4.5 Notes: Measures of Position

Objectives:

8. Can you determine the quartiles of a data set?
9. Can you determine the interquartile range of a data set?
10. Can you create a box-and-whisker plot?
11. Can you interpret other fractiles such as percentiles?
12. Can you determine and interpret a standardized score (z-score)?

Quartiles

- Approximately divide an ordered data set into ___4___ equal parts.
- 1st quartile, $Q1$: About ___25%___ of the data fall on or below $Q1$.
- 2nd quartile, $Q2$: About ___50%___ of the data fall on or below $Q2$ (median).
- 3rd quartile, $Q3$: About ___75%___ of the data fall on or below $Q3$.

Example 1: The test scores of 15 employees enrolled in a CPR training course are listed. Find the first, second, and third quartiles of the test scores.

~~13~~ ~~9~~ ~~18~~ ~~15~~ ~~14~~ ~~21~~ ~~7~~ ~~10~~ ~~11~~ ~~20~~ ~~5~~ ~~18~~ 37 ~~16~~ ~~17~~

Step 1: Order the data. 5 7 9 10 11 13 14 15 16 17 18 18 20 21 37
Step 2: Find the median.
Step 3: Find the quartiles
(the medians of each half.) Q_1 Q_2 median Q_3

Example 2: Using the data from Example 1, answer the questions below.

a) About what % of employees score below 10? 25%

b) About what % of employees score below 18? 75%

c) About what % of employees score above 15? 50%

Interquartile Range (IQR)

- The ___difference___ between the ___third___ and first quartiles.
- IQR = $Q3 - Q1$

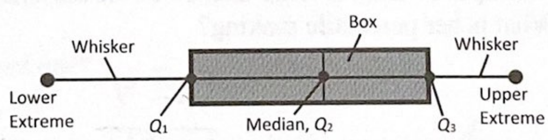
Example 3: Find the IQR for the data in Example 1.

$IQR = Q_3 - Q_1$

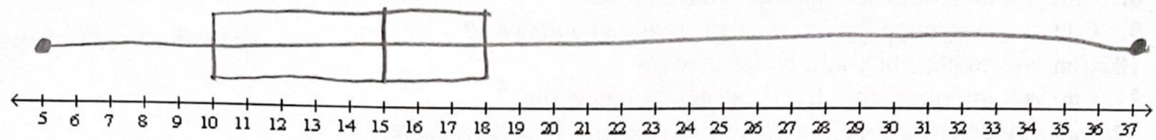
$18 - 10 = 8$

Box-and-whisker plot

- Divides data set into quartiles.
- Requires (___5___ **-number summary**):
  1. Lower extreme (minimum value)
  2. $Q1$ (lower quartile)
  3. $Q2$ (median)
  4. $Q3$ (upper quartile)
  5. Upper extreme (maximum value)



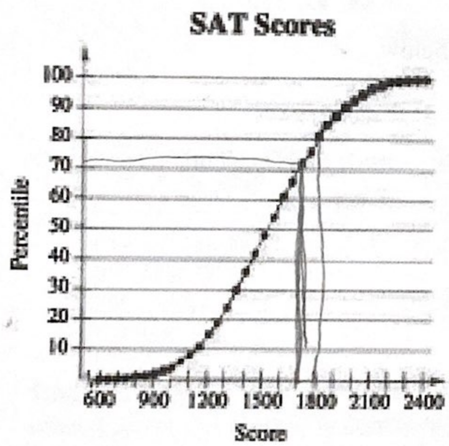
Example 4: Draw a box-and-whisker plot from the data in Example 1. Recall that Min = 5 $Q1 = 10$
$Q2 = 15$ $Q3 = 18$ Max = 37



Fractiles are numbers that partition (divide) an ordered data set into equal parts. The chart below describes different types of fractiles.

Fractiles	Summary	Symbols
Quartiles	Divides data into 4 equal parts	$Q1, Q2, Q3$
Deciles	Divides data into 10 equal parts	$D1, D2, D3, \ldots, D9$
Percentiles	Divides data into 100 equal parts	$P1, P2, P3, \ldots, P99$

Example 5: The ogive shown represents the cumulative frequency distribution for SAT test scores of college-bound students in a recent year. *(Source: College Board Online)*



a) What test score represents the 72nd percentile (P72)? How should you interpret this? 72% ≈ 1700

This means that 72% of the students had an SAT score of 1700 or less

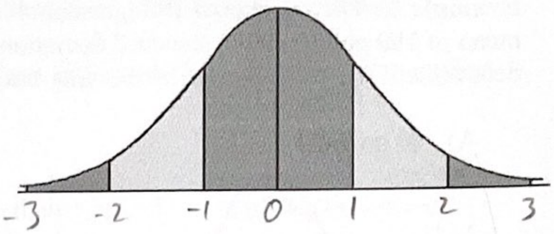
b) What percentile is a score of 1800? Interpret this score.

About 80%, this means that 80% of students had an SAT score of 1800 or less

Example 6: Sara is taller than 28 of the students in her class. There are 32 students in her class. What is her percentile ranking?

$$\frac{29}{32} \approx 0.906 \text{ or } 91\%$$

Standard Normal Distribution



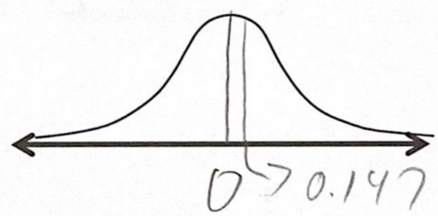
- Mean = μ or $\bar{x}$
- Standard Deviation = σ_x or S_x

Standardized Score (*z*-score)

- Represents the number of standard deviation a given value *x* falls from the mean μ.
- Used to transform any score to fit a **Standard Normal Distribution** (mean = μ and standard deviation = σ.)
- An **unusual score** (or an outlier) is more than 2 standard deviations above or below the mean.
- $z = \dfrac{\text{value - mean}}{\text{standard deviation}} = \dfrac{x-\mu}{\sigma}$

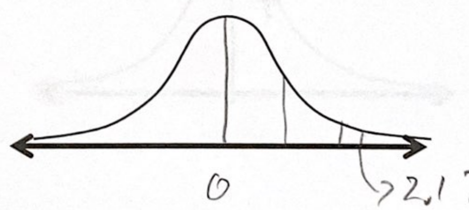
Example 1: In 2007, Forest Whitaker won the Best Actor Oscar at age 45 for his role in the movie *The Last King of Scotland.* Helen Mirren won the Best Actress Oscar at age 61 for her role in *The Queen*. The mean age of all best actor winners is 43.7, with a standard deviation of 8.8. The mean age of all best actress winners is 36, with a standard deviation of 11.5.

a) Find Forest Whitaker's *z*-score for best actor winners.



$z = \dfrac{x-\mu}{\sigma}$ $\dfrac{45-43.7}{8.8} = \dfrac{1.3}{8.8} = 0.147$

b) Find Helen Mirren's *z*-score for best actress winners.



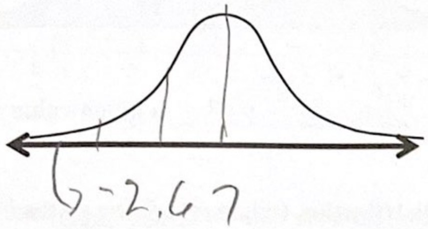
$z = \dfrac{x-\mu}{\sigma} = \dfrac{61-36}{11.5} = \dfrac{25}{11.5} = 2.17$

c) Compare your results. Whose age was a more unusual one?

Helen Mirren's age is more than two Sd's away from the mean.

Example 2: The weights of 19 high school basketball players have a bell-shaped distribution, with a mean of 180 pounds and a standard deviation of 15 pounds. Use standardized scores (*z*-scores) to determine if the weights of the following basketball players are unusual (outliers.)

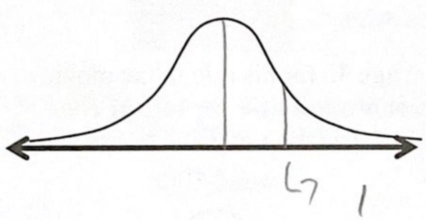
A) 140 pounds



$$z = \frac{140-180}{15} = \frac{-40}{15} = -2.67$$

outlier

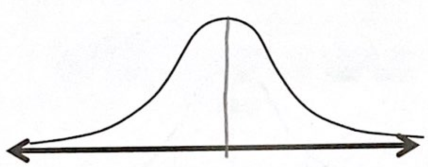
B) 195 pounds



$$\frac{195-180}{15} = 1$$

not an outlier

C) 180 pounds



$$\frac{180-180}{15} = 0$$

not an outlier

<u>Outliers</u>... There are two main methods for determining whether or not a score is an outlier.

1) Find its *z*-score. If it is more than 2 standard deviations from the mean, that score is an outlier.

OR

2) Find the Interquartile Range (IQR). If the score is more than 1.5(IQR) units higher than Q3 or more than 1.5(IQR) units lower than Q1, then it is an outlier.

Example 3: A collection of data has Q1 = 49, Q2 = 53, and Q3 = 57. Are the following values outliers?

(1) Find the IQR $= Q_3 - Q_1$ $57 - 49 = 8$

(2) Find $1.5(IQR)$ $= 1.5(8) = 12$

(3) Add $1.5(IQR)$ to Q_3 & Subtract $1.5(IQR)$ from Q_1

$49 - 12$ $57 + 12$

$\boxed{37 \text{ Normal } 69}$

a) 40

no

b) 30

yes

c) 70

yes