11.2: Number Bases in Positional Systems

Objectives
1. Change numerals in bases other than ten to ba
2. Change base ten numerals to numerals in other

Not every system uses base ten (10 digits). Computers use a base ___two___, system (binary)

Changing Numerals in Bases Other Than Ten to Base Ten

The base of a positional numeration system refers to the number whose powers define the place values.

Example 1: The place values in a base five system are powers of five:

$\ldots, 5^4, 5^3, 5^2, 5^1, 1^\circ$ ○ ○

How do we get a 1 here if it is base 5?
$5^0 = 1$

The digits are: 0, 1, 2, 3, 4 ← one less than the base!

Steps for Changing to Base Ten

To change a numeral in a base other than ten to a base ten numeral,
1. Find the ___place value___ for each digit in the numeral.
2. ___Multiply___ each digit in the numeral by its respective place value.
3. Find the ___sum___ of the products in step 2.

Changing to base ten

Example 2: Convert 4726_{eight} to base ten. (Note: Read "Four, seven, two, six, base eight)

1. Find the place value for each digit in the numeral:

← start at end ☺

Place value: 8^3	Place value: 8^2	Place value: 8^1	Place value: 1
4	7	2	6_{eight}

2. Multiply each digit in the numeral by its respective place value and find the sum.

4726_{eight}

$4 \times 8^3 + 7 \times 8^2 + 2 \times 8^1 + 6 \times 1$

$2048 + 448 + 16 + 6 = \boxed{2518 \text{ base ten}}$

Example 3: Convert 100101_{two} to base ten.

2^5	2^4	2^3	2^2	2^1	2^0
↓	↓	↓	↓	↓	↓
1	0	0	1	0	1

Base two digits : 0, 1

ON/OFF in computer applications

$1 \cdot 32 + 1 \cdot 4 + 1 \cdot 1$

$= 32 + 4 + 1$

$= \boxed{37}$ base ten

For computer programming there is a system called the hexadecimal system or base sixteen. (Why might base sixteen be a problem in comparison to our base ten system?)

We don't have enough digits ... so we use letters :

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
(A = 10, B = 11, C = 12, D = 13, E = 14, F = 15)

↑ one less than base

Example 4: Convert $EC7_{\text{sixteen}}$ to base ten.

16^2	16^1	16^0
↓	↓	↓
E	C	7

$14 \cdot 16^2 + 12 \cdot 16^1 + 7 \cdot 1$

$3584 + 192 + 7 = \boxed{3783}$ base ten

16^2 16^1 1
↓ ↓ ↓

Example 4a: Convert $AD4_{\text{sixteen}}$ to base ten.

$10 \cdot 16^2 + 13 \cdot 16 + 4 \cdot 1$

$2560 + 208 + 4 = \boxed{2772}$ base ten

Changing Base Ten Numerals to Numerals in Other Bases: We use division to determine how many groups of each place value are contained in a base ten numeral.

Example 5: Change 8_{ten} into base five.

-5
3 left over

$1 \cdot 5^1 + 3 \cdot 1$

Answer: 13_{five}

Base five: ..., 5^3, 5^2, 5^1, 5^0
..., 125, 25, 5, 1
Digits: 0, 1, 2, 3, 4

→ Biggest that goes into 8

$8 \div 5 = 1\ R3$

Example 6: Change 6_{ten} to base five.

-5
1 left over

$1 \cdot 5 + 1 \cdot 1$

11_{five}

$5^3, 5^2, 5^1, 5^0$

↓
Goes into 6

$6 \div 5 = 1\ R1$

Example 7: Convert the base ten numeral 299 to a base eight numeral.

4 groups of 64 + 5 groups of 8 + 3

$4 \cdot 8^2 + 5 \cdot 8 + 3 \cdot 1$

453_{eight}

$8^3, 8^2, 8^1, 8^0$
512, 64, 8, 1
Too big
Goes into 299
Start here
Goes into

$64\overline{)299}$ → 4
-256
43 R

$8\overline{)43}$ → 5
40
3 R