example: Finding an Expected Value

At a raffle, 1500 tickets are sold at $2 each for four prizes of $500, $250, $150, and $75. You buy one ticket. What is the expected value of your gain?

- **Solution: Finding an Expected Value**
  To find the gain for each prize, subtract the price of the ticket from the prize:
  - Your gain for the $500 prize is
  - Your gain for the $250 prize is
  - Your gain for the $150 prize is
  - Your gain for the $75 prize is
- If you do not win a prize, your gain is
- **Solution: Finding an Expected Value**
  Probability distribution for the possible gains (outcomes)

Gain, x					
P(x)					

You can expect to lose an average of $____________________ for each ticket you buy.

Section 5.2: Binomial Distributions

Section 5.2 Objectives:

- Determine if a probability experiment is a binomial experiment
- Find binomial probabilities using the binomial probability formula
- Find binomial probabilities using technology and a binomial table
- Graph a binomial distribution
- Find the mean, variance, and standard deviation of a binomial probability distribution

Binomial Experiments:

1. The experiment is repeated for a ___________ number of trials, where each trial is independent of other trials.
2. There are only _________ possible outcomes of interest for each trial. The outcomes can be classified as a success (*S*) or as a failure (*F*).
3. The probability of a success *P*(*S*) is the same for each trial.
4. The random variable *x* counts the number of successful trials.

Notation for Binomial Experiments

Symbol	*Description*
n	The number of times a trial is repeated
$p = P(s)$	The probability of success in a single trial
$q = P(F)$	The probability of failure in a single trial ($q = 1 - p$)
X	The random variable represents a count of the number of successes in n trials: $x = 0, 1, 2, 3, \ldots, n$.

Example: Binomial Experiments

Decide whether the experiment is a binomial experiment. If it is, specify the values of n, p, and q, and list the possible values of the random variable x.

2. A jar contains five red marbles, nine blue marbles, and six green marbles. You randomly select three marbles from the jar, *without replacement*. The random variable represents the number of red marbles.

Binomial Probability Formula

- The probability of exactly x successes in n trials is

$$P(x) = {}_nC_x p^x q^{n-x} = \frac{n!}{(n-x)!x!} p^x q^{n-x}$$

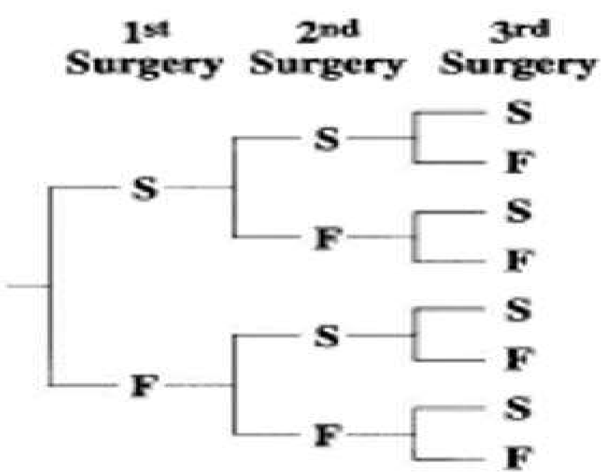
- n = number of trials
- p = probability of success
- $q = 1 - p$ probability of failure
- x = number of successes in n trials

Example: Finding Binomial Probabilities

Microfracture knee surgery has a 75% chance of success on patients with degenerative knees. The surgery is performed on three patients. Find the probability of the surgery being successful on exactly two patients.

Solution: Finding Binomial Probabilities

Method 1: Draw a tree diagram and use the Multiplication Rule



Outcome	Number of Successes	Probability
SSS	3	$\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{3}{4} = \frac{27}{64}$
SSF	2	$\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{4} = \frac{9}{64}$
SFS	2	$\frac{3}{4}\cdot\frac{1}{4}\cdot\frac{3}{4} = \frac{9}{64}$
SFF	1	$\frac{3}{4}\cdot\frac{1}{4}\cdot\frac{1}{4} = \frac{3}{64}$
FSS	2	$\frac{1}{4}\cdot\frac{3}{4}\cdot\frac{3}{4} = \frac{9}{64}$
FSF	1	$\frac{1}{4}\cdot\frac{3}{4}\cdot\frac{1}{4} = \frac{3}{64}$
FFS	1	$\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{3}{4} = \frac{3}{64}$
FFF	0	$\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{4} = \frac{1}{64}$

Method 2: Binomial Probability Formula

$n =$ $p =$ $q =$ $x =$

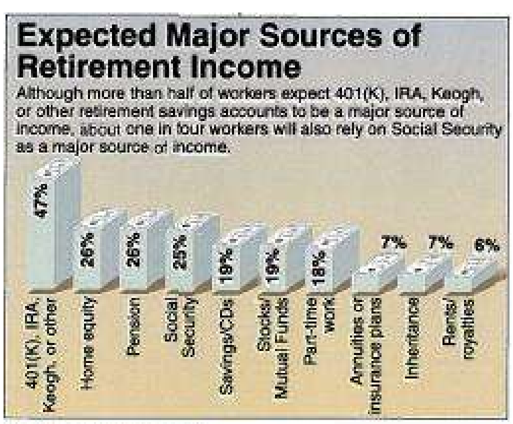
Binomial Probability Distribution

- List the possible values of *x* with the corresponding probability of each.
- Example: Binomial probability distribution for Microfacture knee surgery: *n* = 3, *p* = ¾
  - Use binomial probability formula to find probabilities.

x	0	1	2	3
P(x)				

Example: Constructing a Binomial Distribution:

In a survey, workers in the U.S. were asked to name their expected sources of retirement income. Seven workers who participated in the survey are randomly selected and asked whether they expect to rely on Social Security for retirement income. Create a binomial probability distribution for the number of workers who respond yes.



(Source: The Gallup Organization)

Solution: Constructing a Binomial Distribution:

- _____% of working Americans expect to rely on Social Security for retirement income.

$P(x = 0) = {}_7C_0(\quad)^0(\quad)^7 =$

$P(x = 1) = {}_7C_1(\quad)^1(\quad)^6 =$

$P(x = 2) = {}_7C_2(\quad)^2(\quad)^5 =$

$P(x = 3) = {}_7C_3(\quad)^3(\quad)^4 =$

$P(x = 4) = {}_7C_4(\quad)^4(\quad)^3 =$

$P(x = 5) = {}_7C_5(\quad)^5(\quad)^2 =$

$P(x = 6) = {}_7C_6(\quad)^6(\quad)^1 =$

$P(x = 7) = {}_7C_7(\quad)^7(\quad)^0 =$

x	*P(x)*

All of the probabilities are between 0 and 1 and the sum of the probabilities is 1.00001 ≈ 1.

Example: Finding Binomial Probabilities

A survey indicates that 41% of women in the U.S. consider reading their favorite leisure-time activity. You randomly select four U.S. women and ask them if reading is their favorite leisure-time activity. Find the probability that at least two of them respond yes.

Solution:

- *n* = ____, *p* = _______, *q* = _______
- At least two means ________ ______ __________.
- Find the sum of $P(2)$, $P(3)$, and $P(4)$.

Solution: Finding Binomial Probabilities

$P(x = 2) = {}_4C_2(\quad)^2(\quad)^2 =$

$P(x = 3) = {}_4C_3(\quad)^3(\quad)^1 =$

$P(x = 4) = {}_4C_4(\quad)^4(\quad)^0 =$

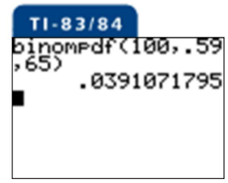
Example: Finding Binomial Probabilities Using Technology

The results of a recent survey indicate that when grilling, 59% of households in the United States use a gas grill. If you randomly select 100 households, what is the probability that exactly 65 households use a gas grill? Use a technology tool to find the probability. *(Source: Greenfield Online for Weber-Stephens Products Company)*

Solution:

- Binomial with n = 100, p = 0.59, x = 65

Solution: Finding Binomial Probabilities Using Technology:



From the display, you can see that the probability that exactly 65 households use a gas grill is about 0.04.

Example: Finding Binomial Probabilities Using a Table

About thirty percent of working adults spend less than 15 minutes each way commuting to their jobs. You randomly select six working adults. What is the probability that exactly three of them spend less than 15 minutes each way commuting to work? Use a table to find the probability. *(Source: U.S. Census Bureau)*

Solution:

- Binomial with n = ____, p = ______, x = _____

Solution: Finding Binomial Probabilities Using a Table

- A portion of Table 2 is shown

		p												
n	x	.01	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60
2	0	.980	.902	.810	.723	.640	.563	.490	.423	.360	.303	.250	.203	.160
	1	.020	.095	.180	.255	.320	.375	.420	.455	.480	.495	.500	.495	.480
	2	.000	.002	.010	.023	.040	.063	.090	.123	.160	.203	.250	.303	.360
3	0	.970	.857	.729	.614	.512	.422	.343	.275	.216	.166	.125	.091	.064
	1	.029	.135	.243	.325	.384	.422	.441	.444	.432	.408	.375	.334	.288
	2	.000	.007	.027	.057	.096	.141	.189	.239	.288	.334	.375	.408	.432
	3	.000	.000	.001	.003	.008	.016	.027	.043	.064	.091	.125	.166	.216
6	0	.941	.735	.531	.377	.262	.178	.118	.075	.047	.028	.016	.008	.004
	1	.057	.232	.354	.399	.393	.356	.303	.244	.187	.136	.094	.061	.037
	2	.001	.031	.098	.176	.246	.297	.324	.328	.311	.278	.234	.186	.138
	3	.000	.002	.015	.042	.082	.132	.185	.236	.276	.303	.312	.303	.276
	4	.000	.000	.001	.006	.015	.033	.060	.095	.138	.186	.234	.278	.311
	5	.000	.000	.000	.000	.002	.004	.010	.020	.037	.061	.094	.136	.187
	6	.000	.000	.000	.000	.000	.000	.001	.002	.004	.008	.016	.028	.047

The probability that exactly three of the six workers spend less than 15 minutes each way commuting to work is 0.185.

Example: Graphing a Binomial Distribution

Fifty-nine percent of households in the U.S. subscribe to cable TV. You randomly select six households and ask each if they subscribe to cable TV. Construct a probability distribution for the random variable *x*. Then graph the distribution. *(Source: Kagan Research, LLC)*

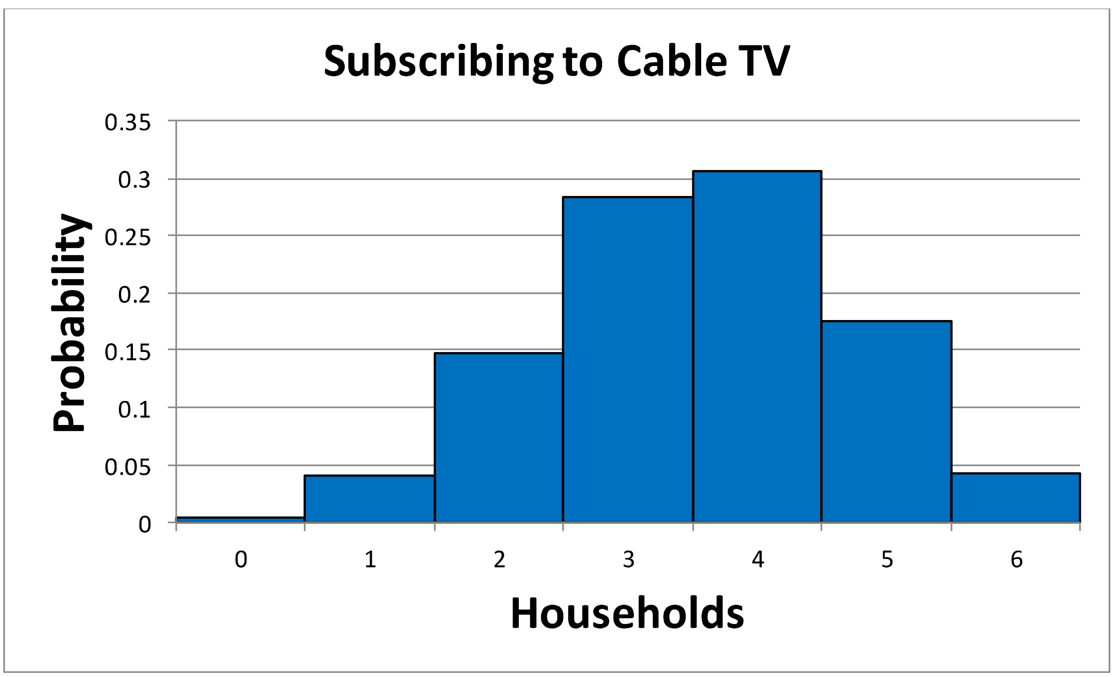
Solution:

- *n* = ____, *p* = _________, *q* = _________
- Find the probability for each value of *x*

Solution: Graphing a Binomial Distribution

x	0	1	2	3	4	5	6
P(x)							

Histogram:



Mean, Variance, and Standard Deviation:

- **Mean:** $\mu = np$
- **Variance:** $\sigma^2 = npq$
- **Standard Deviation:** $\sigma = \sqrt{npq}$

Example: Finding the Mean, Variance, and Standard Deviation

In Pittsburgh, Pennsylvania, about 56% of the days in a year are cloudy. Find the mean, variance, and standard deviation for the number of cloudy days during the month of June. Interpret the results and determine any unusual values. *(Source: National Climatic Data Center)*

Solution: *n* = ____, *p* = _____, *q* = _____

Mean: $\mu = np =$

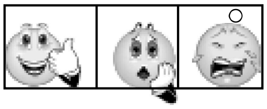
Variance: $\sigma^2 = npq =$

Standard Deviation: $\sqrt{\sigma^2} = \sigma \ or \ \sqrt{npq}$ =

Solution: Finding the Mean, Variance, and Standard Deviation

μ = ____ $\sigma^2 \approx$ ____ σ ≈ ___

- On average, there are _______ cloudy days during the month of June.
- The standard deviation is about _____ days.
- Values that are more than two standard deviations from the mean are considered unusual.
  - , A June with _____ cloudy days would be unusual.
  - , A June with _____ cloudy days would also be unusual.

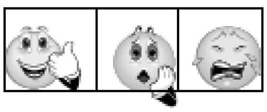


Objective #1: Can you distinguish between discrete random variables and continuous random variables?

In exercises 1 and 2, decide whether the random variable x is discrete or continuous.

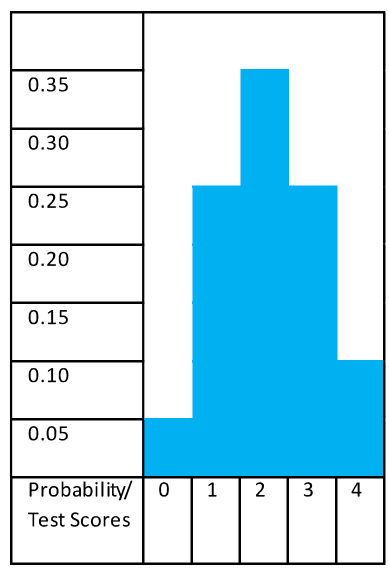
1. x represents the number of motorcycle accidents in one year in California.

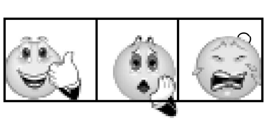
2. x represents the volume of blood drawn for a blood test.



Objective #2: Can you construct a discrete probability distribution and its graph?

1. **Employee Testing**
   A company gave psychological tests to prospective employees. The random variable x represents the possible test scores. Use the histogram to find the probability that a person selected at random from the survey's sample had a test score of (a) more than two and (b) less than four.

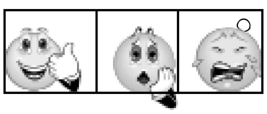




Objective #3: Can you determine if a distribution is a probability distribution?

1. **Tires** A mechanic checked the tire pressures on each car that he worked on for one week. The random variable x represents the number of tires that were underinflated.

X	0	1	3	4	5
P(x)	0.30	0.25	0.25	0.15	0.05

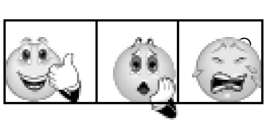


Objective #4: Can you find the mean, variance, and standard deviation of a discrete probability distribution?

For exercise 1, (a) use the frequency distribution to construct a probability distribution, find the (b) mean, (c) variance, and (d) standard deviation of the probability distribution, and (e) interpret the results in the context of the real-life situation.

1. **The number of dogs per household in a small town**

Dogs	0	1	2	3	4	5
Households	1491	425	168	48	29	14

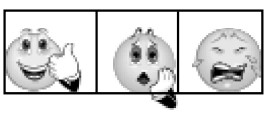


Objective #5: Can you find the expected value of a discrete probability distribution?

For exercise 1, use the probability distribution or histogram to find the (a) mean, (b) variance, (c) standard deviation, and (d) expected value of the probability distribution and (e) interpret the results.

1. **Students in a class take a quiz with eight questions. The number x of questions answered correctly can be approximated by the following probability distribution**

X	0	1	2	3	4	5	6	7	8
P(x)	0.02	0.02	0.06	0.06	0.08	0.22	0.30	0.16	0.08

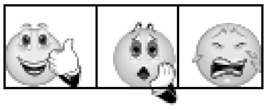


Objective #6: Can you determine if a probability experiment is a binomial experiment?

In exercises 1 and 2, decide whether the experiment is a binomial experiment. If it is, identify a success, specify the values of n, p and q, and list the possible values of the random variable x. If it is not a binomial experiment, explain why.

1. **Cyanosis:** Cyanosis is the condition of having bluish skin due to insufficient oxygen in the blood. About 80% of babies are born with cyanosis recover fully. A hospital is caring for five babies born with cyanosis. The random variable represents the number of babies that recover fully. (*Source: The World Book Encyclopedia*)

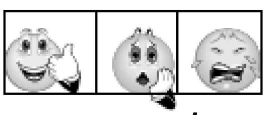
2. **Political Polls:** A survey asks 1000 adults, “Do tax cuts help or hurt the economy?” Twenty-one percent of those surveyed said tax cuts hurt the economy. Fifteen adults who participated in the survey are randomly selected. The random variable represents the number of adults who think tax cuts hurt the economy. (*Source: Rasmussen Reports*)



Objective #7: Can you find binomial probabilities using the binomial probability formula?

In exercise 1, find the indicated probabilities. If convenient, use technology to find the probability.

1. **Answer Guessing:** You are taking a multiple choice quiz that consists of five questions. Each question has four possible answers, only one of which is correct. To complete the quiz, you randomly guess the answer to each question. Find the probability of guessing (a) exactly three questions correctly, (b) at least three answers correctly, and (c) less than three questions correctly.

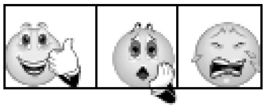


Objective #8: Can you find binomial probabilities using technology and a binomial table?

In exercise 1 and 2, find the indicated probabilities. If convenient, use technology to find the probability.

1. **Baseball Fans:** Fifty-nine percent of men consider themselves professional baseball fans. You randomly select 10 men and ask each if he considers himself a professional baseball fan. Find the probability that the number who consider themselves baseball fans is (a) exactly eight, (b) at least eight, and (c) less than eight. (*Source: Gallup Poll)*

2. **Vacation Purpose:** Twenty-one percent of vacationers say the primary purpose of their vacation is outdoor recreation. You randomly select 10 vacationers and ask each to name the primary purpose of his or her vacation. Find the probability that the number who say outdoor recreation is their primary purpose of their vacation is (a) exactly three, (b) more than three, and (c) at most three. (*Source: Travel Industry Association)*



Objective #9: Can you graph a binomial distribution?

o***In exercises 1 and 2, (a) construct a binomial distribution, (b) graph the binomial distribution using a histogram, (c) describe the shape of the histogram, find the (d) mean, (e) variance, and (f) standard deviation of the binomial distribution, and (g) interpret the results in the context of the real-life situation. What values of the random variable x would you consider unusual? Explain your reasoning.***

1. **Women Baseball Fans**: Thirty-seven percent of women consider themselves professional baseball fans. You randomly select six women and ask each if she considers herself a fan of professional baseball.

2. **Blood Donors:** Five percent of people in the United States eligible to donate blood actually do. You randomly select four eligible blood donors and ask them if they donate blood. (*Adapted from American Association of Blood Banks)*