12.1 Worksheet #11 & 12: Possible to do without retracing steps? Yes – one of them is a circuit, the other begins & ends in diff. places.

12.2 Notes: Euler Paths and Circuits

Pronounce "Oiler"

Objectives

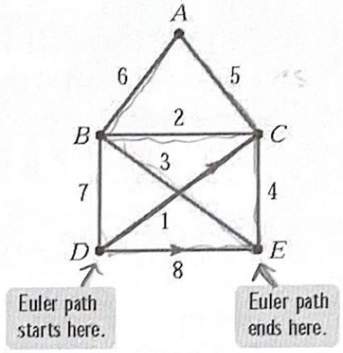
1. **Can you compare and contrast an Euler path and an Euler circuit?**
2. **Can you summarize Euler's Theorem?**
3. **Can you solve problems using Euler's Theorem?**
4. **Can you use Fleury's Algorithm to find possible Euler paths and Euler circuits?**

Vocabulary

❖ An Euler path is a path that travels through every edge of a graph once and only once. Each edge must be traveled and no edge can be retraced.

A graph with an Euler Path has 2 odd vertices and the rest are even vertices.

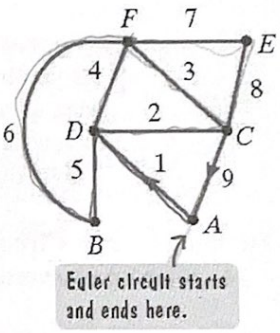
An Euler Path must start and end at the two odd vertices of the graph. Use your pencil to darken the edges and trace an Euler Path.



❖ An Euler circuit is a circuit that travels through every edge of a graph once and only once. Like all circuits, an Euler circuit must begin and end at the same vertex.

A graph with an Euler Circuit has all even vertices.

An Euler Circuit may start at any vertex. Use your pencil to trace an Euler Circuit starting and ending at vertex A.

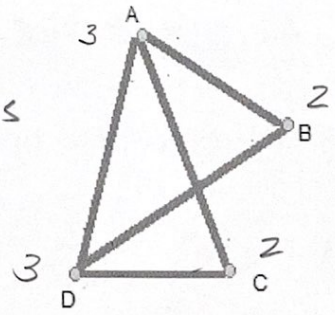


Euler's Theorem: The following are true for connected graphs…

1. If a graph has exactly 2 odd vertices, then it has at least one Euler path, but no Euler circuit.
2. If a graph has no odd vertices (all even vertices), it has at least one Euler circuit.
3. If a graph has more than two odd vertices, then it has no Euler paths and no Euler circuits.

Example:

❖ Explain why the graph in the figure has at least one Euler path. exactly 2 odd vertices



❖ Use trial and error to name one Euler path.

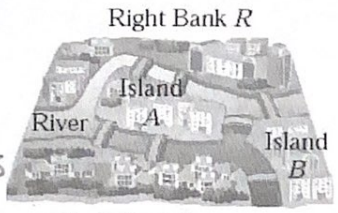
A, B, D, C, A, D

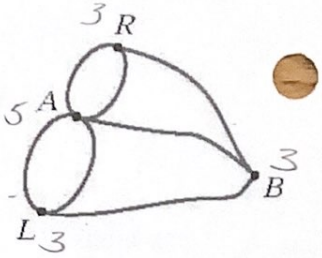
(A: odd; D: odd)

Example: Recall the graph that models the layout of Königsberg.

1) Use Euler's theorem to determine whether or not a person can walk across all of the bridges in Königsberg exactly one time without re-crossing any bridge. Euler Path

No: more than 2 odd vertices (#3)





2) Use Euler's Theorem to decide whether or not a person can start at one land mass of Königsberg, cross all bridges exactly once, and return to the original land mass. Euler Circuit

No: more than 2 odd vertices (#3), not all even (#2)

Fleury's Algorithm to find Euler Paths and Circuits

strategy!

To find an Euler Path:

1. Choose one of the two odd vertices as the starting point. The other odd vertex will be the ending point.

2. List the edges as you trace through the graph according to the following rules:
   - As you travel over an edge, draw hash marks to show that edge has been crossed. Label each edge with a consecutive number.
   - When faced with a choice of edges to trace, first choose any edge that is not a bridge. Travel over a bridge only if there is no alternative.

To find an Euler Circuit:

1. Choose any vertex as the starting point.

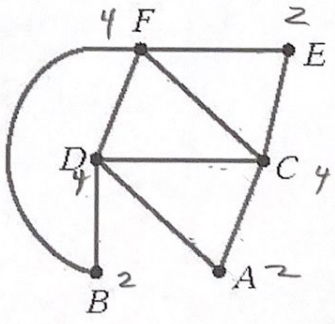
2. List the edges as you trace through the graph according to the following rules:
   a. As you travel over an edge, draw hash marks to show that edge has been crossed. Label each edge with a consecutive number.
   b. When faced with a choice of edges to trace, first choose any edge that is not a bridge.

3. The starting vertex is also the end point.

Example:

a) Explain why the graph shown has an Euler circuit.

All even vertices (all have even # degree)



b) Find one by using Fleury's Algorithm.

A, C, E, F, B, D, F, C, D, A

perimeter — zig-zag through interior

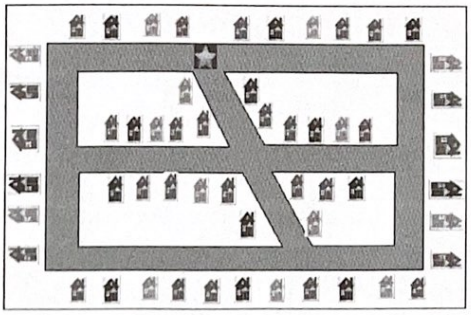
12.2 Notes continued: Euler Paths and Circuits

Objectives

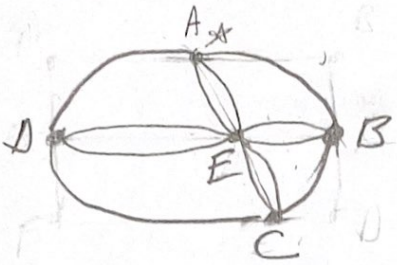
1. **Can you determine whether a situation is asking you to find an Euler Path or Circuit?**
2. **Can you use Fleury's Algorithm in application problems?**

Examples:

Use the graph shown for #1 – 5.



1) A mail carrier parks her truck at the intersection starred in the figure and then walks to deliver mail to each of the houses. The streets on the outside of the neighborhood have houses on side of the street only. Many interior streets have houses on both sides of the street. On those streets, the mail carrier must walk down the street twice, covering each side of the street separately. Draw a graph of this situation.



2) Is it possible for the mail carrier to start at the starred intersection, deliver mail to each house without retracing her route, and then return to the starred intersection? Explain your reasoning.

Euler Circuit? (All even vertices)

Yes: all vertices are even

3) Does this situation describe an Euler Path, an Euler Circuit, or neither or these?

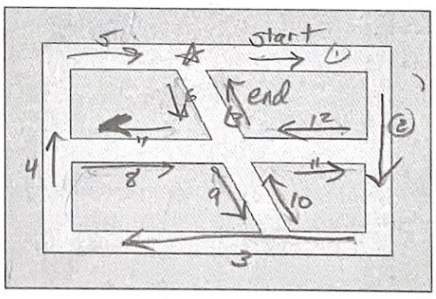
4) Use Fleury's Algorithm to list the route the mail carrier would take for the situation described in #2.

No bridges

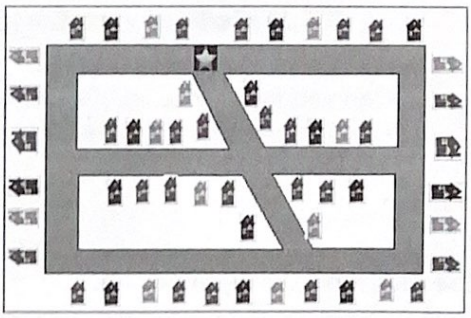
A B C D A E D E C E B E A

perimeter

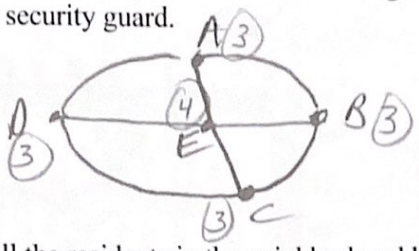
5) Use your answer from #4 to show the mail carrier the route she should follow on the map of the neighborhood. Be sure the route is clearly designated.



For # 6 – 9, use the map of the neighborhood shown.



6) A security guard patrols the streets of the neighborhood. Unlike the mail carrier, the guard is to walk down each street once, whether or not the street has houses on both sides. Draw a graph that models the neighborhood walked by the security guard.



7) Will the residents in the neighborhood be able to establish a route for the security guard so that each street is walked exactly once? If so, where would the security guard begin the walk? Explain your answer.

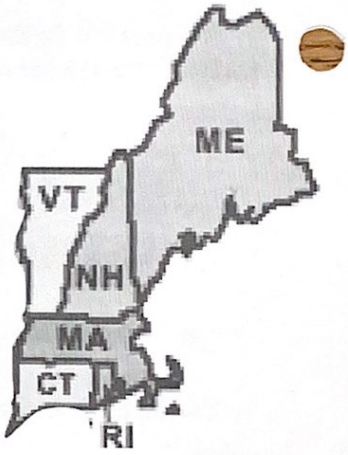
No – there are more than 2 odd vertices

8) Does the situation in #7 describe an Euler Path, an Euler Circuit, or neither of these?

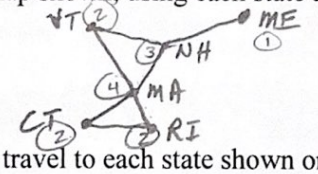
9) Use the graph you drew in #6 to show the route the security guard will need to walk, if this situation is possible.

N/A

For # 10 – 14, use the map shown.



10) Create a graph of the map shown, using each state as a vertex and each border as an edge.



11) A family would like to travel to each state shown on the map while only crossing each common state border exactly once. Does this describe an Euler Path, and Euler Circuit, or neither?

12) Use Euler's Theorem to explain why it is possible for the family to do travel in the way described in #11.

There are exactly 2 odd vertices

13) Use Fleury's Algorithm to find this route on the graph you drew in #10.

ME, NH, VT, MA, CT, RI, MA, NH

14) Use your answer from #13 to show the route on the map of the New England states shown.