12.4 Notes: Trees

Objectives

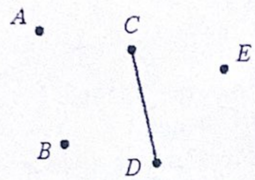
1. **Can you decide if a graph is a tree and justify your reasoning?**
2. **Can you use the properties of a tree?**
3. **Can you find a spanning tree for a connected graph?**
4. **Can you discover the minimum spanning tree for a weighted graph?**

Definition: A *tree* is a graph that is connected and has no circuits. Some other ways to say this are: ✓make path thru every vertex once ✓smallest # of edges

- o A tree is a connected graph in which every edge is a bridge.
- o A tree is a connected graph with n vertices and $n - 1$ edges.

Example: Which of the following graphs are trees? Why or why not?

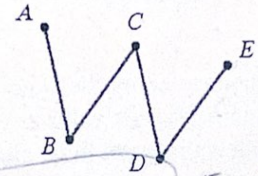
1)



No

not conn. (disconn)

2)

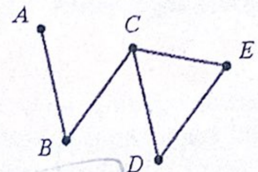


tree

5 vert. 4 edges (5-1)

every edge is bridge

3)



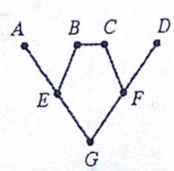
No has circuit CDEC

DE, CE, & CD not bridges

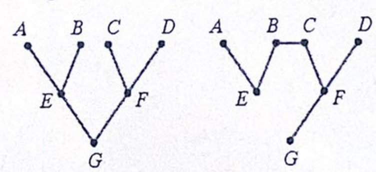
5 vert. & 5 edges

Vocabulary:

- A subgraph is a set of vertices and edges chosen from among those of the original graph.



Original graph



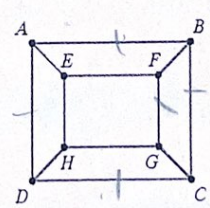
Two possible subgraphs

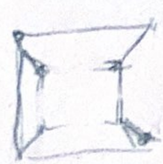
- A subgraph that contains all of a connected graph's vertices, is still connected, and contains no circuits is called a spanning tree.

Are the two subgraphs shown above spanning trees? Explain. Yes - no circuits
- 7 vert. & 6 edges
- every edge is a bridge

marker boards

Example: Find a spanning tree for the graph shown.



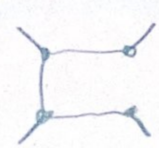


check: 8 vert. 5 ~~0~~ 7 edges

12 edges remove 5

break inner

remove outer edges



Fiber Optics

Vocabulary: The min spanning tree for a weighted graph is a spanning tree with the smallest possible total weight.

Kruskal's Algorithm to find a minimum spanning tree from a weighted graph.

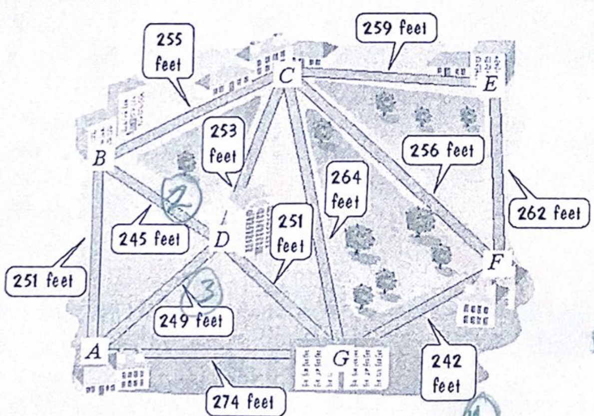
1. Find the edge with the Sm weight in the graph. If there is more than one, pick one at random. Darken (or highlight) that edge. Keep or discard?

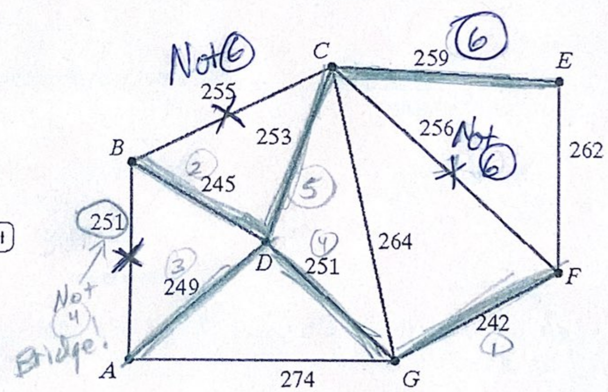
2. Find the next-smallest edge in the graph. If there is more than one, pick one at random. Darken (or highlight) that edge.

3. Find the next-smallest unmarked edge in the graph that does not create a darkened bridge. If there is more than one, pick one at random. Darken (or highlight) that edge.

4. Repeat step 3 until all vertices have been included ($n - 1$ edges). The darkened edges are the desired minimum spanning tree.

Example: Seven buildings on a college are connected by the sidewalks shown in the figure. The weighted graph represents buildings as vertices, sidewalks as edges, and sidewalk lengths as weights. A heavy snow has fallen and the sidewalks need to be cleared quickly. Campus decides to clear as little as possible and still ensure that students walking from building to building will be able to do so along cleared paths. Determine the shortest series of sidewalks to clear. What is the total length of the sidewalks that need to be cleared?





Not 4

Not 6, Not 6

7 vert.
Need 6 edges

242 + 245 + 249 + 251 + 253 + 259

The total minimum length of sidewalks to be cleared is 1499.

Useful in Fiber-Optics