

12.5 Notes: Map Coloring

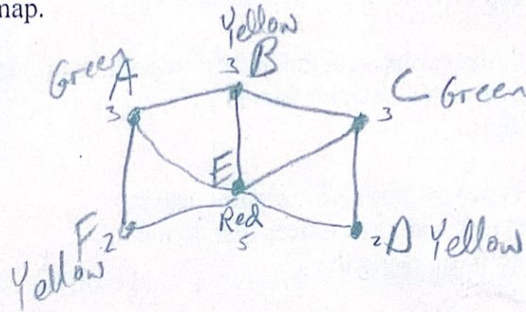
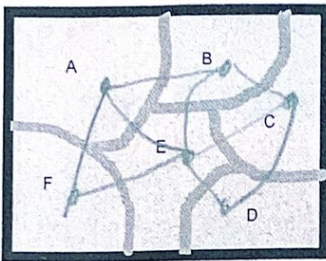
Note: This section is not from your textbook.

Objectives:

- Can you create a diagram to model a coloring situation?
- Can you find the fewest possible number of items needed for a coloring situation?

For centuries, people have wondered what minimum number of colors are required to shade countries on a map so that countries with a common border will not be shaded the same color.

Consider the map shown below. Using a vertex for each region and an edge to represent a shared border, draw a diagram representing the map.



3 colors

To find the minimum number of colors needed for this graph, start by finding the degree of each vertex.

Degree of a vertex: the number of edges using that vertex.

1. Starting with the vertex with the highest degree, assign a color to that vertex.
2. Are there any vertices that are NOT adjacent to the vertex from #1? If so, assign the same color to those vertices, making sure that they are not adjacent to each other either.
 - a. Note: If you have multiple options, start with the vertices with the highest degree.
3. Continue steps 1 and 2, starting with the vertex with the highest degree that has not already been assigned a color.
4. When all vertices have been assigned a color, count the number of colors used. This value is your final answer.

Map Coloring: Avoiding Scheduling Conflicts

The table shows various sports at a school, along with some female students who participate in them.

The coaches want to organize end-of-the-year banquets for each sport. What is the fewest number of nights that will be needed if they are scheduled so that each student shown does not have a scheduling conflict?

| Student | Cross-County | Softball | Basketball | Volleyball |
|---------|--------------|----------|------------|------------|
| Amy | X | | X | X |
| Heather | X | X | | |
| Sara | | | X | X |
| Amber | X | X | X | |

Step 1: Create a graph. Use a vertex for each sport. An edge indicates a conflict (a girl in both sports).

Step 2: Start with the vertex with the highest degree. Assign a day to that vertex and any that do not share an edge with it.

Step 3: Go to a vertex with the next highest degree. Assign a new day to it and any vertices that do not share an edge with it (or each other).

Step 4: Continue until all sports are assigned a day.

