**Prob/Stat/Discrete**
**Unit 12 Guided Notes**

Name_____

# 12.1: Graphs, Paths, and Circuits

## Objectives

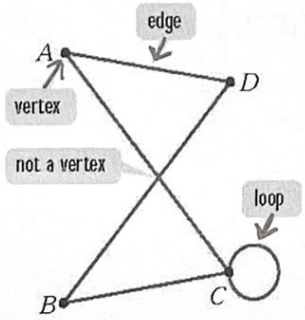
1. Can you model relationships using graphs?
2. Can you use the vocabulary of graph theory?

## VOCABULARY

Ex. A B C D

- ☐ Vertices: a finite set of points (singular is *vertex*)
- ☐ Line segments or curves, called edges.

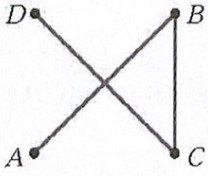
Graph

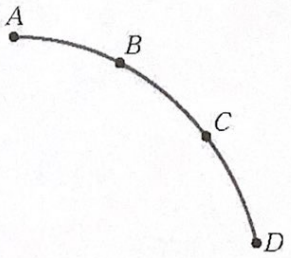



- ➢ An edge that starts and ends at the same vertex is called a loop.
- ➢ A graph consists of vertices and edges that start and end at vertices.

- ☐ Equivalent Graphs: graphs with the same vertices connected by the same number of edges.

**Example:** Are the two graphs shown equivalent? Explain your answer.




same vertices?
✓ A, B, C, D

connected in the same ways?
✓ Edges AB, BC, CD

Yes Same # of vertices, connected in the same way.

Note: Your HW answers might look different than the KEY but still be equivalent. ☺

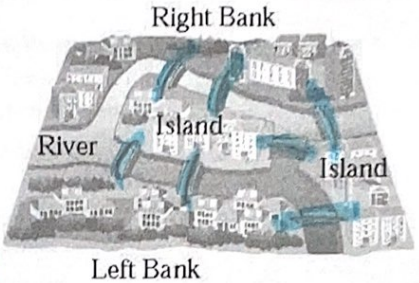
Different meanings to the edges, but all provide paths...

**Modeling Graphs:** We can use graphs to represent relationships in a variety of situations.

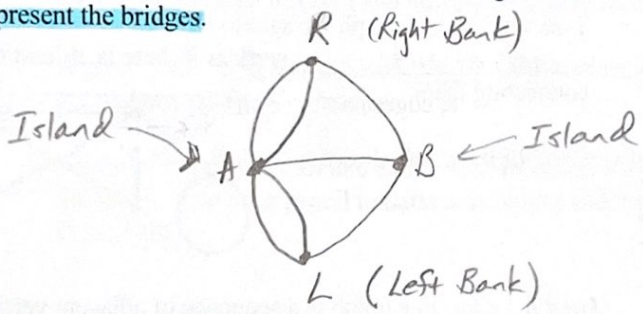
1. **Layout of a city**

Classic graph theory problem

**Example:** In the early 1700's, the city of Königsberg, Germany, was located on both banks and two islands of the Pregel River. The figure shows that the town's sections were connected by seven bridges.

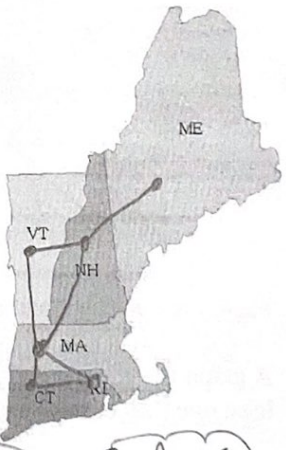


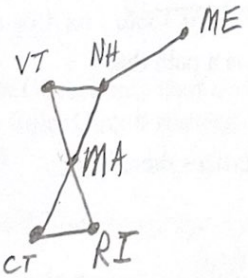

Draw a graph that models the layout of Königsberg. Use vertices to represent the land masses and edges to represent the bridges.




2. **Bordering Relationships**

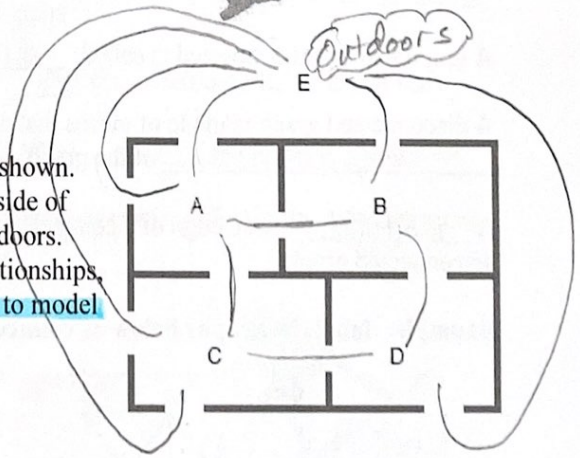
**Example:** The map of New England states are given. Draw a graph that models which New England states share a common border. Use vertices to represent the states and edges to represent common borders.

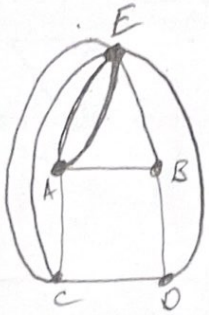



3. **Floor plan of a building**

**Example:** The floor plan of a four-room house is shown. The rooms are labeled A, B, C, and D, and the outside of the house is labeled as E. The openings represent doors. Draw a graph that representing the connecting relationships, using vertices as rooms and the outside, and edges to model the connecting doors.



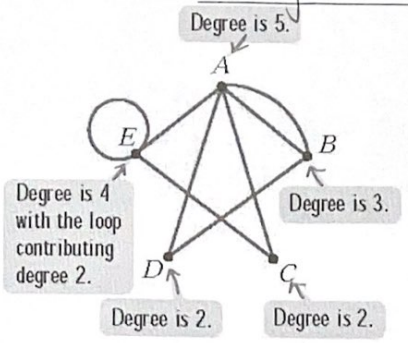

How many ways to get outdoors? Count edges that touch E: 6



vocab →

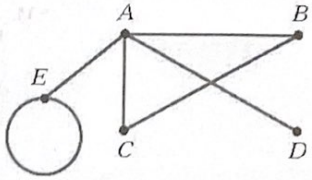
## Vocabulary of Graph Theory

- The degree ***of a vertex*** is the number of edges at that vertex.

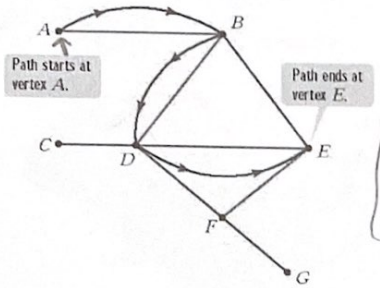



- A vertex with an even number of edges attached to it is an even ***vertex***. Ex. C, D, E are even vertices
- A vertex with an odd number of edges attached to it is an odd ***vertex***. Ex. A & B are odd vertices
- Two vertices in a graph are said to be adjacent ***vertices*** if there is at least one edge connecting them.

**Example:** List the pairs of adjacent vertices for the graph shown.

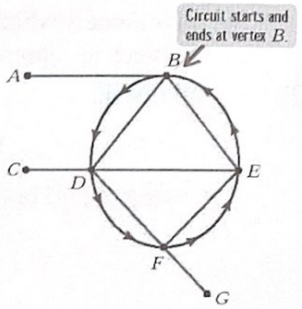



AB AD BC
AC AE EE




A path in a graph is a sequence of adjacent vertices and the edges connecting them.

**Notice the path along this graph is represented by sequential arrows.**
A, B, D, E




A circuit is a path that begins and ends at the same vertex. B, D, F, E, B

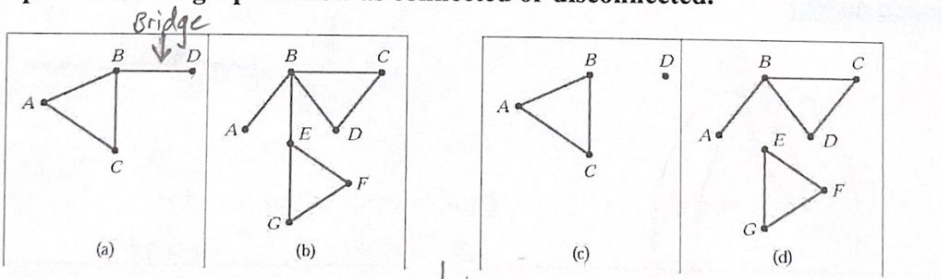
A graph is connected if for any two of its vertices there is at least one path connecting them.

A graph that is not connected is called disconnected.

A disconnected graph is made of pieces that are connected by themselves, called components of the graph.

A BRIDGE is an edge of a connected graph that, if removed, would leave behind a disconnected graph.

**Example: label the graphs below as connected or disconnected.**




connected disconnected