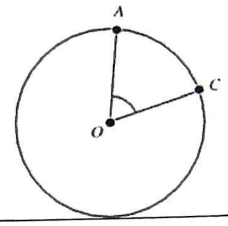
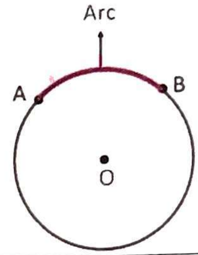
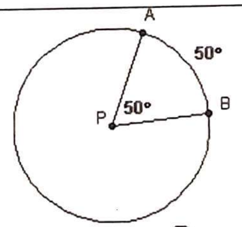
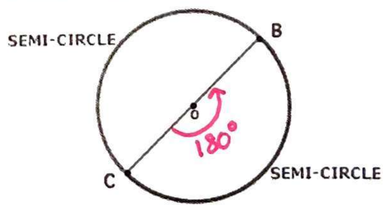
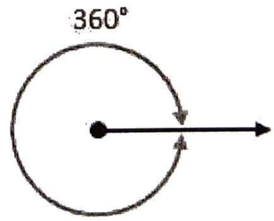
10.1 Notes: Arc and Central Angles in Circles

Objectives:

- **Students will be able to find missing central angles in a circle.**
- **Students will be able to find the measure of arcs in a circle.**

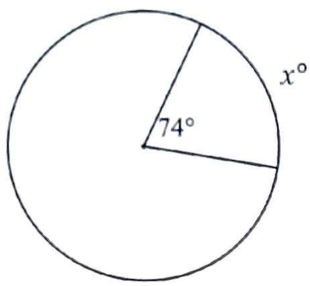
Central Angle of a Circle	**A central angle of a circle has its vertex on the** <u>center</u> **of the circle, and its rays are the** <u>radius</u> **of the circle.**	
Arc of a Circle	**An arc of a circle is a** <u>portion</u> **of the circumference of the circle. In other words, it is part of the curved outer portion of the circle.**	

Exploration: Use the following link to explore the relationship between a central angle and its arc: https://www.geogebra.org/m/pwa6zQtq Move points B and C around, and watch how the measures of the central angle and the enclosed arc change. Make a conjecture about the relationship between a central angle and its arc:

Measure of an arc	**The measure of an arc is** <u>congruent</u> **to the measure of its central angle.**	 If $m\angle APB=50°$, then $m\overset{\frown}{AB}=50°$
Measure of a semi-circle	**A semicircle is an arc formed by the** <u>diameter</u> **of a circle, and its measure is** <u>180°</u> **degrees.**	
Total number of degrees in a circle	**Any circle has a total measure of** <u>360°</u> **degrees.**	

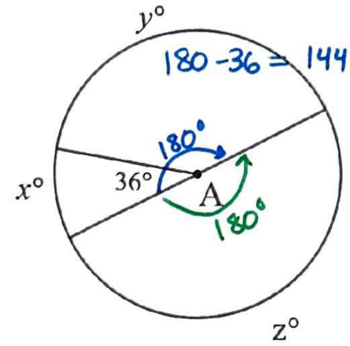
For #1 – 6: Find the measure of the variable(s) for each diagram. Assume a segment that looks like a diameter is a diameter.

1)



x = 74°

2)



180 - 36 = 144

180°

180°

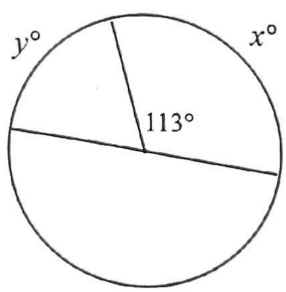
x = 36

y = 144

z = 180

You try #3 – 6!

3)

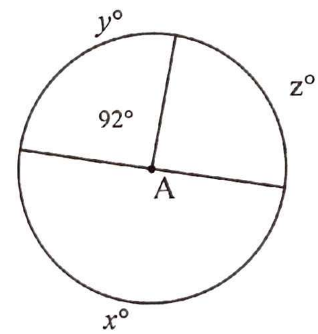


$\boxed{x = 113}$

y = 180 - 113

$\boxed{y = 67}$

4)

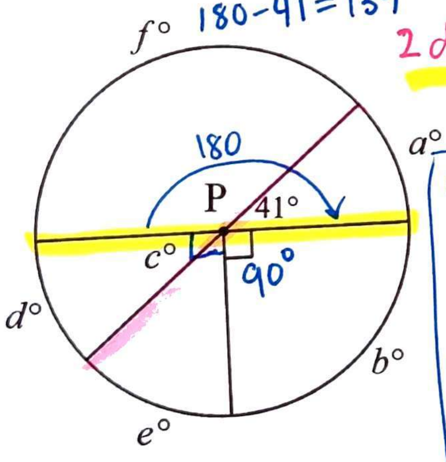


x = 180

y = 92

z = 88

5)



f° 180 - 41 = 139

2 diameters

180

90°

$\boxed{\begin{array}{l} a = 41 \\ b = 90 \\ c = 41 \\ d = 41 \\ e = 49 \\ f = 139 \end{array}}$

c & 41° are vertical <s

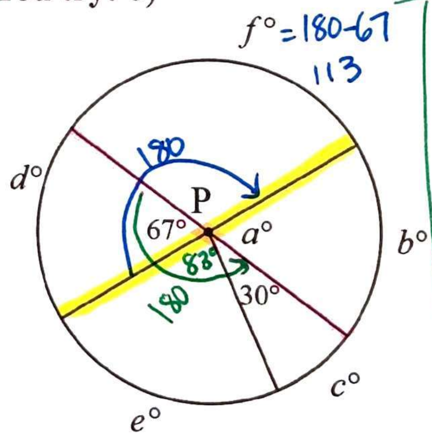
d + e = 90

41 + e = 90

-41 -41

e = 49

You try! 6)



f° = 180 - 67

113

180

83°

180

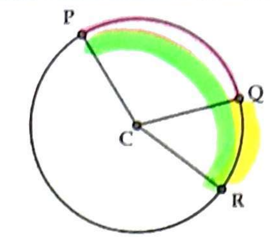
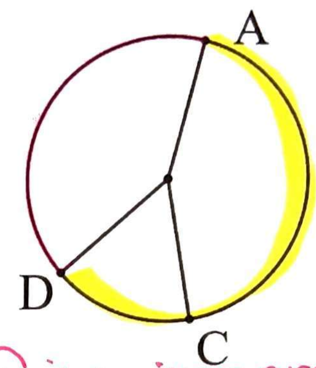
$\boxed{\begin{array}{l} a = 67 \\ b = 67 \\ c = 30 \\ d = 67 \\ e = 83 \\ f = 113 \end{array}}$

67 + e + 30 = 180

e + 97 = 180

-97 -97

e = 83

Arc Addition Postulate	**The Arc Addition Postulate states that arcs that are adjacent can be** added **to find the measure of a larger arc.**	 $m\overset{\frown}{PQR} = m\overset{\frown}{PQ} + m\overset{\frown}{QR}$
Minor Arc	**An arc with a measure** less **than 180°**	 $\overset{\frown}{AD}$ is a minor arc $\overset{\frown}{ACD}$ is a major arc
Major Arc	**An arc with a measure** greater **than 180°.** **To name a major arc** 3 **letters are used.** ***Follow the order in which they are written to calculate measure.***	

7) **Find the measure of each arc or angle (assuming $\overline{KN}$ and $\overline{QT}$ are diameters).**
Note: If an arc is named with three letters, it is called a **major arc**. The measure is calculated the *long way around* the circle. Follow the order of the letters.

A. measure of $\overset{\frown}{QN}$

44°

B. measure of $\overset{\frown}{SQ}$

180
− 86
−44
$\boxed{50°}$

C. measure of $\angle SRN$

50° + 44° =
$\boxed{94°}$

D. measure of $\overset{\frown}{QNK}$

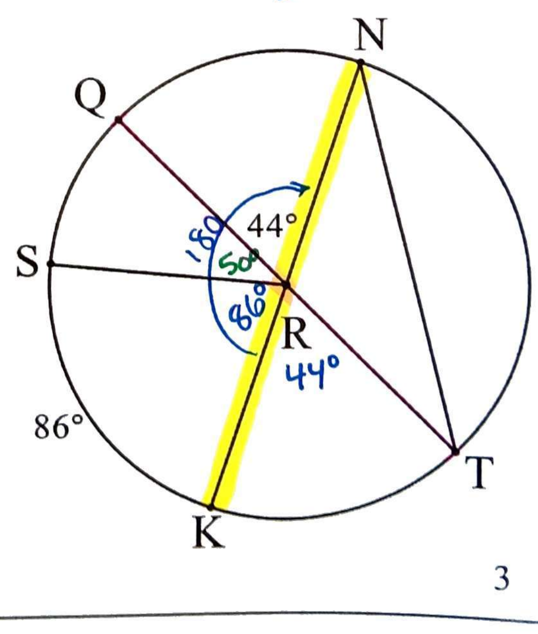
44° + 180° =
$\boxed{224°}$

E. measure of $\angle TRN$

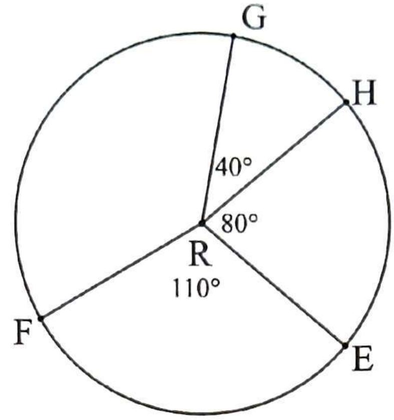
180° − 44° =
$\boxed{136°}$

F. measure of $\overset{\frown}{QNS}$

44 + 180 + 86 = $\boxed{310°}$
or 360° − 50° = 310°



You Try! 8) Find the measure of each arc.



A. measure of $\overset{\frown}{GE}$

$80 + 40 = \boxed{120^\circ}$

B. measure of $\overset{\frown}{GEF}$

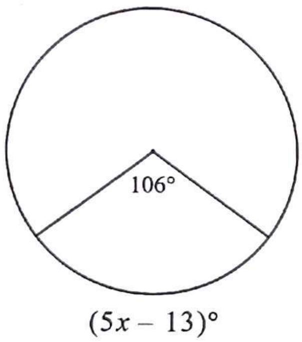
$40 + 80 + 110 = \boxed{230^\circ}$

C. measure of $\overset{\frown}{GF}$

$360 - 230 = \boxed{130^\circ}$

For #9-10: Find the value of the variable.

9)



$5x - 13 = 106$

$\quad +13 \quad +13$

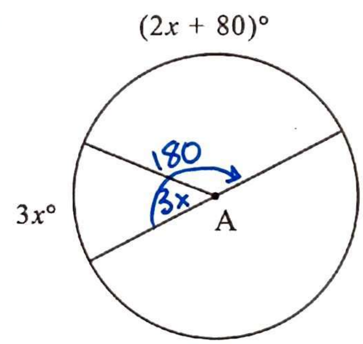
$\dfrac{5x}{5} = \dfrac{119}{5}$

$\boxed{x = 23.8}$

or

$\boxed{\dfrac{119}{5}}$

10)



$3x + 2x + 80 = 180^\circ$

$5x + 80 = 180$

$\quad -80 \quad -80$

$\dfrac{5x}{5} = \dfrac{100}{5}$

$\boxed{x = 20}$

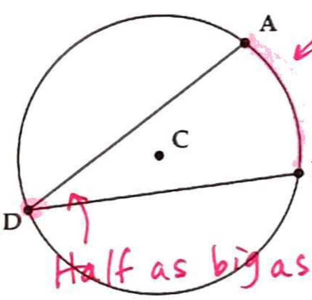
10.2 Notes: Inscribed Angles

Objectives:

- **Students will be able to use the relationship between inscribed angles and arcs to find missing measures.**

Exploration: Use the given link to explore the relationship between inscribed angles and their enclosed arcs: https://www.geogebra.org/m/yX6FgbPA

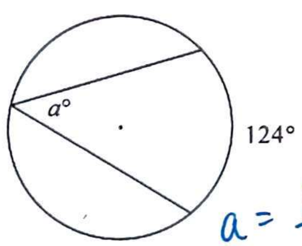
✓ Click on "Inscribed Angle", and move around points B and C.
✓ Make a conjecture comparing the measure of an inscribed angle and its arc.

<table>
<tr><td>Inscribed Angle of a Circle</td><td>An inscribed angle of a circle has its <u>vertex</u> <u>on</u> the circle.</td><td rowspan="2">$\angle ADB = \frac{1}{2}\widehat{AB}$


Twice as big as ∠D
Half as big as $\widehat{AB}$</td></tr>
<tr><td>Measure of an Inscribed Angle</td><td>The measure of an inscribed angle is equal to <u>half</u> of the measure of its enclosed arc.</td></tr>
</table>

For #1 – 6: Find the measure of each variable.

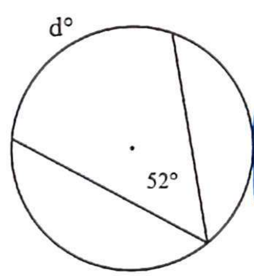
1)



$a = \frac{124}{2}$

$\boxed{a = 62°}$

2)

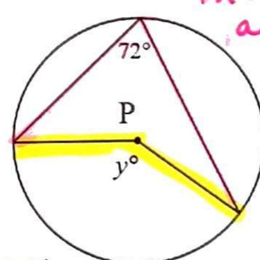


$d = 52 \cdot 2$

$\boxed{d = 104°}$

3)

inscribed angle



central angle

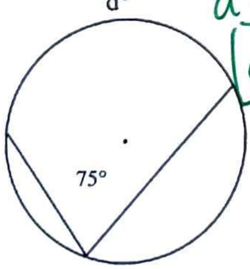
$x = y$

$x = 72 \cdot 2$

$\boxed{x = 144°}$ $\boxed{y = 144°}$

You try #4 – 6!

4)

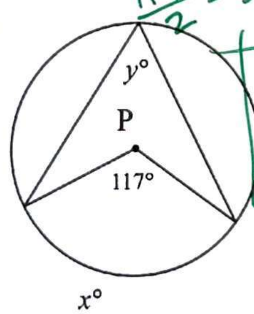


$d = 75 \cdot 2$

$\boxed{d = 150°}$

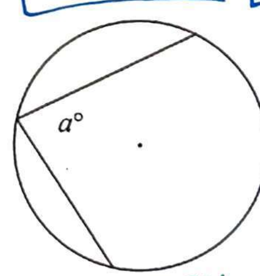
5)

$\frac{117}{2} = 58.5$



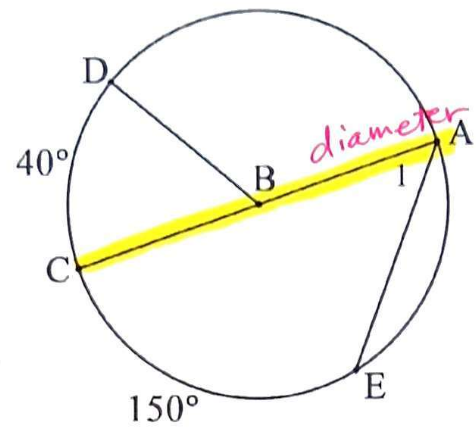
$\boxed{x = 117 \quad y = 58.5}$

6)



$a = \frac{234}{2} = \boxed{117°}$

7) Find the requested measures for circle B, as shown.



A) measure of $\angle 1$

$\frac{150}{2} = \boxed{75°}$

B) measure of $\overset{\frown}{AE}$

$180 - 150 = \boxed{30°}$

C) measure of $\overset{\frown}{DA}$

$180 - 40 = \boxed{140°}$

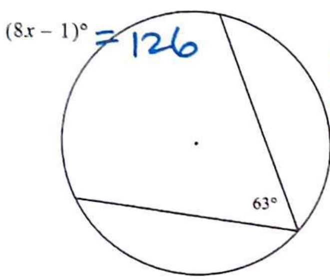
D) measure of $\overset{\frown}{DEA}$

$40 + 150 + 30 = \boxed{220°}$

or $\cancel{180}$ 360 $- 140 = 220$

For #8 – 9, find the value of *x*.

8)



$(8x-1)° = 126$

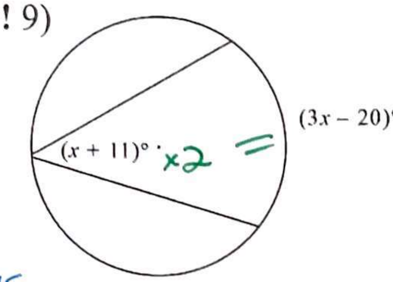
$63 \times 2 = 126$

$8x - 1 = 126$
$+1 \quad +1$
$\frac{8x}{8} = \frac{127}{8}$

$x = 15.875$

$\boxed{15.9}$ or $\boxed{\frac{127}{8}}$

You try! 9)



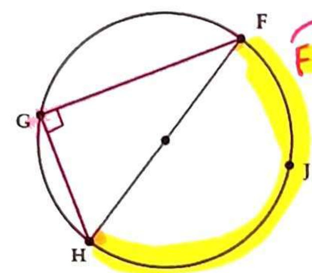
$(x+11)° \times 2 =$

$2(x+11) = 3x - 20$

$2x + 22 = 3x - 20$
$-2x \quad\quad -2x$

$22 = x - 20$
$+20 \quad\quad +20$

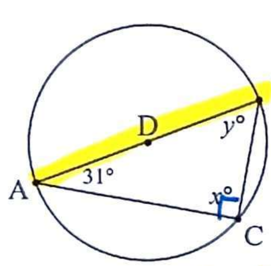
$\boxed{42 = x}$

Angle Inscribed in a Semicircle	**An angle inscribed in a semi-circle has a measure of ~~180°~~ 90° degrees.**	

$\overset{\frown}{FJH} = 180$ so $\angle HGF = \frac{180}{2} = 90°$

For #10 – 11: Find each variable, given that AB is a diameter.

10)

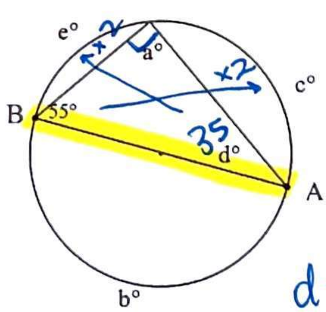


$\boxed{x = 90 \quad y = 59}$

$y + 31 + 90 = 180$

$y + 121 = 180$
$-121 \quad -121$
$y = 59$

11)



$\boxed{a = 90° \quad b = 180° \quad c = 110° \quad d = 35° \quad e = 70°}$

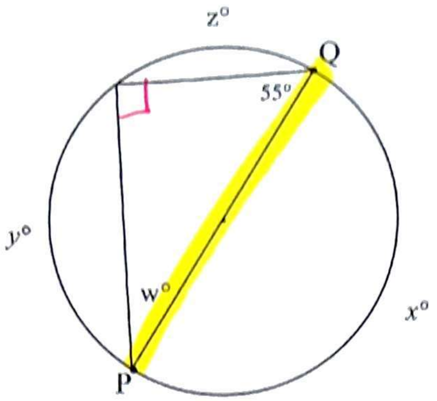
$d + 55 + 90 = 180$

$d = 35$

$e = 35 \cdot 2$

$c = 55 \cdot 2$

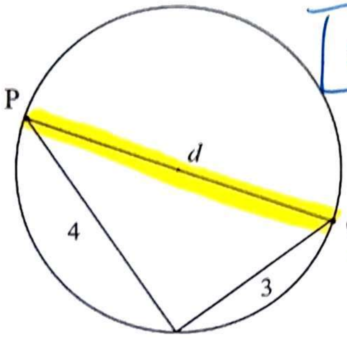
You try! 12) Find the measure of each variable, given that $\overline{PQ}$ is a diameter.



$x = 180°$
$y = 110°$
$z = 70°$
$w = 35°$

For #13 – 16: Find each variable, given that PQ is a diameter.

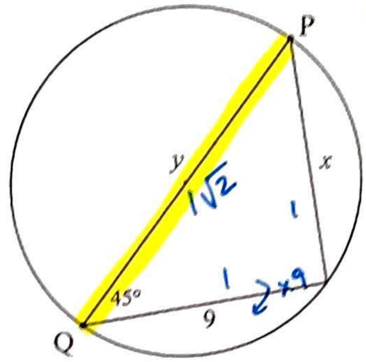
13)



$\boxed{d = 5}$

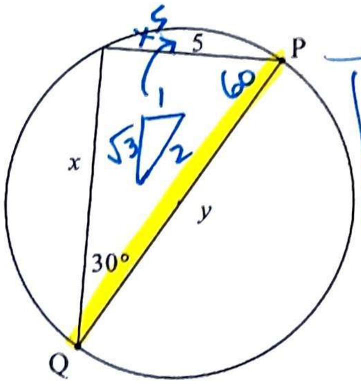
3-4-5
or
$3^2 + 4^2 = d^2$

14)



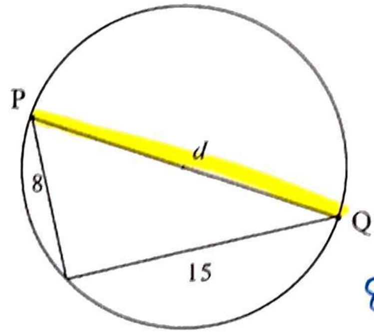
$x = 9$
$y = 9\sqrt{2}$

15)



$x = 5\sqrt{3}$
$y = 10$

16)



8-15-17

$\boxed{d = 17}$

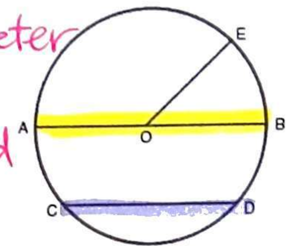
or

$8^2 + 15^2 = d^2$

10.3 Notes: Chords and Tangent Segments in Circles

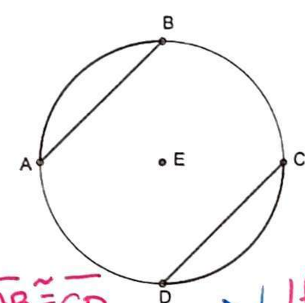
Objectives:

- **Students will be able to solve problems using chords in circles.**
- **Students will be able to solve problems involving tangent segments with circles.**

Chord	**A chord is a segment whose endpoints lie** on **a circle.** **A diameter is a special type of chord that goes through the** center **of a circle.**	O is center AB is diameter CD is a chord 

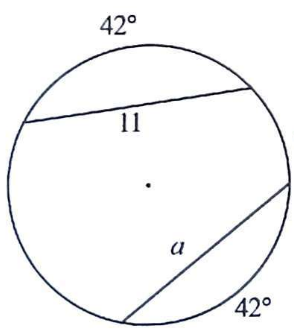
Exploration: Use this link to explore relationships between central angles, chords, and arcs: https://www.geogebra.org/m/U76G7TtB

- ✓ Click on the boxes for ANGLES, CHORDS, and ARCS.
- ✓ Move points A, B, C, and D around the circle.
- ✓ **Write a conjecture about the relationships between chords and arcs that have the same central angle.**

Chord-Arc Property	**In one circle (or two congruent circles), if two chords are** congruent**, then their intercepted arcs are also** congruent**.** **In one circle (or two congruent circles), if two arcs are** ~~ce~~ congruent**, then their chords are also** congruent**.**	If $\overline{AB} \cong \overline{CD}$ then $\widehat{AB} \cong \widehat{CD}$ and If $\widehat{AB} \cong \widehat{CD}$ then $\overline{AB} \cong \overline{CD}$

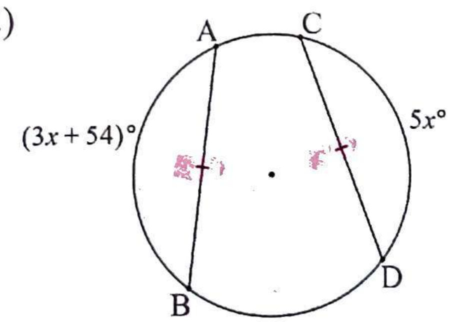
For #1–2: Find each variable.

1)



$\boxed{a = 11}$

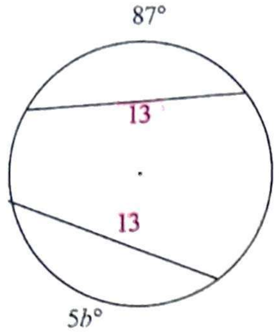
2)



$5x = 3x + 54$
$-3x \quad -3x$
$\frac{\cancel{2}x}{\cancel{2}} = \frac{54}{2}$
$\boxed{x = 27}$

You try #3 – 4! Find each variable.

3)



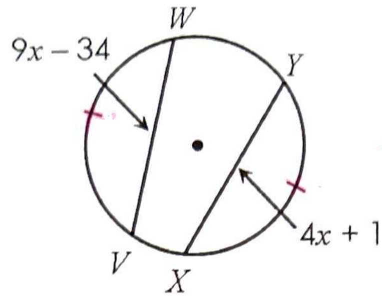
$5b = 87$

$\frac{5b}{5} = \frac{87}{5}$

$\boxed{b = 17.4}$

or $\frac{87}{5}$

4)



$9x - 34 = 4x + 1$

$-4x \qquad -4x$

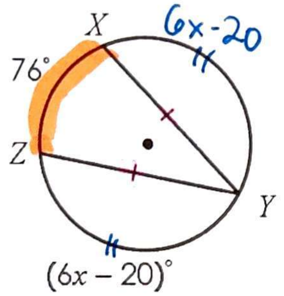
$5x - 34 = 1$

$+34 \quad +34$

$5x = 35$

$\boxed{x = 7}$

5) Find x.



$76 + 6x - 20 + 6x - 20 = 360$

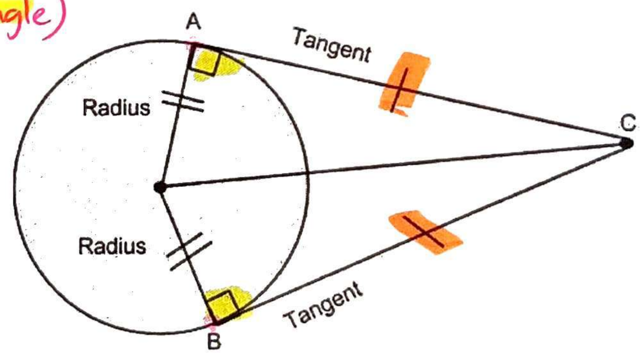
$12x + 36 = 360$

$\frac{12x}{12} = \frac{324}{12}$

$\boxed{x = 27}$

Tangent to a Circle	**A tangent segment will intersect a circle at exactly** one point**.**
Tangent Segments and Radii	(makes a 90° angle) **A tangent segment is always** perpendicular **to the radius drawn from the center of the circle to the point of tangency.**
Tangents Drawn from the Same Point	**Two tangent segments drawn from the same external point are** congruent **to each other.** $\overline{AC} \cong \overline{BC}$

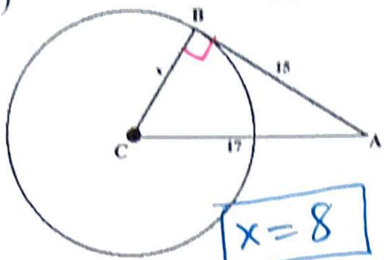
A & B are points on the line and points on the circle



Demonstration of the relationship between tangent segments and radii:
https://www.geogebra.org/m/xbAwK5Pd

For #6–11: Find the variable or segment. Assume that segments that appear to be tangent are tangent.

6) 8-15-17

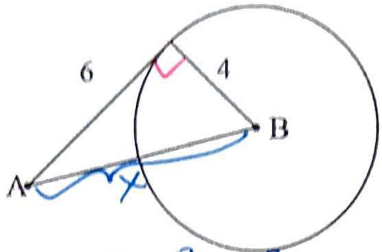


$\boxed{x=8}$

or

$x^2+15^2=17^2$

7) Find AB.



$4^2+6^2=x^2$

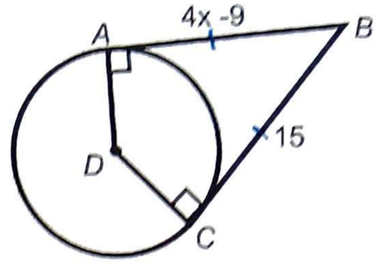
$16+36=x^2$

$\sqrt{52}=\sqrt{x^2}$

$x=\sqrt{52}=\boxed{2\sqrt{13}}$

2, 26, 2, 13

8)



$4x-9=15$

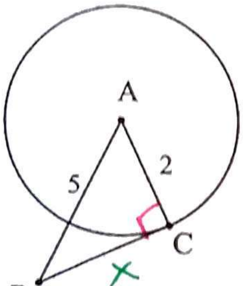
$+9 \quad +9$

$\frac{4x}{4}=\frac{24}{4}$

$\boxed{x=6}$

You try #9-11!

9) Find BC.



$2^2+x^2=5^2$

$4+x^2=25$

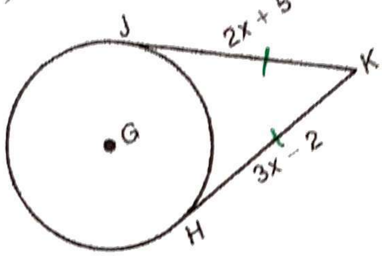
$-4 \quad -4$

$\sqrt{x^2}=\sqrt{21}$

3, 7

$\boxed{x=\sqrt{21}}$

10)



$2x+5=3x-2$

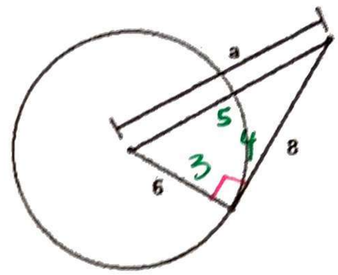
$-2x \quad -2x$

$5=x-2$

$+2 \quad +2$

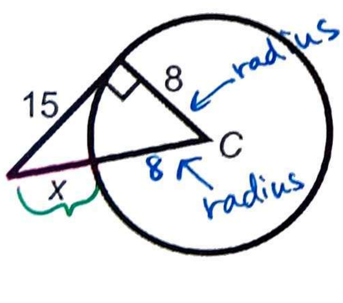
$\boxed{x=7}$

11)



$\boxed{a=10}$

Challenge! 12) Find *x*.



8-15-17

$8+x=17$

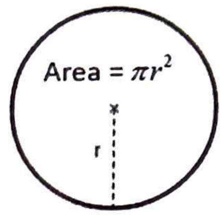
$-8 \quad -8$

$\boxed{x=9}$

10.4 Notes: Area of Circles and Sectors

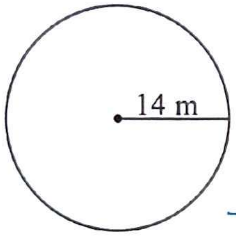
Objectives:

- **Students will be able to find the area of circles and sectors.**
- **Students will be able to find the area of shaded regions.**

Area of a Circle	**Reminder: the area of a circle can be found by using:** $A = \pi r^2$ pi = 3.14...	r = radius **Area of Circle**  diameter is twice the radius $d = 2r$ or $r = \frac{1}{2}d$

For #1–2: Find the area for each circle shown. Write your answers in terms of pi.

1)

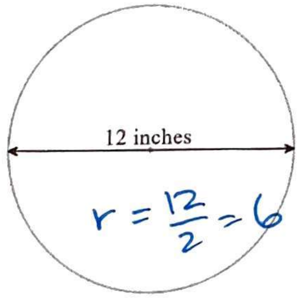


$r = 14m$

$A = \pi(14m)^2$

$\boxed{A = 196\pi\ m^2}$

2)



$A = \pi r^2$

$= \pi(6in)^2$

$= \boxed{36\pi\ in^2}$

3) A circle has an area of $49\pi\ in^2$. Find the length of the radius and diameter of the circle.

$A = \pi r^2$

$\frac{49\cancel{\pi}\ in^2}{\cancel{\pi}} = \frac{\cancel{\pi} r^2}{\cancel{\pi}}$

$\sqrt{49 in^2} = \sqrt{r^2}$

$\boxed{7in = r}$

$7 \cdot 2$

$\boxed{d = 14\ in.}$

You try! 4) A circle has an area of $121\pi\ cm^2$. Find the length of the radius of the circle.

$A = \pi r^2$

$\frac{121\cancel{\pi}\ cm^2}{\cancel{\pi}} = \frac{\cancel{\pi} r^2}{\cancel{\pi}}$

$\sqrt{121\ cm^2} = \sqrt{r^2}$

$\boxed{r = 11\ cm}$

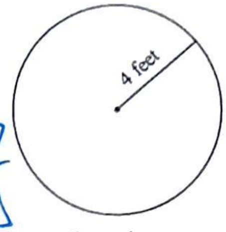
Exploration: Amy is buying a rug with a radius of 4 feet, as shown.

A) What is the area of the rug?

$r = 4\text{ ft.}$

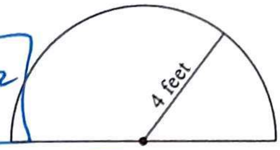
$A = \pi r^2$

$= \pi (4\text{ft.})^2 = \boxed{16\pi \text{ ft}^2}$



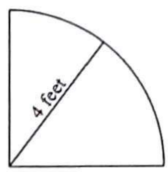
B) Amy finds another rug with the same radius but that is exactly half of a circle (called a semi-circle). What is the area of this rug?

half of the rug $\rightarrow \dfrac{16\pi \text{ ft}^2}{2} = \boxed{8\pi \text{ ft}^2}$

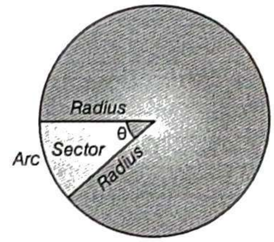


C) Amy now finds a rug with the same radius, but it is only a quarter of a circle. What is the area of this rug?

a quarter $\rightarrow \dfrac{16\pi \text{ ft}^2}{4} = \boxed{4\pi \text{ ft}^2}$

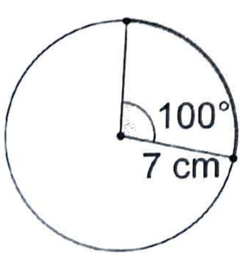


or $\dfrac{8\pi \text{ ft}^2}{2} = \boxed{4\pi \text{ ft}^2}$

Sector of a Circle	**A sector of a circle is a pie-shaped slice of a circle. It is between two** radii (plural of radius) **and an arc of a circle.**	
Area of a Sector	**The area of a sector can be found by finding a** fraction **of the area of a circle.**	$A = \dfrac{m}{360} \cdot \pi r^2$ m: the measure of the arc $\dfrac{m}{360}$: the fraction of the circle

For #5–6: Find the area of each sector, to the nearest tenth.

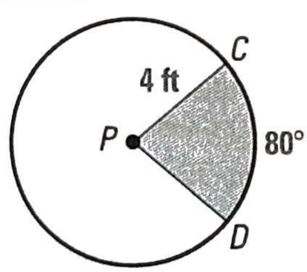
5)



$A = \dfrac{100}{360} \cdot \pi (7\text{cm})^2$

$13.6\pi = \boxed{42.8 \text{ cm}^2}$

6)

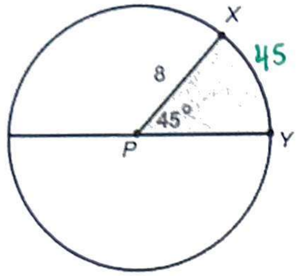


$A = \dfrac{80}{360} \pi (4)^2$

$\boxed{11.2 \text{ ft}^2}$

You try #7–8! Find the area of each sector, to the nearest tenth.

7)

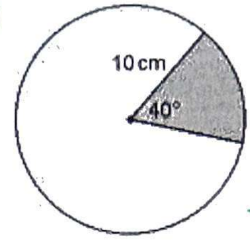


45

$A = \frac{45}{360} \cdot \pi \cdot (8)^2$

$\boxed{A = 25.1}$

8)

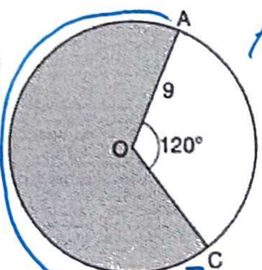


$A = \frac{40}{360} \cdot \pi (10\,cm)^2$

$\boxed{A = 34.9\ cm^2}$

For #9–10: Find the area of each shaded sector, to the nearest tenth.

9)



240°

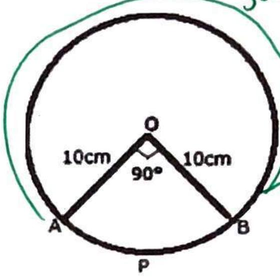
$360 - 120 = 240$

$A = \frac{240}{360} \cdot \pi \cdot 9^2$

$\frac{240 \cdot 81 \cdot \pi}{360}$

$\boxed{A = 169.6}$

You try! 10)



$360 - 90 = 270$

$A = \frac{270}{360} \cdot \pi \cdot (10\,cm)^2$

$A = \frac{270 \cdot 100 \cdot \pi}{360}$

$\boxed{A = 235.6\ cm^2}$

11) A sector has a measure of 60 degrees, and a radius of 9. Find the area of the sector *as an exact answer in terms of pi.*

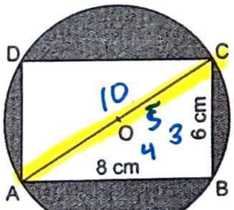
$A = \frac{60}{360} \cdot \pi \cdot 9^2 = \frac{60 \cdot 81 \cdot \pi}{360} = \cancel{42.4\ units^2}$

$\boxed{13.5\pi}$ $units^2$

No rounded decimals

For #12 – 13: Give an *exact answer in terms of pi.*

12) Find the area of the shaded region.



$\overline{AC}$ is diameter

$\overline{AC} = 10$

3-4-5
6-8-10 ×2

Area of circle

πr^2

$\pi (5)^2$

25π

d = 10
r = 5

Area of rectangle

$A = bh$

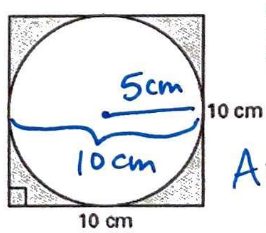
$A = (8cm)(6cm)$

$= 48\,cm^2$

Shaded area is Circle − rectangle

$\boxed{(25\pi - 48)\ cm^2}$

13) Find the area of the shaded region.



Shaded is square − circle

Area of square

$A = 10^2$

$A = 100\,cm^2$

Area of circle

$r = 5\,cm$

$A = \pi r^2$

$\pi 5^2$

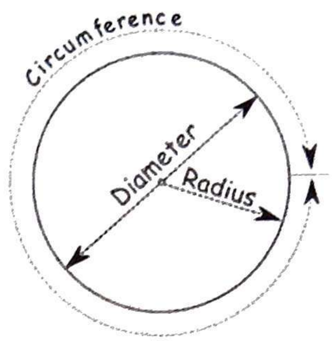
25π

$\boxed{(100 - 25\pi)\ cm^2}$

10.5 Notes: Circumference and Arc Length

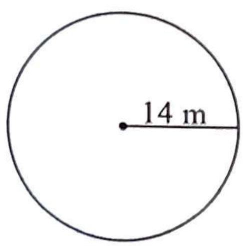
Objectives:

- **Students will be able to use the circumference of circles to solve problems.**
- **Students will be able to find the length of an arc.**

Circumference of a Circle	**Reminder: the circumference of a circle can by found by using:** $2r=d$ $C = 2\pi r$ or $C = d\pi$ r=radius d=diameter	

For #1–2: Find the circumference of each circle shown. Write your answers in terms of pi.

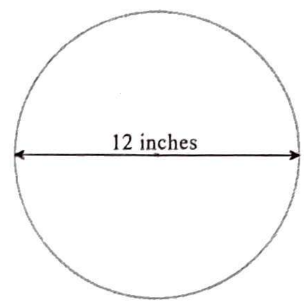
1)



$C = 2\pi r$
$= 2\pi(14m)$
$= \boxed{28\pi\ m.}$

$r = 14m$
$d = 28\ m$

2)



$d = 12in$
$r = 6in$

$C = d\pi$
$= \boxed{12\pi\ in.}$

3) A circle has a circumference of 36π *in*. Find the area of the circle.

$C = d\pi$

$\frac{36\cancel{\pi}\ in}{\cancel{\pi}} = \frac{d\cancel{\pi}}{\cancel{\pi}}$

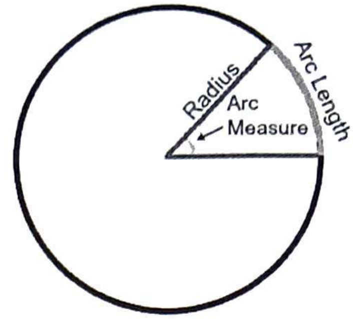
$36in = d$ $\quad r = \frac{36}{2} = 18in$

$A = \pi r^2$
$= \pi(18in)^2$
$= \boxed{324\pi\ in^2}$

You try! 4) A circle has a circumference of 12π *m*. Find the area of the circle.

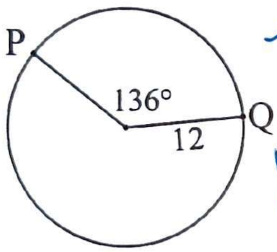
$C = d\pi$
$12\cancel{\pi}m = d\cancel{\pi}$
$d = 12m$
$r = 6m$

$A = \pi r^2$
$= \pi(6m)^2$
$= \boxed{36\pi\ m^2}$

Length of an Arc	**The length of an arc is a distance of a curve between two points on a circle. It is part of the** circumference **of a circle.** Note: The length of an arc is NOT the same as the measure of an arc.	
Arc Length Formula	**The length of an arc (arc length) can be found by finding a** fraction **of the circumference of a circle.**	$\ell = \frac{m}{360} \cdot$ circumference $\ell = \frac{m}{360} \cdot 2\pi r$ or $\ell = \frac{m}{360} \cdot d\pi$

For #5–8: Find the length of $\overset{\frown}{PQ}$. Round answer to the nearest tenth.

5)

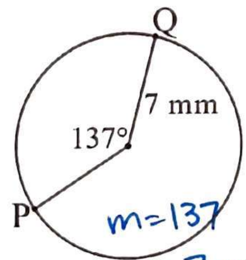


$\ell = \frac{m}{360} \cdot 2\pi r$

$m = 136$
$r = 12$

$\ell = \frac{136 \cdot 2 \cdot \pi \cdot 12}{360} = \boxed{28.5}$

6)



$\ell = \frac{137}{360} \cdot 2 \cdot \pi \cdot 7\text{mm}$

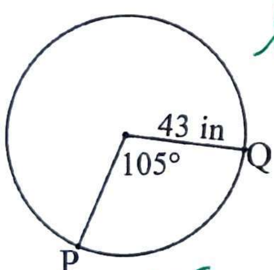
$\frac{137 \cdot 2 \cdot 7 \cdot \pi}{360}$

$m = 137$
$r = 7\text{mm}.$

$\boxed{\ell = 16.7\text{ mm}}$

You try #7 – 8!

7)

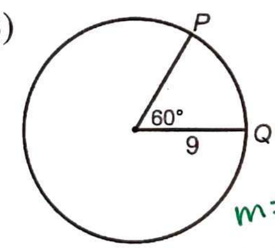


$\ell = \frac{105}{360} \cdot 2 \cdot \pi \cdot 43\text{in}$

$\boxed{\ell = 78.8\text{ in}}$

$m = 105$
$r = 43$

8)

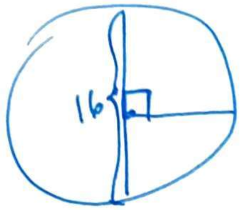


$\ell = \frac{60}{360} \cdot 2 \cdot \pi \cdot 9$

$m = 60$
$r = 9$

$= \boxed{9.4 \text{ units}}$

9) An arc has a measure of 90 degrees, and a *diameter* of 16. Find the length of the arc *as an exact answer in terms of pi.*



$m = 90$
$d = 16$
$r = 8$

$l = \frac{90}{360} \cdot 2 \cdot \pi \cdot 8$ or $\frac{9\cancel{0}}{36\cancel{0}} \cdot 16 \cdot \pi$

$\frac{9 \cdot 16}{36} \pi = \frac{144}{36} \pi$

$\boxed{l = 4\pi \text{ units}}$

You try! 10) An arc has a measure of 60 degrees, and a *radius* of 9. Find the length of the arc *as an exact answer in terms of pi.*

$m = 60$
$r = 9$
$d = 18$

$l = \frac{6\cancel{0}}{36\cancel{0}} \cdot 18\pi$

$\frac{1}{\cancel{6}} \cdot \overset{3}{\cancel{18}}\pi = \boxed{3\pi \text{ units}}$

11) An arc has a length of 10π cm and a radius of 20 cm. Find the measure of the arc.

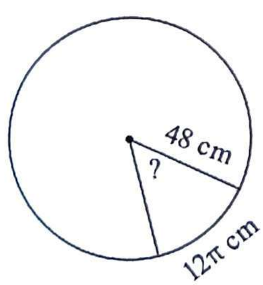
$l = 10\pi$ cm $\qquad r = 20$ cm

$l = \frac{m}{360} \cdot 2 \cdot \pi \cdot r$

$360 \cdot 10\pi \cancel{cm} = \frac{m}{\cancel{360}} \cdot 2 \cdot \pi \cdot 20 \cancel{cm} \cdot \cancel{360}$

$\frac{360\cancel{0}\cancel{\pi}}{4\cancel{0}} = \frac{\cancel{40}\cancel{\pi} m}{\cancel{40}}$ $\qquad \boxed{m = 90°}$

You try! 12) An arc has a length of 12π cm and a radius of 48 cm. Find the measure of the arc.



$l = 12\pi$ cm $\qquad r = 48$ cm

$l = \frac{m}{360} \cdot 2 \cdot \pi \cdot r$

$12\cancel{\pi} cm = \frac{m}{360} \cdot 2\cancel{\pi} \cdot 48cm$

$360 \cdot 12 = \frac{m \cdot 96}{\cancel{360}} \cdot \cancel{360}$

$\frac{4320 \cancel{cm}}{96 \cancel{cm}} = \frac{96m}{96}$

$\boxed{m = 45°}$