Lesson 10.1 Part 1: Solving Equations by Factoring (Review)

In the coming lessons we will be solving much more complex equations by factoring, so it will help to have the basics dialed in:

Zero-Product Property	Let a and b be real numbers. If $ab = 0$, then a = 0 and/or b = 0.

For #1 – 4: Solve each equation for x.

1) $x(x-6) = 0$

$\boxed{x=0}$ $x-6=0$ $\boxed{x=6}$

2) $-2.5x(x+1) = 0$

$-2.5x=0$ $\boxed{x=0}$ $\boxed{x=-1}$

3) $\cancel{3}(x-2)(5x+2) = 0$

$\boxed{x=2}$ $\boxed{x=-2/5}$

4) $4x(2x-3)(x-100) = 0$

$\boxed{x=0}$ $\boxed{x=3/2}$ $\boxed{x=100}$

Steps for Solving Quadratic Equations by factoring:
1) Get a zero on one side of the equation. 2) Factor. 3) Set each factor equal to zero and solve. The solutions are also called **roots, x-intercepts, solutions, or zeros.**

Examples 5 – 10: Solve each equation for the variable by factoring.

5) $x^2 + 3x - 10 = 0$

$(x+5)(x-2)=0$

$x=-5$ $x=2$

6) $0 = x^2 - 9$

$0=(x+3)(x-3)$

$x=-3$ $x=3$

7) $\frac{-9x^2}{-3x} + \frac{6x}{-3x} = 0$

$-3x(3x-2)=0$

$x=0$ $x=2/3$

8) $x^2 - 49 = 0$

$(x+7)(x-7)=0$

$x=\pm 7$

9) $\frac{15x^2}{3x} + \frac{3x}{3x} = 0$

$3x(5x+1)=0$

$\boxed{x=0}$ $\boxed{x=-\frac{1}{5}}$

10) $0 = x^2 + 4x - 12$

$0=(x+6)(x-2)$

$x=-6$ $x=2$

3·1 4·2 ~~8·1~~

Example 11: Solve for a by factoring: $3a^2 - 10a - 8 = 0$

$$(3a + 2)(a - 4) = 0$$

$$a = -\frac{2}{3} \qquad a = 4$$

Examples 12 – 15: Solve each equation for the variable by factoring.

2·1 5·3 15·1

12) $0 = 2x^2 + 11x + 15$

$$0 = (2x + 5)(x + 3)$$

$$x = -\frac{5}{2} \qquad x = -3$$

2b·2b 7·7

13) $4b^2 - 49 = 0$

$$(2b + 7)(2b - 7) = 0$$

$$b = \pm 7/2$$

5y·5y 9·9

14) $0 = 25y^2 - 81$

$$0 = (5y + 9)(5y - 9)$$

$$y = \pm 9/5$$

2·1

15) $5x^2 + 9x - 2 = 0$

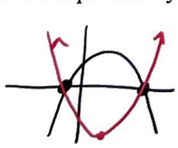
$$(5x - 1)(x + 2) = 0$$

$$x = 1/5 \qquad x = -2$$

Hints for solving equations by factoring:

1. Get a ___0___ on one side of the equal sign. → standard
2. Put the terms in ___$ax^2+bx+c=0$___ order.
3. **It is usually easiest to factor if the leading coefficient is ___positive___. If needed, move the terms to the other side of the equal sign in order to change their signs (or multiply both sides by -1).**

Example 16: Solve the equation by factoring: $-2x^2 + 16x = 14$.



$$\frac{-2x^2 + 16x - 14}{-2} = 0$$

$$x^2 - 8x + 7 = 0$$

$$(x - 7)(x - 1) = 0$$

$$\boxed{x = 7 \qquad x = 1}$$

For #17 – 22: Solve the following equations.

17) $x^2 - 30 = x$

$-x \quad -x$

$x^2 - 1x - 30 = 0$

$(x-6)(x+5) = 0$

$x=6 \quad x=-5$

18) $-3a^2 + 18a + 45 = \cancel{-3}$

$+3 \quad \cancel{+3}$

$\frac{-3a^2 + 18a + 48}{-3} = 0$

$a^2 - 6a - 16 = 0$

$(a-8)(a+2) = 0$

$a=8 \quad a=-2$

19) $10 = 9a^2 + 9$

$-10 \quad -10$

$0 = \overset{3a\cdot 3a}{9a^2} - \overset{1\cdot 1}{1}$

$0 = (3a+1)(3a-1)$

$a = \pm 1/3$

$11 \cdot 5$

20) $-4x^2 + 12 = 8\cancel{x}$

$-8x \quad \cancel{-8x}$

$\frac{-4x^2 - 8x + 12}{-4} = 0$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x=-3 \quad x=1$

21) $3x^2 + 28x - 55 = 0$

$(3x-5)(x+11) = 0$

$x = 5/3 \quad x = -11$

$5x^2 - x^3$

22) $-4x^2 - 2x = -6$

$+6 \quad +6$

$\frac{-4x^2 - 2x + 6}{-2} = 0$

$2x^2 + x - 3 = 0$

$(2x+3)(x-1) = 0$

$x = -3/2 \quad x = 1$

For #23 – 26: Factor each expression using the GCF.

23) $5\cancel{(x-2)^2} - \cancel{(x-2)^3}$

$\cancel{(x-2)^2} \quad \cancel{(x-2)^2}$

gcf: $(x-2)^2$

$(x-2)^2(5-(x-2))$

$(x-2)^2(5-x+2)$

$(x-2)^2(-x+7)$

$\boxed{-(x-2)^2(x-7)}$

24) $\cancel{(x+3)^3} + 4\cancel{(x+3)^2}$

$\cancel{(x+3)^2} \quad \cancel{(x+3)^2}$

$(x+3)^2(x+3+4)$

$\boxed{(x+3)^2(x+7)}$

25) $x^2\cancel{(x-6)} + 14\cancel{(x-6)}$

$\cancel{(x-6)} \quad \cancel{(x-6)}$

$x \cdot x \quad \sqrt{14}\cdot\sqrt{14}$

$\boxed{(x-6)(x^2+14)}$

$\cancel{(x+i\sqrt{14})(x-i\sqrt{14})}$

26) $x^2\cancel{(x-10)} + 8\cancel{(x-10)}$

$\cancel{(x-10)} \quad \cancel{(x-10)}$

$\boxed{(x-10)(x^2+8)}$

For #27 – 30: Factor completely or state that the polynomial is prime.

27) $y^5 - 625y$

$y(y^4 - 625)$

$y(y^2 + 25)(y^2 - 25)$

$\boxed{y(y^2 + 25)(y + 5)(y - 5)}$

28) $x^4 - 81$

$(x^2 + 9)(x^2 - 9)$

$\boxed{(x^2 + 9)(x + 3)(x - 3)}$

29) $x^2 - 2x + 1 - 4y^2$

$(x - 1)(x - 1) - 4y^2$

$(x - 1)^2 - 4y^2$

$(x - 1 + 2y)(x - 1 - 2y)$

30) $x^2 - 10x + 25 - 4y^2$

$(x - 5)(x - 5) - 4y^2$

$(x - 5)^2 - 4y^2$

$(x - 5 + 2y)(x - 5 - 2y)$

Lesson 10.1 Part 2: Reconstruct the Values in the Unit Circle (Review)

In the coming lessons, and also in math courses that follow this one, it is important that you know the unit circle solidly. So let's review it to put that topic into your long-term memory.