Prob/Stat/Discrete **Unit 11: Number Representation**

11.1: Our Hindu-Arabic System & Early Positional Systems

Objectives

1. Evaluate an exponential expression.
2. Write a Hindu-Arabic numeral in expanded form.
3. Express a number's expanded form as a Hindu-Arabic numeral.
4. Understand and use the Babylonian numeration system.
5. Understand and use the Mayan numeration system.

A number is an abstract idea that addresses the question, "How many?"

A numeral is a symbol used to represent a number. Write down as many numerals as you can think of to represent the number "nine."

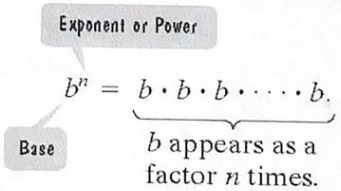
𓏺𓏺𓏺𓏺 𓏺 IX 9 ← Hindu-Arabic symbol

A system of numeration consists of a set of basic numerals and rules for combining them to represent numbers.

Our numerals are called Hindu-Arabic numerals. Why are they called this?

- invented in India → 9 Hindu digits
- brought to Europe by Arabs → Arab symbol 0

Exponential Notation – We need to be able to understand exponents in order to understand our numeration system.



Example 1: Exponential Expression

Evaluate the following: $10^8 = 100,000,000$

Note:

$10^2 = 10 \cdot 10 = 100$

$10^3 = 10 \cdot 10 \cdot 10 = 1000$

$10^4 = 10 \cdot 10 \cdot 10 \cdot 10 = 10,000$

exponent matches # of zeroes for powers of 10

every time there's another zero, there's another place value

Our Hindu-Arabic Numeration System

An important characteristic is that we can write the numeral for any number, large or small, using only ten digits: 0,1,2,3,4,5,6,7,8,9

Hindu-Arabic numerals can be written in expanded form, in which the value of the digit in each position is made clear.

Hindu Arabic numeration system is called a positional value, or place value, system. The positional values in the system are based in the powers of 10: $\ldots, 10^5, 10^4, 10^3, 10^2, 10^1, 1$

Example 2: Write 3407 in expanded form.

$3407 = 3000 + 400 + 7$

$= (3 \times 10^3) + (4 \times 10^2) + (0 \times 10^1) + (7 \times 1)$

Recall: $10^0 = 1$

(0×10^2)

Example 3: Express the expanded form as a Hindu-Arabic numeral: $(7 \times 10^3) + (5 \times 10^1) + (4 \times 1)$.

line up place values

$$\begin{array}{r} 7000 \\ 50 \\ 4 \\ \hline 7054 \end{array}$$

$= 7000 + 50 + 4$

$= 7054$

0 in hundreds place

The Babylonian Numeration System

Based on powers of 10

Babylonian	V	<
Hindu-Arabic	1	10

← Based on powers of 60

The place values in the Babylonian system use powers of 60. The place values are:

$\ldots, \quad 60^3, \quad 60^2, \quad 60^1, \quad 1.$

$60^3 = 60 \times 60 \times 60 = 216{,}000$

$60^2 = 60 \times 60 = 3600$

What other positional system uses 60 as a power? Time!

1 min = 60 sec

1 hr = 60 min. = 60×60 sec = 60^2 sec

The Babylonians left a space to distinguish the various place values in a numeral from one another: < or V

The Babylonian Numeration System:
Converting from a Babylonian Numeral to a Hindu-Arabic Numeral

Babylonian	V	<
Hindu-Arabic	1	10

What does V mean? 1 What does < mean? 10

Example 4: Write VV (60²) < (60¹) <<VV (60⁰ = 1) as a Hindu-Arabic numeral.

Note: $60^2 = 3600$

$= (1+1)\cdot 60^2 + (10+1)\cdot 60^1 + (10+10+1+1)\cdot 1$

$= 2\times 60^2 + 11\times 60^1 + 22\times 1$

$= 7200 + 660 + 22 = \boxed{7882}$

Example 5: Write <V (60²) <VV (60¹) <VV (1 place) as a Hindu-Arabic numeral.

$(10+1)\cdot 60^2 + (10+1+1)\cdot 60^1 + (10+1+1)\cdot 1$

$= 11\times 60^2 + 12\times 60 + 12\times 1$

$= 39600 + 720 + 12 = \boxed{40,332}$

Example 6: Write 4223 as a Babylonian number.

4223 − 3600 = 623 ← One group of $60^2 = 3600$

← How many groups of 60^1? Ten

$\frac{623}{60} = 10$ R23

23 in ones place

$1\cdot 60^2 + 10\cdot 60^1 + 23\cdot 1$

$\boxed{\text{V} \quad < \quad \text{<<VVV}}$

**A major disadvantage of the Babylonian system is that there is no symbol for zero. This lead to large gaps and confusion. **

The Mayan Numeration System

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19

First to have a symbol for zero!

The place values in the Mayan system are

..., 18×20^3, 18×20^2, 18×20, 20, 1

$18 \times 20 \times 20 \times 20 = 144{,}000$ $18 \times 20 \times 20 = 7200$ $18 \times 20 = 360$

Instead of using 20^2 as the third position, the Mayans used 18x20, probably so that this system would include their calendar of 360 days.

...144,000, 7200, 360, 20, 1

Numerals in the Mayan system are expressed vertically. The place value at the bottom of the column is 1

Example 7

Write  as a Hindu-Arabic numeral.

Mayan numeral		Hindu-Arabic numeral		Place value		
	=	14	×	7200	=	100,800
	=	0	×	360	=	0
	=	7	×	20	=	140
	=	12	×	1	=	12
						100,952

figure out this way!

Example 8: Write the following as a Hindu-Arabic numeral.

	4	× 360	=	1440
	15	× 20	=	300
	8	× 1	=	8
				1748