9.1 Notes: Angles of Quadrilaterals and Polygons
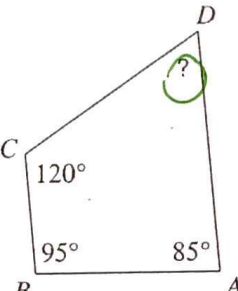
Objectives:
- Students will be able to find missing angles in a quadrilateral.
- Students will be able to find the sum of the angles in a polygon.

Exploration: Use the following **TWO** links: https://www.geogebra.org/m/XjSKUQBz and https://www.geogebra.org/m/xwbvZyhv. Move the vertices of the quadrilateral around and observe what happens to the angles. Make a conjecture about the sum of the angles in a quadrilateral.
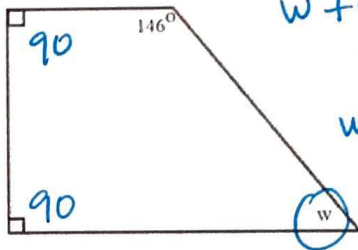
Sum of the Angles in a Quadrilateral	The sum of the angles of a quadrilateral is always ___$360°$___.	

For #1–4: Find the missing angle in each quadrilateral.

1)
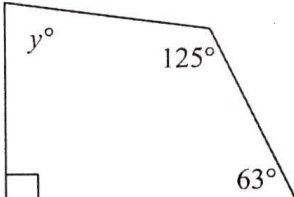
$? + 85 + 95 + 120 = 360$

$? + 300 = 360$
$\quad -300 \quad -300$
$\boxed{? = 60°}$

2)

$w + 90 + 90 + 146 = 360$

$w + 326 = 360$
$\quad -326 \quad -326$
$\boxed{w = 34°}$

You try #3 – 4!

3)

$y = 82°$
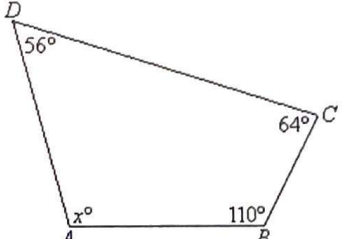
4)

$x = 130°$

For #5–8: Find the value of the variable.

5)

S →————→ T
70° 23x − 5
110° 14x
R ←———— U
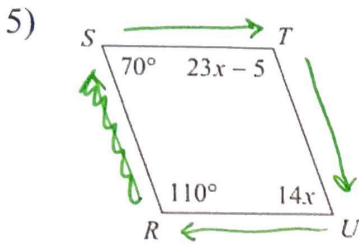
$70 + 23x - 5 + 14x + 110 = 360$

$37x + 175 = 360$
$\quad\ -175\quad -175$
$\dfrac{37x}{37} = \dfrac{185}{37}$ $\boxed{x = 5}$

6)

W
90
11x − 1   V
75° 12x + 10
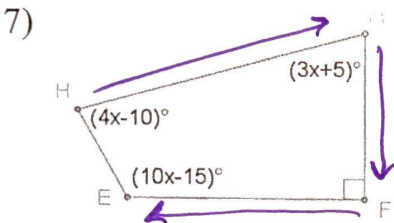X ←——— U
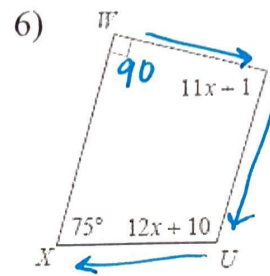
$90 + 11x + 1 + 12x + 10 + 75 = 360$

$23x + 176 = 360$
$\quad\ -176\quad -176$
$\dfrac{23x}{23} = \dfrac{184}{23}$

$\boxed{x = 8}$

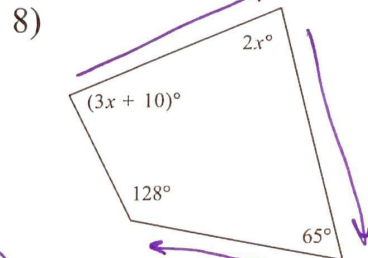**You try #7 – 8! Round to one decimal place, if needed.**

7)

H
(3x+5)°
(4x-10)°
(10x-15)°
E ←———— F

$4x - 10 + 3x + 5 + 90 + 10x - 15 = 360$

$17x + 70 = 360$
$\quad\ -70\quad\ -70$
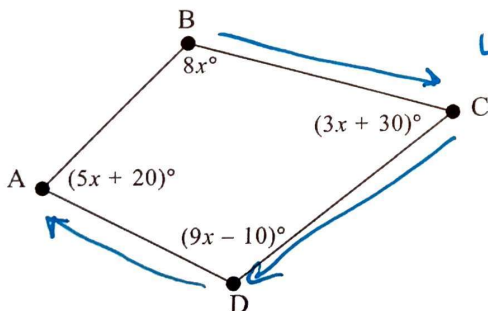$\dfrac{17x}{17} = \dfrac{290}{17}$ $\boxed{x = 17.1}$

8)

2x°
(3x + 10)°
128°
65°

$3x + 10 + 2x + 65 + 128 = 360$

$\boxed{x = 31.4}$

9) Find the measure of the largest angle in the quadrilateral shown below.

B
8x°
(3x + 30)°   C
A   (5x + 20)°
(9x − 10)°
D

$8x + 3x + 30 + 9x - 10 + 5x + 20 = 360$

$25x + 40 = 360$
$\quad\ -40\quad -40$
$\dfrac{25x}{25} = \dfrac{320}{25}$
$x = 12.8$

$\angle B = 8(12.8) = 102.4$
$\angle C = 3(12.8) + 30 = 68.4$
$\boxed{\angle D = 9(12.8) - 10 = 105.2}$
$\angle A = 5(12.8) + 20 = 84$

10) All four angles of a quadrilateral are congruent to each other. Find the measure of each angle in the quadrilateral.

$\angle 1 = \angle 2 = \angle 3 = \angle 4 = x$

$x + x + x + x = 360$

$\dfrac{4x}{4} = \dfrac{360}{4}$ $\boxed{x = 90°}$

2

Sum of the Angles in a Polygon	The sum of the angles of a polygon can be found by using the formula $\underline{180(n-2)}$, where n is the number of sides.

For #11 – 16, find the sum of the angles in each polygon.

11) octagon $= 8$

$$180(8-2) =$$

$$\boxed{1080°}$$

12) hexagon $= 6$

$$180(6-2)$$

$$\boxed{720°}$$

13) nonagon $= 9$

$$180(9-2)$$

$$\boxed{1260°}$$

You try #14 – 16!

14) pentagon $= 5$

$$180(5-2)$$

$$\boxed{540°}$$

15) decagon $= 10$

$$180(10-2)$$

$$\boxed{1440°}$$

16) quadrilateral $= 4$

$$180(4-2)$$

$$\boxed{360°}$$

The Measure of One Interior Angle of a Regular Polygon	The measure of one interior angle of a regular polygon can be found by using the formula $\dfrac{180(n-2)}{n}$, where n is the number of sides.

17) Assume all the angles of a hexagon are congruent (the hexagon is *regular*). Find the measure of *one* interior angle of the hexagon.

$$\frac{180(6-2)}{6} = \frac{720°}{6} = \boxed{120°}$$

You Try!

18) Find the measure of one angle of a regular pentagon.
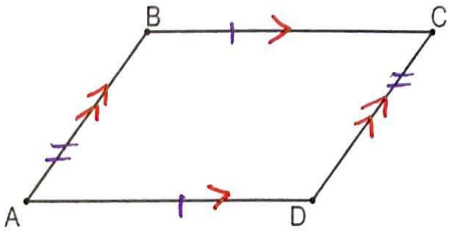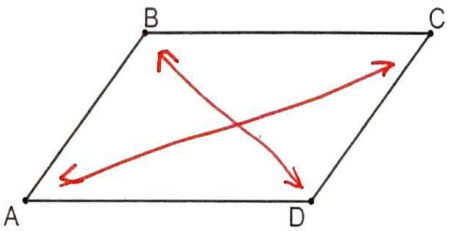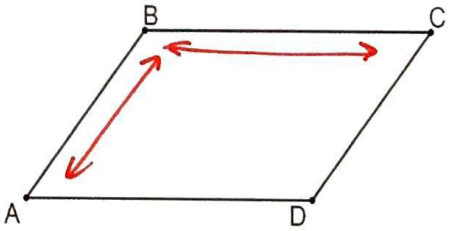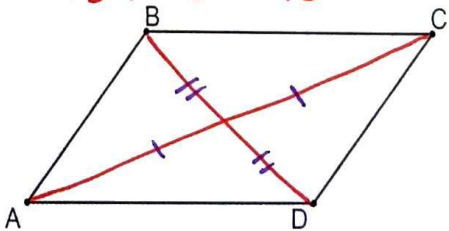
$$\boxed{108°}$$

9.2 Notes: Parallelograms

Objective:
- Students will be able to use properties of parallelograms to solve problems.

Exploration: **A parallelogram is a quadrilateral that has both pairs of opposite sides parallel.** Use the given link to fill in the properties of a parallelogram in the table below:
https://www.geogebra.org/m/amdzUqFu

Opposite Sides of a Parallelogram	The **opposite sides** of a parallelogram are ___Congruent___ and ___parallel___ .	
Opposite Angles of a Parallelogram	The **opposite angles** of a parallelogram are ___congruent___ .	
Consecutive Angles of a Parallelogram	The **consecutive angles** of a parallelogram are ___Supplementary___ .	$<A + <B = 180°$ $<B + <C = 180°$
Diagonals of a Parallelogram	The **diagonals** of a parallelogram ___bisect___ each other.	

1) Find the measure of the missing angles and the lengths of the missing sides.

$\angle A = \underline{48°}$

$\angle B = \underline{132°}$ $\angle B + \angle C = 180$
                  $\angle B + 48 = 180$

$\angle D = \underline{132°}$ $-48 \quad -48$

$\overline{BC} = \underline{12\ cm}$

$\overline{CD} = \underline{8\ cm}$

B   12 cm   C
132°   48°
8 cm   8cm
48°   132°
A   12 cm   D
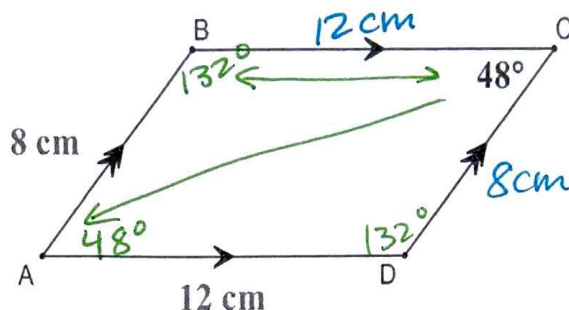
You Try!

2) Find the measure of the missing angles and the lengths of the missing sides.

$\angle A = \underline{50°}$

$\angle B = \underline{130°}$

$\angle C = \underline{50°}$

$\overline{AD} = \underline{14}$

$\overline{AB} = \underline{6}$

B   14   C
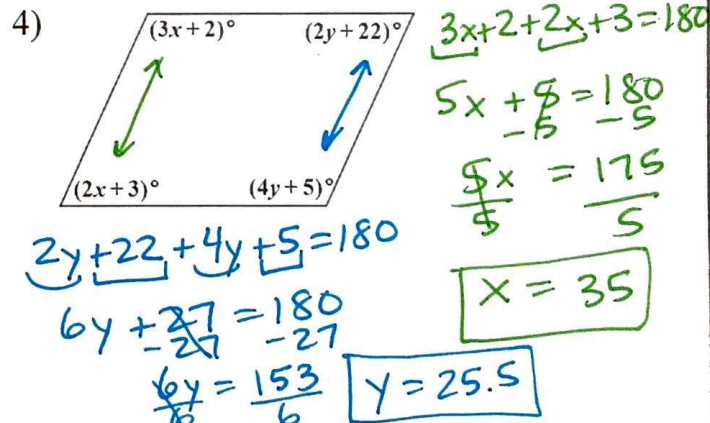   6
       130°
A   D
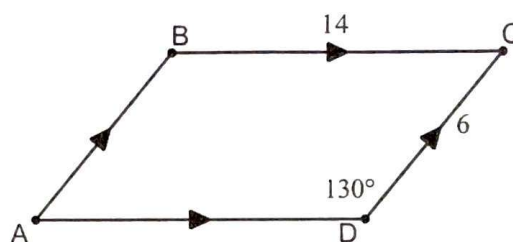
For #3–6: Given that each quadrilateral shown is a parallelogram, find the value of the variable(s). Use the properties that opposite angles are congruent and consecutive angles are supplementary.

3)

45° $a°$
    $5b°$

$\dfrac{45}{5} = \dfrac{5b}{5}$

$\boxed{b = 9}$

$45 + a = 180$
$-45 \qquad -45$
$\boxed{a = 135°}$

You try #5 – 6!

5)

$(4x)°$

$(3x + 14)°$

$4x = 3x + 14$
$-3x \quad -3x$

$x = 14$

$x = 14$

4)

$(3x+2)°$ $(2y+22)°$

$(2x+3)°$ $(4y+5)°$
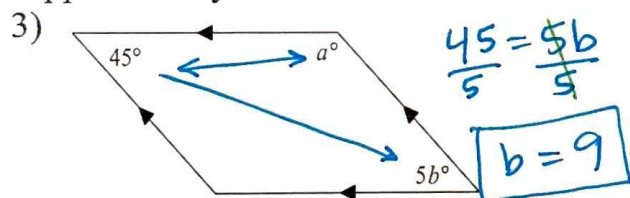
$3x+2+2x+3=180$

$5x + 5 = 180$
$\quad -5 \quad -5$

$\dfrac{5x}{5} = \dfrac{175}{5}$

$\boxed{x = 35}$

$2y+22+4y+5=180$

$6y + 27 = 180$
$\quad -27 \quad -27$

$\dfrac{6y}{6} = \dfrac{153}{6}$ $\boxed{y = 25.5}$

6)

A   B
$x°$ $y°$

Start

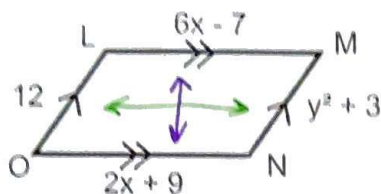$(z-15)°$ $(2z)°$
D   here   C

$x = 130$

$y = 50$

$z = 65$
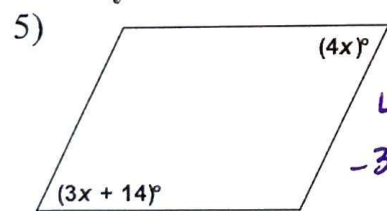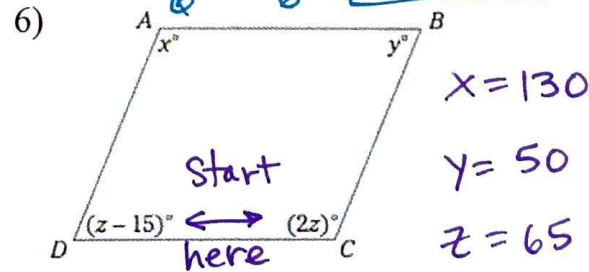
5

For #7 – 8: For each parallelogram shown below, find each variable. Use the property that states that opposite sides are congruent.

7)

L $6x - 7$ M

12 $y^2 + 3$

O $2x + 9$ N

$y^2 + 3 - 12$
$\quad - 3 \quad - 3$

$\sqrt{y^2} = \sqrt{9}$

$\boxed{y = 3}$

$6x - 7 = 2x + 9$
$-2x \qquad -2x$

$4x - 7 = 9$
$\quad +7 \quad +7$

$\dfrac{4x}{4} = \dfrac{16}{4}$ $\boxed{x = 4}$

You try! 8)

$20 + 4x$

$7y$ $6y + 9$

$x + 26$
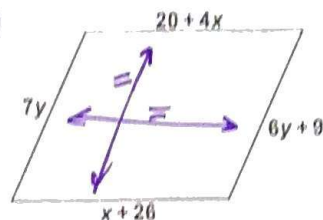
$x = 2$ $y = 9$

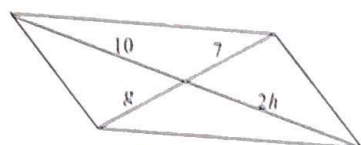**For #9–11: Find the value of each variable, given that the quadrilateral is a parallelogram.** Use the property that states the diagonals bisect each other.
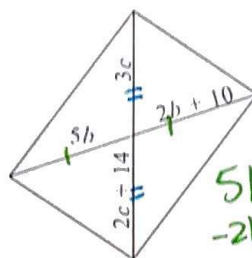
9)

10 7

g $2h$

$\boxed{g = 7}$ $\dfrac{2h}{2} = \dfrac{10}{2}$

$\boxed{h = 5}$

10)

$3c$

$5b$ $2b + 10$

$2c + 14$

$5b = 2b + 10$
$-2b \quad -2b$

$\dfrac{3b}{3} = \dfrac{10}{3}$

$3c = 2c + 14$
$-2c \quad -2c$

$\boxed{c = 14}$ $\boxed{b = \dfrac{10}{3}}$

$\boxed{\text{or } 3.\overline{3}}$

You try! 11)

$3x - 2$
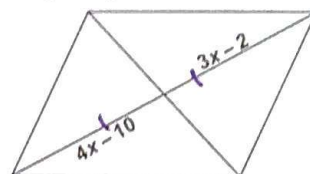
$4x - 10$

$4x - 10 = 3x - 2$

$\boxed{x = 8}$

12) What is the measure of $\angle F$ in Parallelogram $FGHJ$?
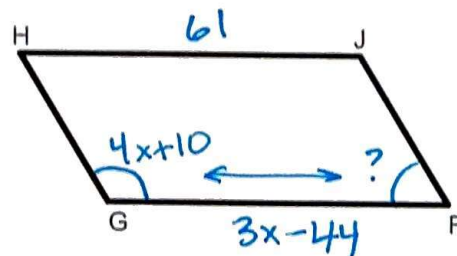
$\overline{FG} = 3x - 44$

$\overline{HJ} = 61$

$m\angle G = (4x + 10)°$

61

$4x + 10$?

G $3x - 44$ F

$4x + 10 + \angle F = 180$
↑
what is x?

$4(35) + 10 + \angle F = 180$
$140 + 10 + \angle F = 180$
$150 + \angle F = 180$
$-180 \qquad -150$
$\boxed{\angle F = 30}$

$3x - 44 = 61$
$\quad +44 \quad +44$
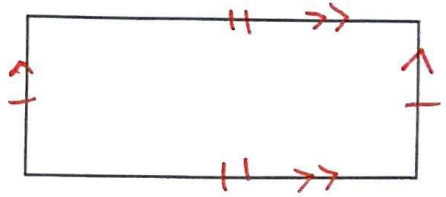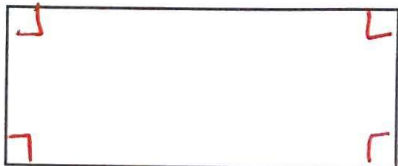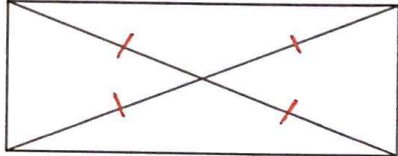$\dfrac{3x}{3} = \dfrac{105}{3}$
$\boxed{x = 35}$

9.3 Notes: Rectangles and Squares

Objectives:

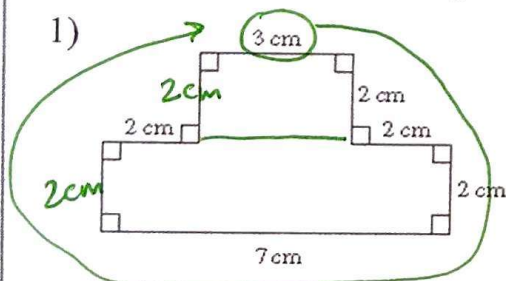- **Students will be able to solve problems using properties of rectangles & squares.**

Exploration: Use this link to fill in the properties of rectangles in the table below:
https://www.gcogebra.org/m/RCAX5KZa

Opposite Sides of a Rectangle	The opposite sides of a rectangle are ___parallel___ and ___congruent___.	
Angles of a Rectangle	The angles of a rectangle are each a ___90^0___ angle.	
Diagonals of a Rectangle	The diagonals of a rectangle are ___congruent___ and ___bisect___ each other.	

For #1–3: Find the perimeter of each shape shown below. Use the property that says opposite sides of a rectangle are congruent.
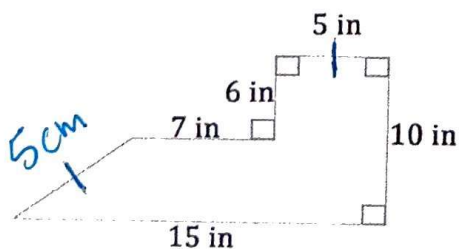
1)

3 cm
2 cm   2 cm
2 cm   2 cm
2 cm
2 cm
7 cm

$3 + 2 + 2 + 2 + 7 + 2 + 2 + 2 =$

22 cm

You Try! 2)

3.1 cm
7.1 { 3.4   3.4 cm
       3.8 cm
3.7   3.7 cm
6.9

$3.1 + 3.4 + 3.8 + 3.7 + 6.9 + 7.1 =$

61.3 cm

You Try! 3) Pay attention to congruent segments.

5 in
6 in
7 in   10 in
5 cm
15 in

$5 + 10 + 15 + 5 + 7 + 6 + 5 =$

32 in

7
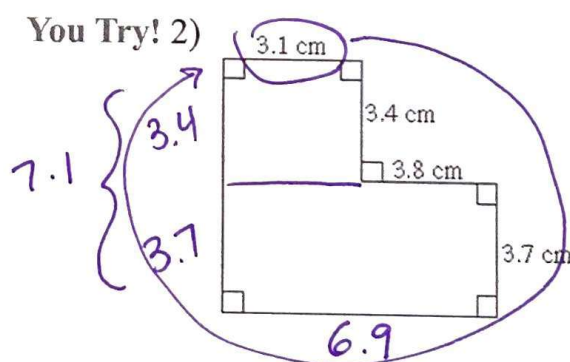
For #4 – 5: Find the missing variables for each rectangle. Use the properties that the diagonals of a rectangle are congruent and bisect each other.

4)
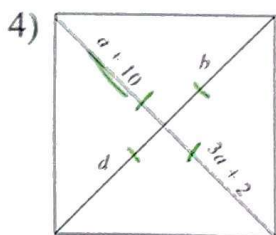
$a + 10 = 3a + 2$
$-a \qquad -a$

$10 = 2a + 2$
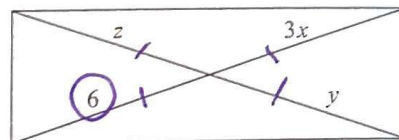$-2 \qquad -2$

$\dfrac{8}{2} = \dfrac{2a}{2}$

$\boxed{4 = a}$

$a + 10 =$
$4 + 10 = 14$
$\boxed{d = 14}$
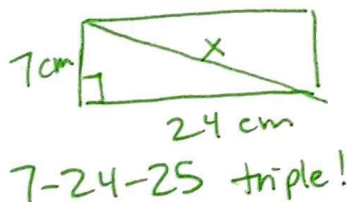$\boxed{b = 14}$

You try! 5)

$\boxed{y = 6}$
$\boxed{z = 6}$

$\dfrac{3x}{3} = \dfrac{6}{3}$

$\boxed{x = 2}$

6) A rectangle has a length of 7 cm and a width of 24 cm. Find the length of one diagonal. Hint: draw a diagram.
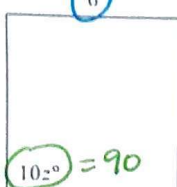
7cm, x, 24 cm

7-24-25 triple!
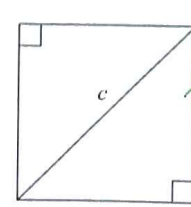
$7^2 + 24^2 = x^2$
$49 + 576 = x^2$
$\sqrt{625} = \sqrt{x^2}$
$\boxed{x = 25}$
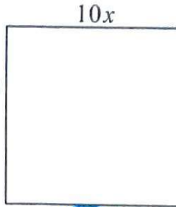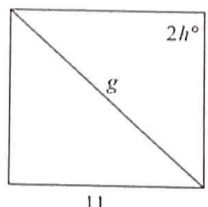
Sides of a Square	The sides of a square are all ___congruent___.	
Angles of a Square	The angles of a square are all ___90°___ angles.	
Diagonals of a Square	The diagonals of a square are ___congruent___ and ___perpendicular___ ___bisectors___ of each other.	

8
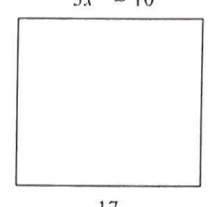
For #7–11: Find the missing variables in each square.

7)

6

w $2x$

$(10z°) = 90$

$3y + 1$

$\frac{2x = 6}{2 \quad 2}$

$\boxed{x = 3}$

$\boxed{w = 6}$

$\frac{10z = 90}{10 \quad 10}$

$\boxed{z = 9}$

You Try #10-11!

$3y + 1 = 6$
$\quad -1 \quad -1$

$\frac{3y}{3} = \frac{5}{3}$

$\boxed{y = \frac{5}{3}}$

8)

c (10)

b

$1-1-\sqrt{2}$
$10-10-10\sqrt{2}$
or
$10^2 + 10^2 = c^2$

$\boxed{b = 10}$

$\boxed{c = 10\sqrt{2}}$

9)

$10x$

$5y^2$

20

$\frac{10x = 20}{10 \quad 10}$

$\boxed{x = 2}$

$\frac{5y^2 = 20}{5 \quad 5}$

$\sqrt{y^2} = \sqrt{4}$

$\boxed{y = 2}$

10)

$2h°$

g

11

$\boxed{h = 45}$

$\boxed{g = 11\sqrt{2}}$

11)

$3x^2 - 10$

$5y$

17

$\boxed{x = 3}$
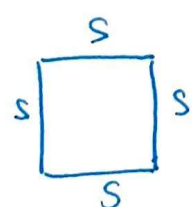
$\boxed{y = \frac{17}{5}}$

12) A square has a perimeter of 32 inches. Find the area of the square. (Reminder: $A = s^2$)

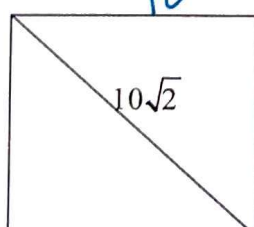s s s s

$\frac{4s}{4} = \frac{32}{4}$

$s = 8$

$A = s^2$
$A = 8^2$
$\boxed{A = 64 \ in^2}$

13) Find the perimeter of the square shown below.

10

$10\sqrt{2}$ 10

$1-1-\sqrt{2} \quad_{\times 10}$
$10-10-10\sqrt{2}$

$10 + 10 + 10 + 10 = \boxed{40}$

14) Given that a rectangle and a square both have a perimeter of 24 mm. If the length and width of the rectangle is 3 cm and 9 cm, then which has the larger area, the square or the rectangle?

S

$4S = 24$ mm

$$\frac{4S}{4} = \frac{24}{4}$$

$S = 6$ mm

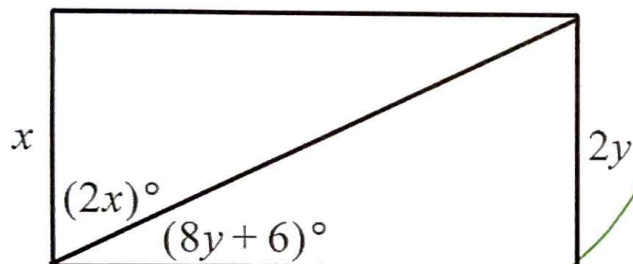$A = 27$ mm^2
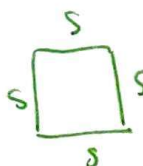
$A = 6^2$

$A = 36$ mm^2

Square has larger area

Challenge! Solve for the variables in the rectangle below.

x

$(2x)°$

$(8y+6)°$

$2y$

$2y = x$ — sides

$2x + 8y + 6 = 90°$ — angles

solve the system using substitution

$x = 2y$

$2(2y) + 8y + 6 = 90$

$4y + 8y + 6 = 90$

$12y + 6 = 90$

$-6 \quad -6$

$$\frac{12y}{12} = \frac{84}{12}$$

$y = 7$

Substitute again

$2(7) = x$

$14 = x$

9.4 Notes: Rhombi and Kites

Objectives:
- Students will be able to name regular polygons by the sides.
- Students will be able to find the area of a regular polygon.

Properties of a Rhombus		
	Sides	A rhombus is a parallelogram where all four sides are __congruent__ .
	Diagonals	The diagonals of a rhombus are __perpendicular__ __bisectors__ of each other.
	Diagonals and Angles	The diagonals of a rhombus __bisect__ the angles of the rhombus.
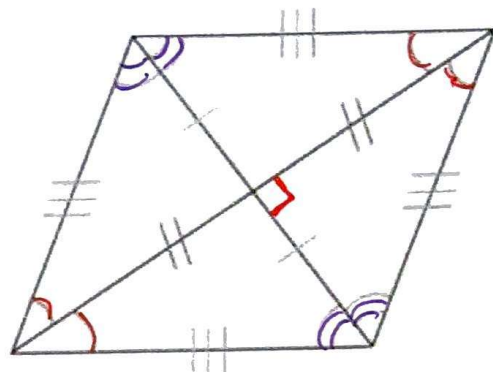
For #1-3: Find the variables in each rhombus.

1)

P   Q
2y + 3
S   5y – 6   R

$5y - 6 = 2y + 3$
$-2y \quad -2y$
$3y - 6 = 3$
$+6 \quad +6$
$\dfrac{3y}{3} = \dfrac{9}{3}$
$\boxed{y = 3}$

2)

5x – 24
3x – 2
y

$5x - 24 = 3x - 2$
$-3x \quad\quad -3x$
$2x - 24 = -2$
$+24 \quad +24$
$\dfrac{2x}{2} = \dfrac{22}{2}$
$\boxed{x = 11}$

$3(11) - 2$
$33 - 2$
31

$\boxed{y = 31}$

You try! 3)

2a + 10
4a – 8
b

$\boxed{a = 9}$

$\boxed{b = 28}$

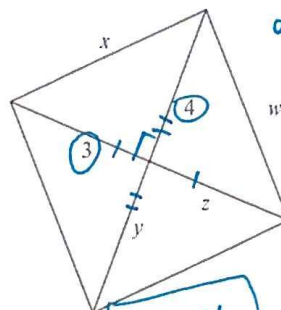**For #4-6: Find the missing variable in each rhombus.**

4)

$$\frac{3g}{3} = \frac{90}{3}$$

$$g = 30$$

$3g°$ $10h°$
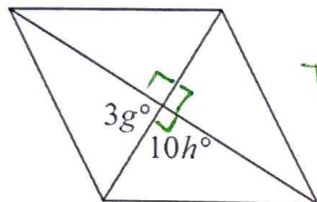
$$\frac{10h}{10} = \frac{90}{10}$$
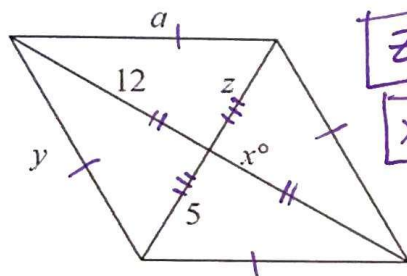
$$h = 9$$

5)

3-4-5 -triple
or $3^2 + 4^2 = x^2$

x ③ ④ w z y

$$x = 5$$
$$w = 5$$

$$y = 4$$
$$z = 3$$

You Try! 6)

a 12 z y $x°$ 5
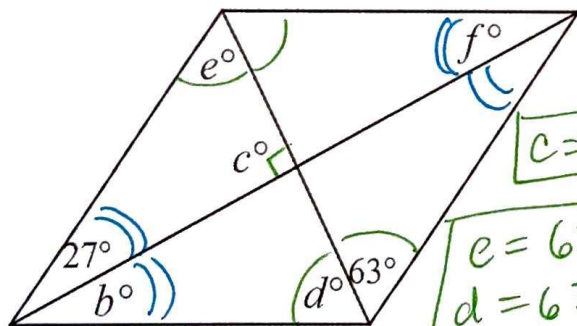
$$z = 5$$
$$x = 90$$
$$a = y$$

5-12-13 triple

$$a = 13$$
$$y = 13$$

or $5^2 + 12^2 = a^2$
$25 + 144 = a^2$
$\sqrt{169} = \sqrt{a^2}$
$a = 13$

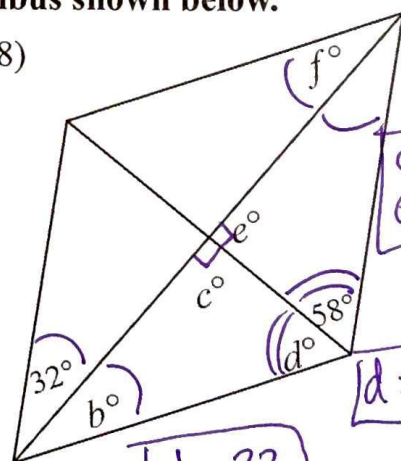**For #7-8: Find the measure of each variable for each rhombus shown below.**

7)

$e°$ $f°$ $c°$ 27° $b°$ $d°$ 63°

$$c = 90$$

$$e = 63$$
$$d = 63$$

$$b = 27$$
$$f = 27$$

You Try! 8)

$f°$ $e°$ $c°$ 58° 32° $b°$ $d°$

$$c = 90$$
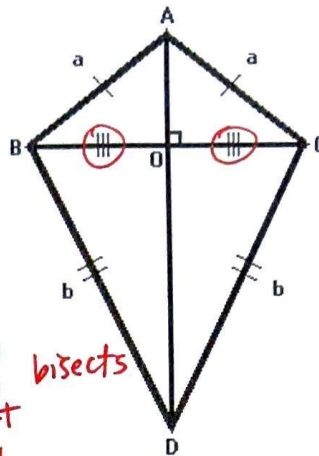$$e = 90$$

$$d = 58$$

$$b = 32$$
$$f = 32$$

9) **Find the perimeter of the rhombus shown below.**

7 $x = 25$ 24
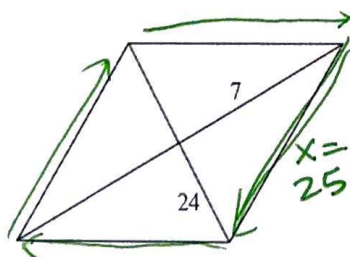
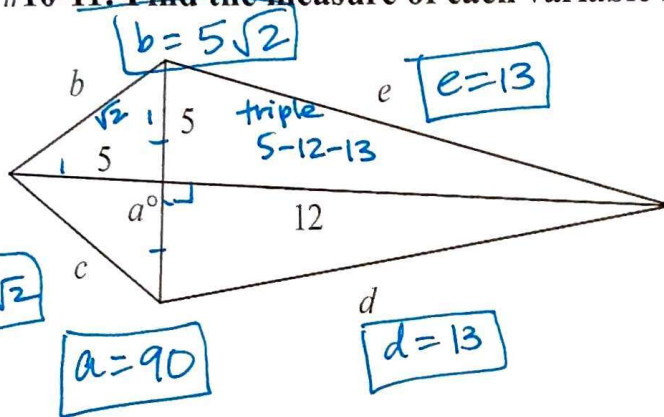P = distance around outside

$7^2 + 24^2 = x^2$ or 7-24-25 triple

$25 + 25 + 25 + 25 = 100$ units

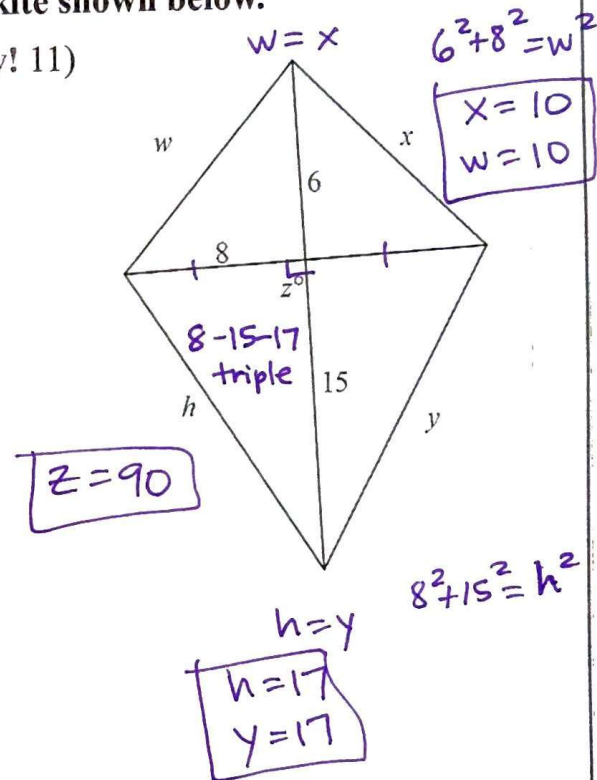|		A kite is a quadrilateral that has two pairs of consecutive sides that are _congruent_.	
Properties of a Kite	**Sides**		
	Diagonals	One diagonal of a kite is the _perpendicular bisector_ of the other.	the long diagonal bisects the short diagonal

Note: there are other properties about kites (specifically about the angles) that we are not studying this year.

For #10-11: Find the measure of each variable for each kite shown below.

10)

$b = 5\sqrt{2}$

$e = 13$

triple 5-12-13

$c = 5\sqrt{2}$

$a = 90$

$d = 13$

You Try! 11)

$w = x$

$6^2 + 8^2 = w^2$

$x = 10$

$w = 10$

8-15-17 triple

$z = 90$

$8^2 + 15^2 = h^2$

$h = y$

$h = 17$

$y = 17$

12) Find the perimeter of the kite shown below.

triple 3-4-5

$1 - \sqrt{3} - 2$

$5 + 5 + 8 + 8 =$

26 units