11.1 Notes: Volume of Prisms and Cylinders

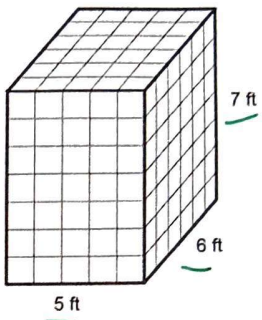
Objectives:

- **Students will be able to find the volume of a prism.**
- **Students will be able to find the volume of a cylinder.**

Exploration: Watch this video about the volume of a rectangular prism: https://www.youtube.com/watch?v=hytONaoJjc0

Then explain in words how you would find the volume of the prism shown:

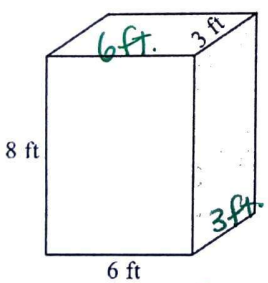
Find the area of the base rectangle and multiply by the height



Prism	**A prism is a solid with two** congruent **parallel bases. The segment connecting the bases is called the** height**. Each lateral face of a prism is a** side ✓ rectangle**.**	Prism bases Triangular Prism Pentagonal Prism Rectangular Prism
Volume of a Prism	**Volume of a prism:** $V = Bh$ (B: Area of the base) For a rectangular prism, this would become $V = lwh$ (base is a rectangle)	

For #1–3: Find the volume of each rectangular prism. You Try #2-3!

1)

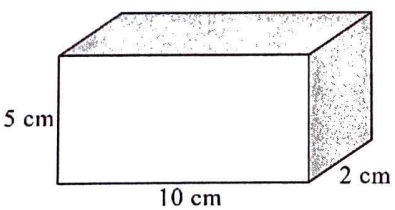


Base

$(6\text{ ft} \cdot 3\text{ ft}) \cdot 8\text{ ft}$

144 ft^3

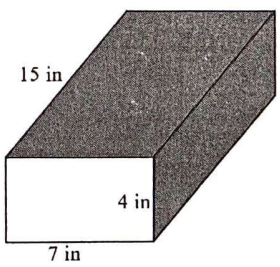
2)



$5 \cdot 10 \cdot 2 =$

100 cm^3

3)

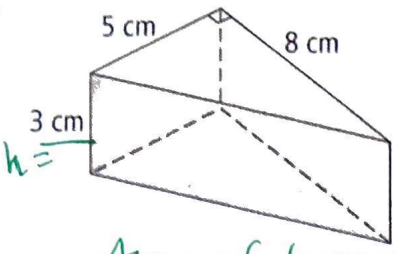


$7 \cdot 4 \cdot 15$

420 in^3

For #4–7: Find the volume of each triangular prism.

4)

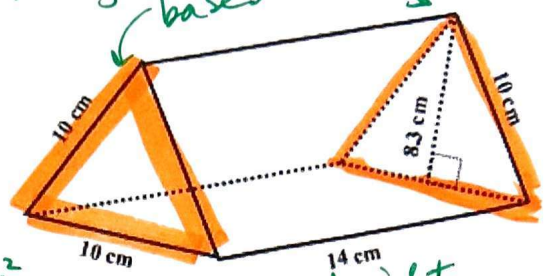


base is a triangle

Area of base = $\frac{1}{2}(5\text{cm})(8\text{cm}) = 20\text{cm}^2$

$(20\text{ cm}^2)(3\text{ cm}) = 60\text{ cm}^3$

5)

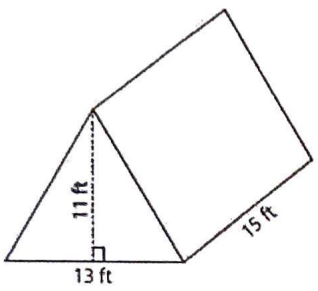


$B = \frac{1}{2} \cdot 10 \cdot 8.3 = 41.5$

$(41.5)(14) = 581\text{ cm}^3$

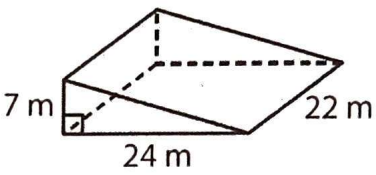
You try #6–7!

6)



1072.5 ft^3

7)



1848 m^3

8) A rectangular prism has volume of 24 cm^3. The length and width of the prism is 5 cm and 3 cm. Find the height of the prism.

$V = lwh$

$24\text{cm}^3 = 5 \cdot 3 \cdot h$

$24\text{ cm}^3 = 15h$

$\frac{24}{15} = \frac{15h}{15}$

1.6 cm

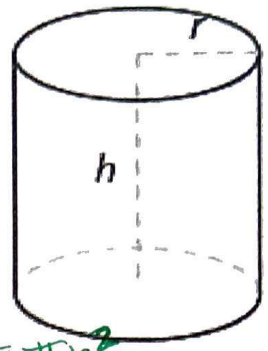
9) Jessica made soup, and the soup had a volume of 30 in^3. She poured the soup into a container shaped like a rectangular prism, and the length and width of the container is 2 inches by 3 inches. The soup completely filled the container perfectly (wow!) 😊 What is the height of the container?

$V = lwh$

$30 = 2 \cdot 3 \cdot h$

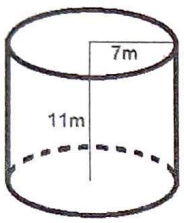
$\frac{30}{6} = \frac{6h}{6}$

$h = 5\text{ in}$

Cylinder	**A cylinder is a solid figure with two Congruent circular bases that are parallel.**	
Volume of a Cylinder	$V = \pi r^2 h$ Base Area of circle $= \pi r^2$	

For #10–13: Find the volume of each solid, in the requested form.

10) in terms of pi



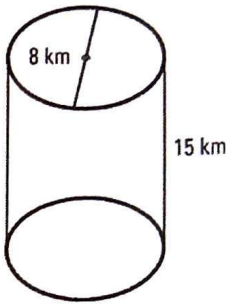
$\pi 7^2 \cdot 11$

$539\pi \text{ m}^3$

11) rounded to one decimal place

$d = 8$ km

$r = 4$ km



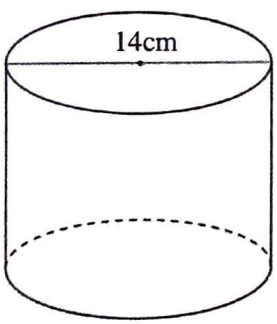
$\pi 4^2 \cdot 15 =$

753.98

754.0 km^3

You try #12 – 13!

12) in terms of pi



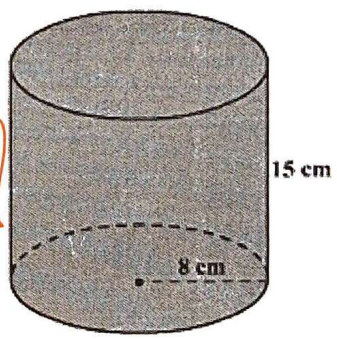
$d = 14$ cm

$r = 7$

$\pi (7)^2 \cdot 10$

$490\pi \text{ cm}^3$

13) rounded to one decimal place



3015.9 cm^3

14) A cylinder has volume of $100\pi \text{ in}^3$ and a height of 4 inches. Find the radius of the cylinder.

$V = \pi r^2 h$

$\frac{100\pi}{4\pi} = \frac{\pi r^2 \cdot 4}{4\pi}$

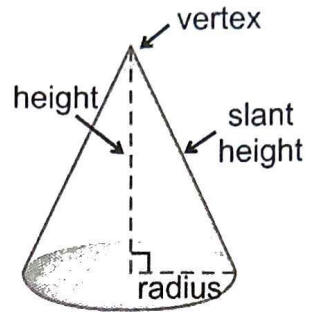
$\sqrt{25} = \sqrt{r^2}$

$r = 5$ in.

11.2 Notes: Volume of Pyramids and Cones

Objective:

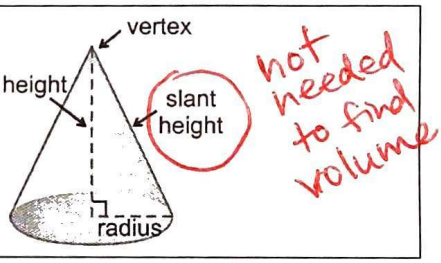
- **Students will be able to find the volume of cones and pyramids.**

Cone	**A cone is a solid with one** circular **base and a vertex.** **The height, radius, and slant height of a cone form a** right **triangle.**	

Exploration: Watch this video (until time 3:24) to observe the relationship between the volume of a cylinder and the volume of a cone. https://www.youtube.com/watch?v=xwPiA0COi8k Given that the volume of a cylinder is $V = \pi r^2 h$, then how could you find the volume of a cone? What formula could you use?

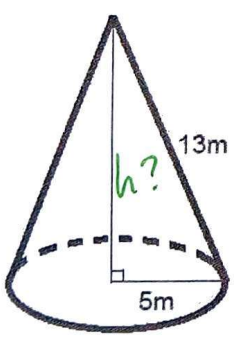
They filled 3 cones with 1 cylinder

Cone = $\frac{1}{3}$ · Volume of cylinder

Volume of a Cone	**The volume of a cone can be found by the formula:** $\frac{1}{3}\pi r^2 h$	

For #1-2: Find the volume of each cone, in the desired form.

1) in terms of pi



$V = \frac{1}{3}\pi r^2 h$

$r = 5m$

$h = 12m.$

5-12-13

$\frac{1}{3}\pi(5)^2(12)$

multiply

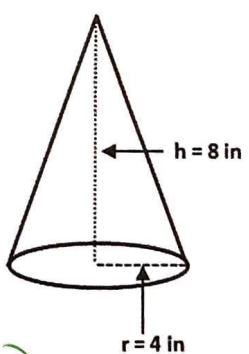
$100\pi\ m^3$

2) rounded to one decimal place

$\frac{1}{3}\pi r^2 h$

$r = 4$ in.

$h = 8$ in



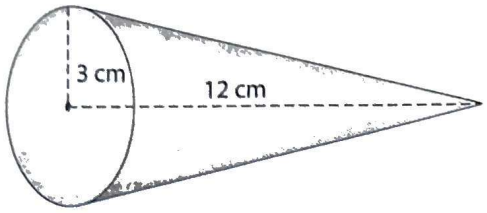
$\frac{1}{3}\pi(4)^2(8)$

134.04

$134.0\ in^3$

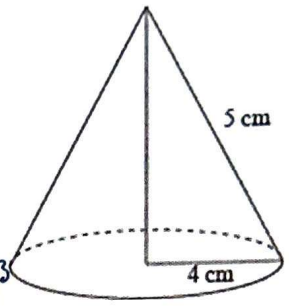
You try #3–4! Find the volume of each cone, in the desired form.

3) rounded to one decimal place



$113.1\ cm^3$

4) in terms of pi



$16\pi\ cm^3$

5) A cone has volume of 12 mm^3 and a height of 4 mm. Find the radius of the cone.

$V = 12\,mm^3$ $\quad V = \frac{1}{3}\pi r^2 h$

$h = 4\,mm$

$r = ?$

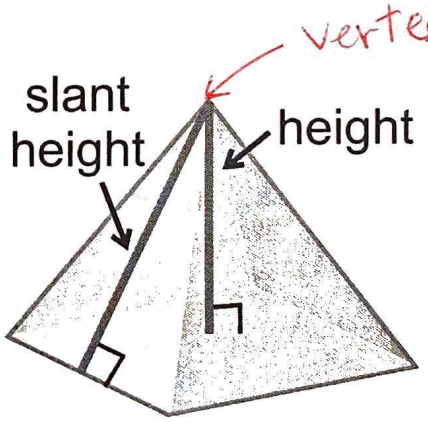
$3.12 = \frac{1}{3}\pi r^2 (4) \cdot 3$

$\frac{36}{4\pi} = \frac{\pi r^2 (4)}{4\pi}$

$\sqrt{2.865} = \sqrt{r^2}$

$r = 1.69\ mm$

Pyramid	**A pyramid has one base, and all the lateral faces are** triangles**. The lateral faces all meet at one point, called the** vertex**.**
Square Pyramid	**A square pyramid has one base, shaped like a** square**.**
Volume of a Pyramid	**The volume of a pyramid can be found by using:** $V = \frac{1}{3}Bh$



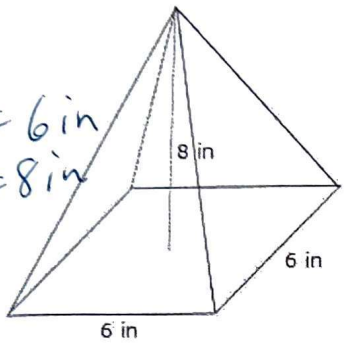
$V = \frac{1}{3}s^2 \cdot h$ for square pyramid

Area of the Base

$A = s^2$ for square

For #6 –11: Find the volume of each square pyramid. If needed, round to one decimal place.

6)



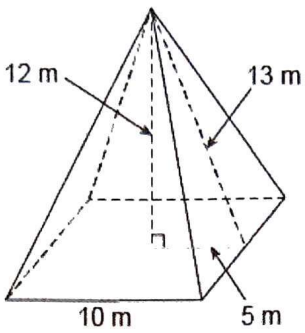
$s = 6$ in
$h = 8$ in

$V = \frac{1}{3} s^2 \cdot h$

$\frac{1}{3}(6)^2 \cdot 8$

96 in^3

7)

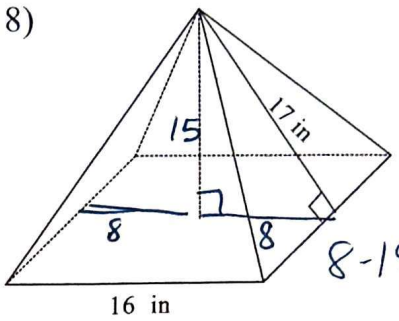


$s = 10$ m
$h = 12$ m

$\frac{1}{3}(10)^2 \cdot 12$

400 m^3

8)



8-15-17

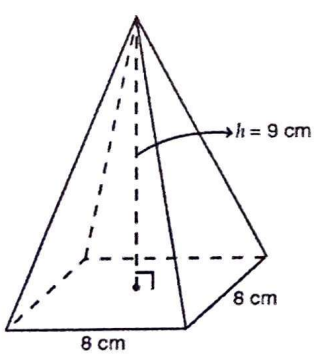
$s = 16$ in
$h = 15$ in.

$\frac{1}{3}(16)^2 \cdot 15$

1280 in^3

You try #9-11!

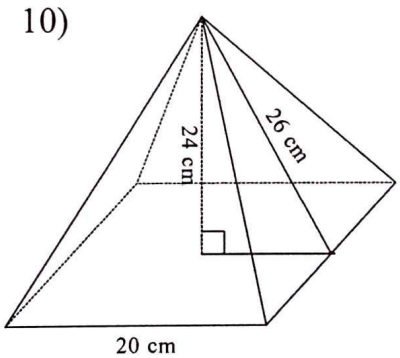
9)



$s = 8$cm
$h = 9$cm

192 cm^3

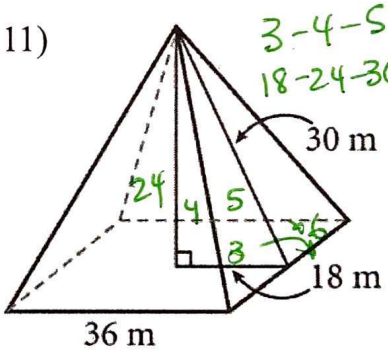
10)



$s = 20$ cm
$h = 24$ cm

3200 cm^3

11)



3-4-5 ×6
18-24-30

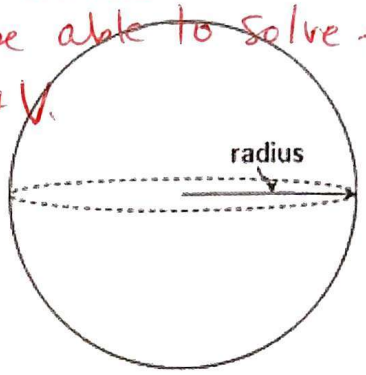
$s = 36$m
$h = 24$ m

$10,368 \text{ cm}^3$

11.3 Notes: Spheres and Cubes

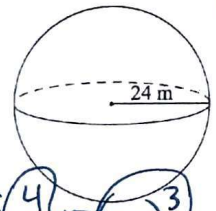
Objectives:

- **Students will be able to find the surface area and volume of a sphere.**
- **Students will be able to find the surface area and volume of a cube.**

Surface Area of a Sphere	**The Surface Area of a sphere can be found with the formula:** $SA = 4\pi r^2$	All we need is radius to be able to solve for SA & V. 
Volume of a Sphere	**The Volume of a sphere can be found with the formula:** $V = \frac{4}{3}\pi r^3$	

For #1 – 4: Find the volume and surface area of each sphere, in the requested form.

1) in terms of pi



r = 24m

$SA = 4\pi(24)^2$

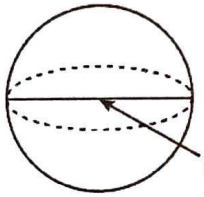
multiply these

$V = \frac{4}{3}\pi(24)^3$

$= 18432\pi\ m^3$

$2304\pi\ m^2$ ← Area, not Volume

2) rounded to one decimal place



r = 4mm

d = 8mm

$SA = 4\pi(4)^2$

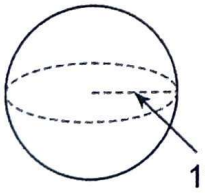
$SA = 201.1\ mm^2$

$V = \frac{4}{3}\pi(4)^3$

$V = 268.1\ mm^3$

You try #3-4!

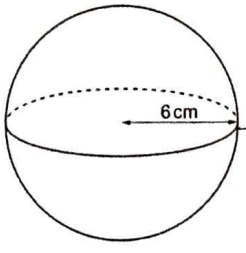
3) rounded to one decimal place



$SA = 1520.5\ in^2$

$V = 5575.3\ in^3$

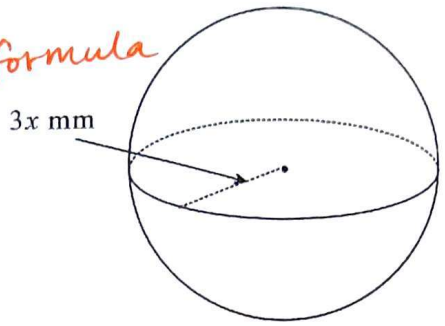
4) in terms of pi



$SA = 144\pi\ cm^2$

$V = 288\pi\ cm^3$

5) **Multiple Choice:** What is the surface area of the sphere shown, in terms of *x* and *pi*? "just set up the formula"



A) $12x\pi\ mm^2$

B) $36x^2\pi\ mm^2$

C) $144x^2\pi\ mm^2$

D) $27x^3\pi\ mm^3$

$SA = 4\pi(3x)^2$

$4\pi\ 9x^2$

$36x^2\pi\ mm^2$

6) A sphere has surface area of $100\pi\ in^2$. What is the radius of the sphere?

$SA = 4\pi r^2$

$r = ?$

$SA = 100\pi\ in^2$

$\frac{100\pi}{4\pi} = \frac{4\pi r^2}{4\pi}$

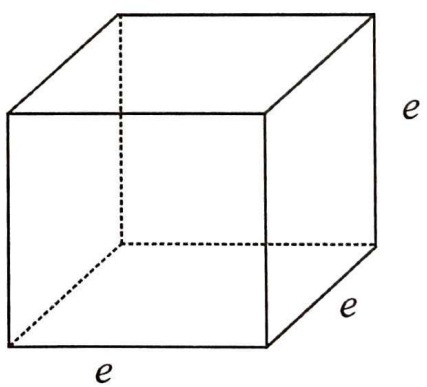
$\sqrt{25} = \sqrt{r^2}$

$r = 5$ in.

7) For the sphere referenced in #6, what is the volume of the sphere, rounded to one decimal place?

$V = \frac{4}{3}\pi(5)^3$

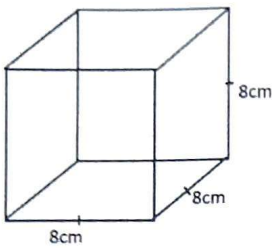
$= 523.6\ in^3$

Cube	**A cube is a rectangular prism where the length, width, and height are all congruent.**	
Volume of a Cube	**The volume of a cube can be found with the formula:** $V = e^3$	
Surface Area of a Cube	**The surface area of a cube can be found with the formula:** $SA = 6e^2$	6 faces each face is e^2 $6e^2$

For #8 – 9: Find the surface area and volume of each cube.

$V = e^3$ and $SA = 6e^2$

8)



$SA = 6e^2$

$6 \cdot (8)^2$

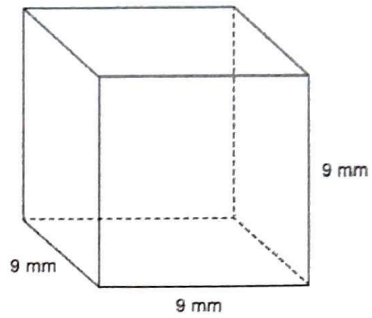
$384\ cm^2$

$s = 8cm$

$V = e^3$

$(8)^3 = 512\ cm^3$

You Try! 9)



$s = 9mm$

$SA = 486\ mm^2$

$V = 729\ mm^3$

10) A cube has a surface area of 54 ft^2. Find the length of one side of the cube.

$SA = 54\ ft^2$ $s = ?$

$SA = 6e^2$

$\frac{54}{6} = \frac{6e^2}{6}$

$\sqrt{e^2} = \sqrt{9}$

$e = 3\ ft.$

11) Use the cube described in #12. Find the volume of the cube.

$V = e^3$

$(3)^3 = 27\ ft^3$

Challenge: A cube has volume of 125 cm^3. Find the surface area of the cube.

$V = 125\ cm^3$

$V = e^3$

$\sqrt[3]{125} = \sqrt[3]{e^3}$

$5 = e$

$SA = 6e^2$

$6(5)^2$

$6(25)$

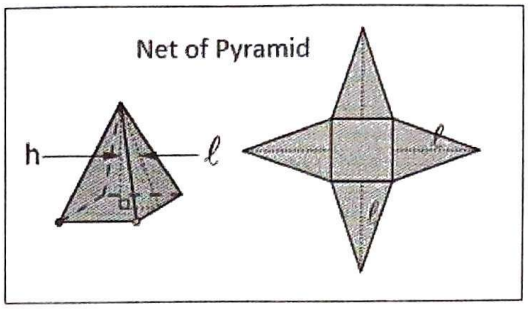
$150\ cm^2$

11.4 Notes: Surface Area

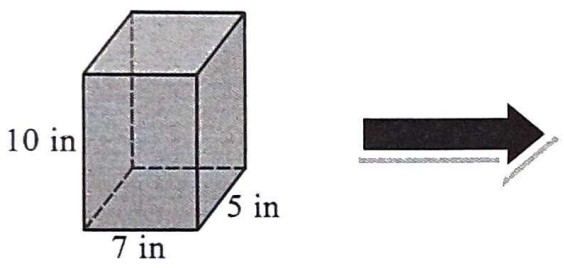
Objectives:

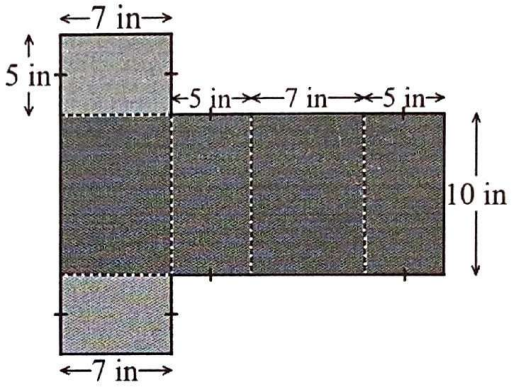
- **Students will be able to find the surface area of a solid by using nets.**
- **Students will be able to find the surface area of a solid by using formulas.**

Exploration: Go to this link to explore surface area of various solids:
https://www.geogebra.org/m/wP2xJt64#material/c8ptXZ2w
Click on various shapes on the left side of the screen. Explore the page as you "unfold" the surface area of each solid figure.

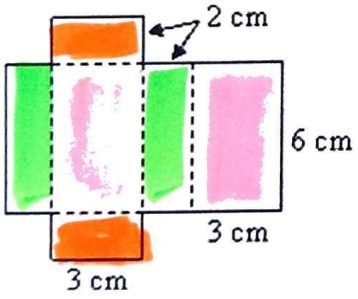
Surface Area	**The Surface Area of a solid is the** Sum **of the areas of each face of the figure.**	

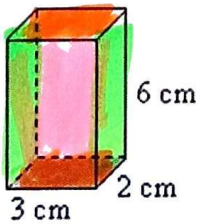
1) Find the surface area of the rectangular prism shown by examining its net.



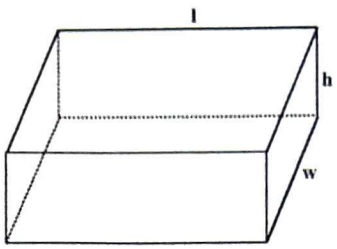
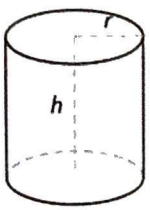
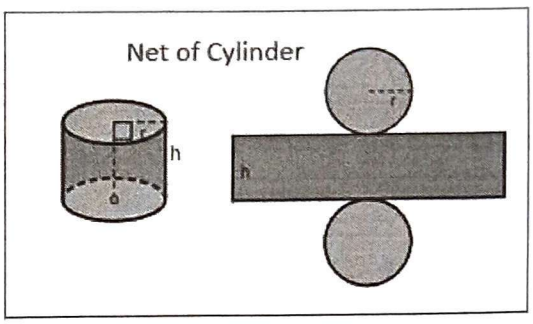


You Try! 2) Find the surface area of the rectangular prism shown by examining its net.



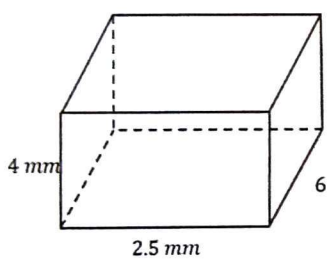


orange $3 \times 2 = 6\ cm^2$
2 orange sides $6\ cm^2 \times 2 = 12\ cm^2$
pink $6 \times 3 = 18\ cm^2$
2 pink sides = 36 cm^2
green $6 \times 2 = 12\ cm^2$
2 green sides $= 24\ cm^2$
Orange + pink + green
$12\ cm^2 + 36\ cm^2 + 24\ cm^2 = 72\ cm^2$

Surface Area of a Rectangular Prism	$SA = 2lw + 2wh + 2lh$ Alternate formula: $SA = 2B + Ph$	 $SA = 2lw + 2wh + 2lh$
Surface Area of a Cylinder	$SA = 2\pi r^2 + 2\pi rh$ 	

For #3–6: Find the surface area of each solid. For cylinders, leave your answer in terms of pi.

3)

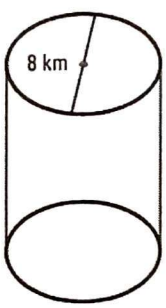


$2lw + 2wh + 2lh$

$l = 2.5$ mm
$w = 6$ mm
$h = 4$ mm

$2(2.5)(6) + 2(6)(4) + 2(2.5)(4)$
$30 + 48 + 20 = \boxed{98\ mm^2}$

4)

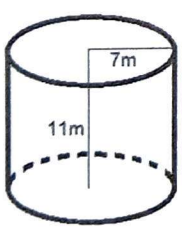


$2\pi r^2 + 2\pi rh$
$2\pi(4)^2 + 2\pi(4)(15)$
$32\pi + 120\pi$
$\boxed{152\pi\ km^2}$

$r = 4$ km
$h = 15$ km

You try #5 – 6!

5)

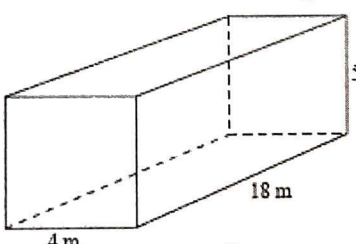


$2\pi r^2 + 2\pi rh$

$r = 7$
$h = 11$

$2\pi(7)^2 + 2\pi(7)(11)$
$98\pi + 154\pi$
$\boxed{252\pi\ m^2}$

6)



$2lw + 2wh + 2lh$
$2(18)(4) + 2(4)(5) + 2(18)(5)$
$144 + 40 + 180$
$\boxed{364\ m^2}$

$l = 18$
$w = 4$
$h = 5$

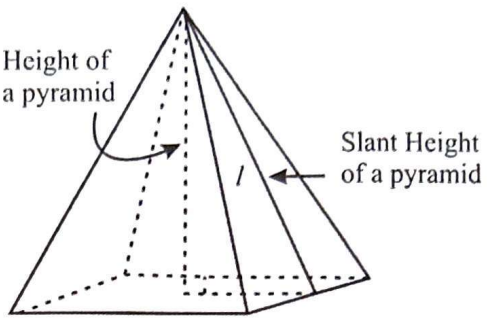
Surface Area of a Pyramid

$$SA = B + \frac{1}{2}Pl$$

slant height

area of the base

Perimeter around the base



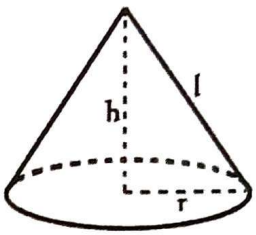
Surface Area of a Cone

$$SA = \pi r^2 + \pi r l$$

slant height

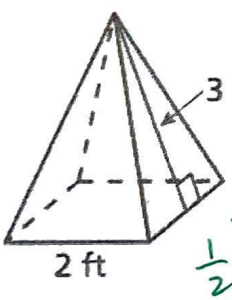
area of the circle

area of the top cone part



For #7–10: Find the surface area of each solid. For cones, leave your answer in terms of pi.

7)



$SA = B + \frac{1}{2}Pl$

$B = 2ft \times 2ft = 4\,ft^2$

$P = 8\,ft$

$l = 3\,ft$

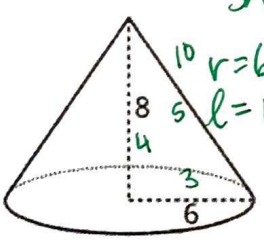
$\frac{1}{2}(8)(3) = 12\,ft^2$

add

$16\,ft^2$

8)

$SA = \pi r^2 + \pi r l$



$r = 6$

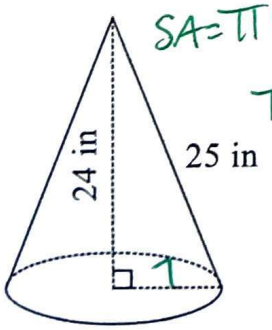
$l = 10$

$\pi(6)^2 + \pi(6)(10)$

$36\pi + 60\pi$

96π units^2

9)



$SA = \pi r^2 + \pi r l$

$\pi(7)^2 + \pi(7)(25)$

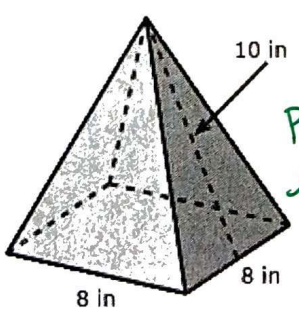
$49\pi + 175\pi$

$224\pi\ in^2$

$r = 7$ in.

$l = 25$ in

10)



$P = 32in$

$l = 10in$

$B = 8 \times 8 = 64in^2$

$\frac{1}{2}(32)(10) = 160\,in^2$

$224\,in^2$