

11.3: Computation in Positional Systems

Objectives

1. Add in bases other than ten.
2. Subtract in bases other than ten.
3. Multiply in bases other than ten.
4. Divide in bases other than ten.

Addition

Example 1: Addition in Base Four

$4^3, 4^2, 4^1, 1$

digits: 0, 1, 2, 3

$$\begin{array}{r} 33_{\text{four}} \\ + 13_{\text{four}} \\ \hline \end{array}$$

6_{ten} too big! Go to next place value!

$4 \overline{) 6}$
1 group of 4'
 $\underline{-4}$
2 groups in 1's place

$6_{\text{ten}} = 12_{\text{four}}$
must carry!

$$\begin{array}{r} 1 \\ 33_{\text{four}} \\ + 13_{\text{four}} \\ \hline \end{array}$$

$5_{\text{ten}} 2$

too big! $\Rightarrow$

$4 \overline{) 5}$
1 group of 4'
 $\underline{-4}$
1 group of 1's

$5_{\text{ten}} = 11_{\text{four}}$

112_{four}
Answer

$$112_{\text{four}} = 1 \cdot 4^2 + 1 \cdot 4^1 + 2 \cdot 1 = 22_{\text{ten}}$$

Check

$$33_{\text{four}} = 3 \cdot 4 + 3 \cdot 1 = 15_{\text{ten}}$$

$$13_{\text{four}} = 1 \cdot 4 + 3 \cdot 1 = 7_{\text{ten}}$$

$$\text{Sum: } 15 + 7 = 22 \checkmark$$

Base Five

digits: 0, 1, 2, 3, 4

Place Value:

 $5^3, 5^2, 5^1, 1$
↳ 125

Example 2:

$$\begin{array}{r} 1 \\ 32_{\text{five}} \\ + 44_{\text{five}} \end{array}$$

 131_{five} sum in 1's place: 6_{ten} too big!

$$6_{\text{ten}} = 1 \cdot 5^1 + 1 \cdot 1 = 11_{\text{five}} \quad (\text{must carry})$$

sum in 5's place: 8_{ten} too big!

$$8_{\text{ten}} = 1 \cdot 5^1 + 3 \cdot 1 = 13_{\text{five}}$$

$$\rightarrow \text{Check: } 131_{\text{five}} = 1 \cdot 5^2 + 3 \cdot 5^1 + 1 \cdot 1 = 41_{\text{ten}}$$

Check

$$32_{\text{five}} = 3 \cdot 5 + 2 \cdot 1 = 17_{\text{ten}}$$

$$44_{\text{five}} = 4 \cdot 5 + 4 \cdot 1 = 24_{\text{ten}}$$

$$\text{Sum: } 41_{\text{ten}}$$

Subtraction

Example 3: Subtraction in Base Four

Subtract

$$\begin{array}{r} 2 \quad 5 \leftarrow \text{write in base ten} \\ 31_{\text{four}} \\ - 12_{\text{four}} \end{array}$$

$$13_{\text{four}}$$

Borrow a group of 4'

Digits: 0, 1, 2, 3

Check

$$31_{\text{four}} = 3 \cdot 4 + 1 \cdot 1 = 13_{\text{ten}}$$

$$12_{\text{four}} = 1 \cdot 4 + 2 \cdot 1 = 6_{\text{ten}}$$

$$7_{\text{ten}} \text{ diff.}$$

$$13_{\text{four}} = 1 \cdot 4^1 + 3 \cdot 1 = 7_{\text{ten}}$$

Example 4:

$$\begin{array}{r} 3 \quad 6 \leftarrow \text{write in base ten} \\ 41_{\text{five}} \\ - 23_{\text{five}} \end{array}$$

 13_{five}

Borrow a group of 5'

You can always check by converting to base ten!

Multiplication

Example 5:

$$\begin{array}{r}
 34_{\text{six}} \\
 \times 2_{\text{six}} \\
 \hline
 112_{\text{six}}
 \end{array}$$

$4 \cdot 2 = 8$ too big!
 $8 = 1 \cdot 6 + 2 \cdot 1$
 12_{six}
 $3 \cdot 2 + 1 = 7$ too big!
 $7 = 1 \cdot 6 + 1 \cdot 1$
 11_{six}

Example 6: 45_{seven}
 $\times 3_{\text{seven}}$

$$\begin{array}{r}
 45_{\text{seven}} \\
 \times 3_{\text{seven}} \\
 \hline
 201_{\text{seven}}
 \end{array}$$

$3 \cdot 5 = 15$
 $15 = 2 \cdot 7 + 1 \cdot 1 = 21_{\text{seven}}$
 $3 \cdot 4 + 2 = 14 \rightarrow 2 \text{ groups of } 7$
 $14 = 2 \cdot 7 + 0 \cdot 1 = 20_{\text{seven}}$

Division

Example 7: Use the table, showing products in base four, to perform the following division:

$$4^3, 4^2, 4^1, 1$$

Digits:

x	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	10	12
3	0	3	12	21

$$2 \cdot 4^1 + 1 \cdot 1 = 9_{\text{ten}}$$

$$\begin{array}{r}
 32_{\text{four}} \\
 3_{\text{four}} \overline{) 222_{\text{four}}} \\
 \underline{21} \downarrow \\
 12 \\
 \underline{-12} \\
 0
 \end{array}$$

Example 8: Use the table, showing products in base four, to perform the following division:

x	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	10	12
3	0	3	12	21

$$\begin{array}{r}
 221_{\text{four}} \\
 2_{\text{four}} \overline{) 1102_{\text{four}}} \\
 \underline{-10} \downarrow \\
 10 \\
 \underline{-10} \downarrow \\
 02 \\
 \underline{-2} \\
 0
 \end{array}$$