Name _____ Date _____ Class _____

LESSON 12-1

Probability

Practice and Problem Solving: A/B

Determine the probability of each event. Write *impossible, unlikely, as likely as not, likely,* or *certain*. Then, tell whether the probability is 0, close to 0, $\frac{1}{2}$, close to 1, or 1.

1. randomly picking a blue card from a bag containing all blue cards

_____

2. rolling an odd number on a number cube containing numbers 1 through 6

_____

3. picking a red marble from 4 white marbles and 7 green marbles

_____

Find each probability. Write your answer in simplest form.

4. A bag holds 6 tiles: 2 lettered and 4 numbered. Without looking, you choose a tile. What is the probability of drawing a number? _____

5. The names Phil, Angelica, Yolanda, Mimi, and Ed are on slips of paper in a hat. A name is drawn without looking. What is the probability of **not** drawing Ed? _____

6. A standard deck of cards contains 13 of each suit: red hearts, red diamonds, black clubs, and black spades. What is the probability of drawing a red card without looking? _____

A board game includes the 9 cards below.

Move back 2.	Move up 1.	Move up 4.	Move back 3.	Move up 3.	Move up 6.	Move back 2.	Move up 5.	Move up 2.

7. Mia says the probability of moving back is the same as the probability of moving up. Is she correct? What is the probability of moving back? Explain.

_____

8. Gavin needs to move up more than 4 spaces to win the game. Is he likely to win on his next turn? What is the probability that he will **not** win on his next turn? Explain.

_____

Name _____ Date _____ Class _____

LESSON 12-1

Probability

Practice and Problem Solving: C

Find each probability. Write your answer in simplest form.

1. picking a blue shirt from a drawer with 8 blue shirts and 2 white shirts

_____

2. drawing a vowel from letter tiles that spell out MATHEMATICS

_____

3. A spinner is divided into 8 equal sections: 4 red, 2 white, 1 green, and 1 blue. What is the probability that the spinner lands on blue or white?

_____

There are 6 cans of soup in a kitchen cabinet: 2 chicken noodle, 3 tomato, and 1 vegetable.

4. You select a can without looking. What is the probability that you will **not** choose chicken noodle soup?

_____

5. Suppose you use a can of chicken noodle soup from the original 6 cans. Then your father adds 2 cans of vegetable soup and 1 can of tomato soup to those left in the kitchen cabinet. What is the probability that you will choose tomato soup now?

_____

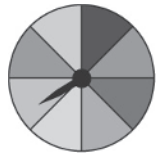
6. Later, your mother adds 7 more cans of soup to the cabinet, some chicken noodle and some vegetable. Now the probability of not choosing chicken noodle soup is $\frac{4}{5}$. How many cans of chicken noodle soup did your mother add to those already in the cabinet? Explain.

_____

_____

Use the picture at the right.

7. Write one number in each section of the spinner at right. Then write a probability problem about the spinner. The answer to your problem should be between $\frac{1}{2}$ and 1.



_____

_____

_____

Name _____ Date _____ Class _____

LESSON 12-1

Probability

Practice and Problem Solving: D

Match each event to its likelihood. The first one is done for you.

1. rolling a number less than 6 on a number cube labeled 1 through 6 — **A**
2. flipping a coin and getting heads — _____
3. spinning a number less than 3 on a spinner with 8 equal sections marked 1 through 8 — _____
4. drawing a red or blue marble from a bag of red marbles and blue marbles — _____
5. rolling a number greater than 6 on a number cube labeled 1 through 6 — _____

A. likely
B. unlikely
C. as likely as not
D. impossible
E. certain

Solve. Write your answer in simplest form. The first one is done for you.

6. A bag contains 4 red marbles, 3 green marbles, and 2 yellow marbles. The probability of randomly picking a yellow marble is $\frac{2}{9}$. What is the probability of not picking a yellow marble? **$\frac{7}{9}$**

7. A number cube is labeled 1 through 6. The probability of randomly rolling a 5 is $\frac{1}{6}$. What is the probability of not rolling a 5? _____

Tell whether the event is *impossible*, *unlikely*, *as likely as not*, *likely*, or *certain*. Explain your choice. The first one is done for you.

8. Tyrone rides his bicycle to school if he gets up by 7:15 A.M. Tyrone gets up by 7:15 about half the time. Estimate the probability that Tyrone will ride his bicycle to school.

as likely as not; Since he gets up by 7:15 about half the time, he will ride his bicycle about half the time. The probability is about $\frac{1}{2}$, or as likely as not.

9. There are 10 shirts in a drawer. Eight of the shirts have short sleeves. Two shirts have long sleeves. Estimate the probability that you get a short-sleeved shirt if you select one out without looking.

_____

_____

Name _____ Date _____ Class _____

LESSON 12-1

Probability

Reteach

Picturing a thermometer can help you rate probability.

At right are 8 letter tiles that spell AMERICAN.

A	M	E	R
I	C	A	N

If something will always happen, its probability is **certain**. If you draw a tile, the letter will be in the word "American."

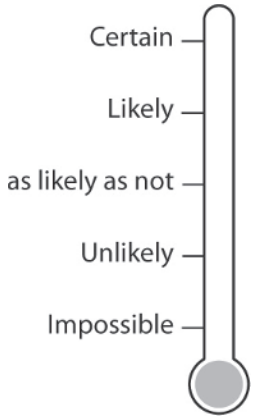
$P(\text{A, M, E, R, I, C, or N}) = 1$

If something will never happen, its probability is **impossible.** If you draw a tile, you cannot draw a "Q."

$P(\text{Q}) = 0$

The probability of picking a vowel is **as likely as not** because there are 4 vowels and 4 consonants.

$$P(\text{a vowel}) = \frac{4\ \textit{vowels}}{8\ \textit{letters}} = \frac{1}{2}$$



Picking the letter "C" is **unlikely** because there is only one "C."

$$P(\text{C}) = \frac{1\ \text{“}c\text{”}}{8\ \textit{letters}} = \frac{1}{8}$$

Picking a letter besides "A" is **likely** because there are 6 letters that are not "A".

$$P(\text{not A}) = \frac{6\ \textit{letters}}{8\ \textit{letters}} = \frac{3}{4}$$

Another way to find $P(\text{not A})$ is to subtract $P(\text{A})$ from 1.

$$P(\text{not A}) = 1 - P(\text{A}) = 1 - \frac{1}{4} = \frac{3}{4}$$

Tell whether each outcome is *impossible*, *unlikely*, *as likely as not*, *likely*, or *certain*. Then write the probability in simplest form.

1. choosing a red crayon from a box of 24 different colored crayons, including red crayons

_____

2. rolling an odd number on a number cube containing numbers 1 through 6

_____

3. randomly picking a white card from a bag containing all red cards

_____

Name _____ Date _____ Class _____

LESSON 12-1

Probability

Reading Strategies: Use a Table

Creating a table can help you solve probability problems.

You are to choose one of the cards at right without looking.

Consider the probability of three outcomes: 1) choosing a vowel, 2) choosing a B, or 3) choosing a letter in the word *MATH.*

Complete the table by writing whether each of the desired outcomes is *impossible, unlikely, as likely as not, likely,* or *certain.*

	Desired Outcomes		
Possible Outcomes	**Vowel**	**B**	**Letter in *MATH***
M	no	no	yes
A	yes	no	yes
T	no	no	yes
H	no	no	yes
Results	1 out of 4	0 out of 4	4 out of 4
Probability	1. _____	2. _____	3. _____

4. You spin the spinner at the right. Complete the table. Tell whether each of the desired outcomes is *impossible, unlikely, as likely as not, likely,* or *certain.*

	Desired Outcomes		
Possible Outcomes	**6**	**Factor of 4**	**Greater than 0**
Results	____ out of ____	____ out of ____	____ out of ____
Probability			

Name _____ Date _____ Class _____

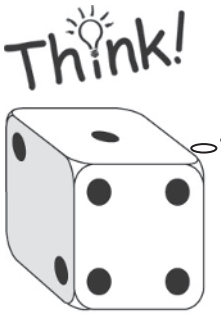
LESSON 12-1

Probability

Success for English Learners

Problem 1

A number cube can help you understand probability.



Possible outcomes:
1 2 3 4 5 6

Is it **likely** that you will roll a 1 every time?

No. The cube has 6 sides. Only one side is a 1. It is **unlikely** that I will roll a 1 every time.

This means the probability of rolling a 1 every time is low.

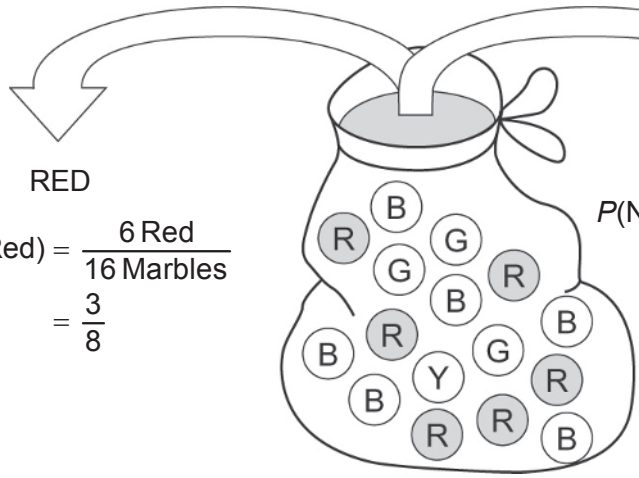
Problem 2

There are 16 marbles in a bag.

To find the probability of **not** drawing a red marble, first find the probability of drawing a red marble.

***P*(Red)** means "the probability of drawing a red marble."
***P*(Not Red)** means "the probability of drawing any marble that is NOT red."

R = Red G = Green B = Blue



RED

$$P(\text{Red}) = \frac{6\ \text{Red}}{16\ \text{Marbles}} = \frac{3}{8}$$

NOT RED

$$P(\text{Not Red}) = 1 - P(\text{Red}) = 1 - \frac{3}{8} = \frac{5}{8}$$

1. In Problem 1, is it more likely, less likely, or as likely as not to roll an even number? Why?

_____

_____

2. In Problem 2, how likely is it that you will select a purple marble? Why?

_____

_____

Name _____ Date _____ Class _____

LESSON 12-2

Experimental Probability of Simple Events

Practice and Problem Solving: A/B

Solve.

1. Jolene is playing basketball. She scored 11 baskets in 15 free throws. What is the experimental probability that she will score a basket on her next free throw?

_____

2. Sarah has gone to work for 60 days. On 39 of those days, she arrived at work before 8:30 A.M. On the rest of the days she arrived after 8:30 A.M. What is the experimental probability she will arrive after 8:30 A.M. on the next day she goes to work?

_____

3. For the past four weeks, Micah has been recording the daily high temperature. During that time, the high temperature has been greater than 45°F on 20 out of 28 days. What is the experimental probability that the high temperature will be below 45°F on the twenty-ninth day?

_____

4. After the movie premiere 99 out of 130 people surveyed said they liked the movie.
   a. What is the experimental probability that the next person surveyed will say he or she liked the movie?

   _____

   b. What is the experimental probability that the next person surveyed will say he or she did not like the movie?

   _____

Find each experimental probability. Write your answer as a fraction, as a decimal, and as a percent.

5. For the past 40 days, Naomi has been recording the number of customers at her restaurant between 10:00 A.M. and 11:00 A.M. During that hour, there have been fewer than 20 customers on 25 out of the 40 days.
   a. What is the experimental probability there will be fewer than 20 customers on the forty-first day?

   _____

   b. What is the experimental probability there will be 20 or more customers on the forty-first day?

   _____

Name _____ Date _____ Class _____

LESSON 12-2

Experimental Probability of Simple Events

Practice and Problem Solving: C

1. A factory makes bicycles. Out of 300 bicycles, 2 were found to have defective brakes.
   a. What is the experimental probability that the next bike manufactured will have defective brakes?

   _____

   b. Predict how many bikes out of 2,100 will have defective brakes.

   _____

2. A factory makes light bulbs. Out of 400 light bulbs, 18 were found to have defective filaments.
   a. What is the experimental probability that the next light bulb manufactured will have a defective filament?

   _____

   b. Predict how many bulbs out of 6,000 will have defective filaments.

   _____

3. A factory makes ceramic bowls. Out of 200 bowls, 8 were chipped.
   a. What is the experimental probability that the next bowl made will **not** be chipped?

   _____

   b. Predict how many bowls out of 10,000 will be chipped

   _____

4. A manufacturer of sparkplugs has a goal of producing less than 2% defective ones. Of the last 8,000 sparkplugs, 13 were defective.
   a. What is the experimental probability that the next sparkplug will be defective?

   _____

   b. Did the manufacturer reach its goal? Explain.

   _____

5. A manufacturer of electric switches has a goal of producing less than 1.5% defective switches. Of the last 300 switches, 23 were defective.
   a. What is the experimental probability that the next switch made will be defective?

   _____

   b. Did the manufacturer reach its goal? Explain.

   _____

Name _____ Date _____ Class _____

LESSON 12-2

Experimental Probability of Simple Events

Practice and Problem Solving: D

Find each experimental probability. The first one is done for you.

1. Kathy played a game of darts. She threw 15 darts and hit the target 9 times. What is the experimental probability that Kathy will hit the target the next time she throws a dart?

   a. What is the number of favorable outcomes? **9**

   b. What is the total number of trials? **15**

   c. What is the experimental probability that Kathy will hit the target the next time she throws a dart?

   $$\frac{9}{15} = \frac{3}{5}$$

2. Between 10 A.M. and 11 A.M., 48 people came into Brad's store. 40 of them made a purchase. What is the experimental probability that the next person to come into the store will make a purchase?

   a. What is the number of favorable outcomes? _____

   b. What is the total number of trials? _____

   c. What is the experimental probability the next person to come into the store will make a purchase?

3. Sharona kept track of the colors of cars that passed her house one afternoon. She collected her data in the table below.

Car Color	Number	Car Color	Number
red	12	white	42
blue	9	silver	36
black	32	yellow	1

   What is the experimental probability that the next car will be silver?

   a. What is the number of favorable outcomes? _____

   b. What is the total number of trials? _____

   c. What is the experimental probability that the next car to pass Sharona's house will be silver?

   d. What is the experimental probability that the next car to pass Sharona's house will **not** be silver?

Name _____ Date _____ Class _____

LESSON 12-2

Experimental Probability of Simple Events

Reteach

Experimental probability is an estimate of the probability that a particular event will happen.

It is called *experimental* because it is based on data collected from experiments or observations.

$$\text{Experimental probability} \approx \frac{\text{number of times a particular event happens}}{\text{total number of trials}}$$

JT is practicing his batting. The pitcher makes 12 pitches. JT hits 8 of the pitches. What is the experimental probability that JT will hit the next pitch?

- A favorable outcome is hitting the pitch.
- The number of favorable outcomes is the number JT hit: 8.
- The number of trials is the total number of pitches: 12.
- The experimental probability that JT will hit the next pitch is $\frac{8}{12} = \frac{2}{3}$.

1. Ramon plays outfield. In the last game, 15 balls were hit in his direction. He caught 12 of them. What is the experimental probability that he will catch the next ball hit in his direction?

   a. What is the number of favorable events? _____

   b. What is the total number of trials? _____

   c. What is the experimental probability that Ramon will catch the next ball hit in his direction?

   _____

2. In one inning Tori pitched 9 strikes and 5 balls. What is the experimental probability that the next pitch she throws will be a strike?

   a. What is the number of favorable events? _____

   b. What is the total number of trials? _____

   c. What is the experimental probability that the next pitch Tori throws will be a strike?

   _____

3. Tori threw 5 pitches for one batter. Kevin, the catcher, caught 4 of those pitches. What is the experimental probability that Kevin will **not** catch the next pitch? Show your work.

   _____

Name _____ Date _____ Class _____

LESSON 12-2

Experimental Probability of Simple Events

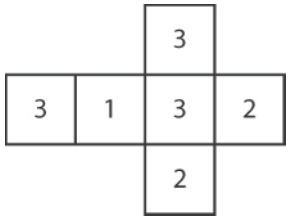
Reading Strategies: Make Predictions

Experimental probability is a ratio. The ratio compares the number of times an event occurs to the total number of trials.

A trial is the number of times that an experiment is carried out or an observation is made.

$$\text{Experimental probability} \approx \frac{\text{number of times a favorable event happens}}{\text{total number of trials}}$$

The net of a number cube is shown below. Use the net to complete Exercises 1–2.



1. Predict which number you will land on most often. Explain.

_____

2. Predict which number you will land on least often. Explain.

_____

Actual events in an experiment may or may not match your prediction. The table shows the outcomes of tossing the above number cube 100 times

Outcome	1	2	3
Number of Tosses	39	28	33

Use the table to complete Exercises 3–4.

3. Did your prediction for landing on 1 match the outcome shown in the table? Explain.

_____

4. Did your prediction for landing on 3 match the outcome shown in the table? Explain.

_____

Name _____ Date _____ Class _____

LESSON 12-2

Experimental Probability of Simple Events

Success for English Learners

Problem 1

Nikos flipped a coin 10 times. It landed heads up 6 times.

What is the experimental probability that the coin will land heads up on the next toss?

$$\text{Experimental probability} \approx \frac{\text{number of times a favorable event happens}}{\text{total number of trials}}$$

favorable event: the outcome you want

total number of trials: how many times the coin was tossed

$$\text{Experimental probability} \approx \frac{6}{10} \approx \frac{3}{5}$$

Problem 2

Nikos flipped the coin 20 times. It landed tails up 11 times.

What is the experimental probability that the coin will **not** land tails up on the next toss?

$$\text{Experimental probability (tails)} \approx \frac{11}{20}$$

$$\text{Experimental probability (not tails)} \approx 1 - \frac{11}{20} \approx \frac{9}{20}$$

1. Marco counted 40 cars in the parking lot. 28 were silver. What is the experimental probability that the next car in the lot will be silver?

   a. What is the number of events? _____

   b. What is the number of trials? _____

   c. What is the experimental probability that the next car in the parking lot will be silver?

   _____

2. Janine flipped a coin 52 times. The coin landed heads up 18 times. What is the experimental probability that the coin will land tails up on the next flip?

   _____

3. Write your own experimental probability problem. Give the answer.

   _____

   _____

   _____

Name _____ Date _____ Class _____

LESSON 12-3

Experimental Probability of Compound Events

Practice and Problem Solving: A/B

Solve.

1. A coin was tossed and a spinner with three equal sections numbered 1 to 3 was spun. The results are shown in the table.

	Heads	Tails
1	**53**	**65**
2	**49**	**71**
3	**54**	**62**

What is the experimental probability that the next toss and spin will result in 3 and Tails?

_____

2. A receptionist recorded the number of people who took an elevator up from his floor and the number who took an elevator down. He also noted the number of men and women. The table shows the results.

	Elevator Up	Elevator Down
Men	**36**	**43**
Women	**39**	**42**

What is the experimental probability that the next person will be a woman taking the elevator up?

_____

3. Sandwich shop customers can choose the bread and meat they want. The table shows the sandwiches that were sold on a given day.

	White Bread	Wheat Bread
Ham	22	24
Turkey	21	22
Tuna	25	23

What is the experimental probability that the next sandwich sold will be tuna on wheat bread?

_____

4. A store sells a coat in three sizes: small, medium, and large. The coat comes in red, navy, and tan. Sales numbers are shown in the table.

	Small	Medium	Large
Red	18	21	19
Navy	24	22	20
Tan	19	25	22

What is the experimental probability that the next coat sold is **not** a large navy?

_____

Name _____ Date _____ Class _____

LESSON 12-3

Experimental Probability of Compound Events

Practice and Problem Solving: C

Solve.

1. Two brands of paint—Durable and Forever—are each sold by the gallon in three different grades: good, better, and best quality. The sales manager at a hardware store tracks all the paint sales. She started making a table, but did not finish it.

	Good	Better	Best
Durable	48	45	35
Forever		39	33

The experimental probability that the next gallon of paint sold will be better-quality Durable paint is $\frac{9}{50}$.

a. How many gallons of good-quality Forever were sold?

_____

b. What is the experimental probability that the next gallon of paint sold will **not** be the best quality paint?

_____

2. A horseback riding club is sending one individual, one pair, and one team of vaulters to the championships. These performers will be judged against others in each class. They will be awarded 1 to 5 points for artistry, and 1 to 6 points for precision. Explain how to use a simulation to find the experimental probability that each of the club's entries will score 11 points.

_____

_____

_____

3. Give your own example of a compound event that could be tested through a simulation.

_____

_____

4. Give your own example of a compound event that could **not** be tested through a simulation.

_____

_____

Name _____ Date _____ Class _____

LESSON 12-3

Experimental Probability of Compound Events

Practice and Problem Solving: D

Solve each problem. The first one is done for you.

1. Peter tossed a dime and a quarter at the same time. He did this 100 times. The results are shown in the table.

		Quarter	
		Heads	**Tails**
Dime	**Heads**	18	30
	Tails	32	20

What is the experimental probability that the next time he tosses the coins he will get a tails on the dime and a heads on the quarter?

a. What is the number of favorable events? **32**

b. What is the total number of trials? **18 + 30 + 32 + 20 = 100**

c. What is the experimental probability that the next time Peter tosses both coins he will get a tails on the dime and a heads on the quarter?

$$\frac{32}{100} = \frac{8}{25}$$

2. Aimee tossed a coin and spun a spinner that is divided into 3 equal sections. She did this 50 times. The results are shown in the table.

		Spinner		
		1	**2**	**3**
Coin	**Heads**	4	7	8
	Tails	12	8	11

What is the experimental probability that the next time Aimee tosses the coin and spins the spinner she will get a Tails and a 2?

_____

3. The Reliable Car dealership sells cars and trucks. The cars and trucks come in red, white, and silver. Damon made this table to show the cars and trucks that are on the lot today.

	Red	**White**	**Silver**
Car	45	41	46
Truck	21	24	23

What is the experimental probability that the next vehicle that comes on the lot will be a red car?

_____

Name _____ Date _____ Class _____

LESSON 12-3

Experimental Probability of Compound Events

Reteach

A **compound event** includes two or more simple events.

The possible outcomes of flipping a coin are heads and tails.

A spinner is divided into 4 equal sections, each one a different color. The possible outcomes of spinning are red, yellow, blue, and green.

If you toss the coin and spin the spinner, there are 8 possible outcomes.

2 possible **coin outcomes**

4 possible **spinner outcomes**

8 possible **compound outcomes**

	Red	Yellow	Blue	Green
Heads	9	11	11	14
Tails	10	12	7	6

To find the experimental probability that the next trial will have an outcome of Tails and Blue:

a. Find the number of times Tails and Blue was the outcome: 7.

b. Find the total number of trials: $9 + 11 + 11 + 14 + 10 + 12 + 7 + 6 = 80$.

c. Write a ratio of the number of tails and blue outcomes to the number of trials: $\frac{7}{80}$.

A store hands out yogurt samples: peach, vanilla, and strawberry. Each flavor comes in regular or low-fat. By 2 P.M. the store has given out these samples:

	Peach	Vanilla	Strawberry
Regular	16	19	30
Low-fat	48	32	55

Use the table to answer the questions.

1. What is the total number of samples given out? _____

2. What is the experimental probability that the next sample will be regular vanilla?

   _____

3. What is the experimental probability that the next sample will be strawberry?

   _____

4. What is the experimental probability that the next sample will **not** be peach?

   _____

Name _____ Date _____ Class _____

LESSON 12-3

Experimental Probability of Compound Events

Reading Strategy: Make a Table

Making a table is often a good way to organize information.

If you are doing an experiment where you flip a coin and toss a number cube, recording the results in a list can be difficult to tabulate later. A table is much easier to read.

Tables work well for experiments that include one or two events.

> Ezekiel tosses a coin and rolls a number cube that has sides labeled 1 to 6. He does this 8 times. Using tick marks, he recorded his results in the table below.

	1	2	3	4	5	6
Heads			//	/		/
Tails			/		//	/

Complete.

1. Jalayne tosses a coin and spins a spinner divided into three equal sections (1, 2, and 3). She does this 20 times. The results of Jalayne's 20 trials are shown below. Make a table to display her results.

1H	3H	3T	2H	2T	1T	2H	1H	3H	1T
3H	2T	1T	3H	1H	2T	3T	3H	1T	3T

Use the data in your table to find each experimental probability.

2. The next trial will have the outcome Tails and 3. _____

3. The next trial will have the outcome Heads and 2. _____

4. The next trial will have the outcome **not** Heads and **not** 2. _____

5. The next trial will have the outcome **not** Tails. _____

Name _____ Date _____ Class _____

LESSON 12-3

Experimental Probability of Compound Events

Success for English Learners

Problem

A **compound event** includes 2 or more simple events.

Compound Event → **Event 1:** flip a coin
Compound Event → **Event 2**: toss a cube

both **must** happen to create a compound event

A coin is flipped **and** a number cube is tossed 50 times.

compound event number of **trials**

The results are shown in the table below.

2 possible outcomes from the coin

This is an outcome for the number cube *and* the coin. The outcome was **Heads *and* 2** a total of **3 times** out of 50.

6 possible outcomes from the number cube

	1	**2**	**3**	**4**	**5**	**6**
Heads	4	3	6	4	4	5
Tails	3	5	2	5	3	6

1. What is the experimental probability that the next outcome will be Tails *and* 4?

   a. Look in the table to find the number for Tails *and* 4. _____

   b. Write that number over the total number of trials: _____

2. What is the experimental probability that the next outcome will **not** be Heads?

   a. Look in the table to find all the outcomes for heads. Add the numbers to find the total.

   _____

   b. In a fraction, write that number over the total number of trials.

   _____

   c. Subtract that fraction from 1 to find the outcomes for "**not** heads."

   _____

Name _____ Date _____ Class _____

LESSON 12-4

Making Predictions with Experimental Probability

Practice and Problem Solving: A/B

Make a prediction based on experimental probability.

1. A bowler knocks down at least 6 pins 70 percent of the time. Out of 200 rolls, how many times can you predict the bowler will knock down at least 6 pins?

_____

2. A tennis player hits a serve that cannot be returned 45 percent of the time. Out of 300 serves, how many can you predict will not be returned?

_____

3. West Palm Beach, Florida, gets rain about 16 percent of the time. On how many days out of 400 can residents of West Palm Beach predict they will get rain?

_____

4. Rob notices that 55 percent of the people leaving the supermarket choose plastic bags instead of paper bags. Out of 600 people, how many can Rob predict will carry plastic bags?

_____

5. A baseball player reaches base 35 percent of the time. How many times can he expect to reach base in 850 at-bats?

_____

6. Fredericka can make 65 percent of her shots from the free-throw line. If she shoots 75 times, how many shots can she expect to make?

_____

7. In a current-events class, a professor predicted that at least 78 percent of students prefer getting their news from a digital source rather than from a print source. He polled 3 classes. The results are shown in the table below.

	Class 1	Class 2	Class 3
Digital	20	14	30
Print	5	10	7

In which class(es) did his prediction hold true? Explain.

_____

Name _____ Date _____ Class _____

LESSON 12-4

Making Predictions with Experimental Probability

Practice and Problem Solving: C

Solve each problem.

1. The Arno family is planning a 14-day April vacation. The location they've chosen has an average of 10 rainy days every April. The Arnos would like at least 7 days without rain. Should they keep their current plan? Explain.

_____

_____

2. Advertisements claim that the train is on-time 90 percent of the time. The bus has a record of being on-time 56 out of 64 days. Which form of transportation provides more reliable service? Explain.

_____

_____

3. During February and March, Jack is spending 7 days in the Yukon observing endangered species. Historically, the region has snowfall that blocks roads 20 days during these months. Can Jack expect to be able to get around at least 5 of the days? Explain.

_____

_____

4. ABC Airlines has had delays on 18 of 126 recent flights. DEF Airlines has had delays 13 percent of the time.
   a. Which airline would you expect to provide more reliable service? Why?

_____

   b. Over the last 7 days at one airport, DEF Airlines maintained the record shown in this table.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
On Time	5	7	9	10	9	11	6
Delayed	2	6	3	1	1	2	2

On which days, if any, did DEF do better than its average? Explain.

_____

_____

Name _____ Date _____ Class _____

LESSON 12-4

Making Predictions with Experimental Probability

Practice and Problem Solving: D

Solve each problem. The first one is done for you.

1. In 1951, Odessa, Texas had high temperatures of at least 95°F for 11 percent of the year. During that year, how many days could residents predict would have highs of at least 95°F? Show your work.

   Use the proportion to solve. Round to the nearest whole number.

   $\frac{11}{100} = \frac{x}{365}$ $x =$ **40**

   The residents of Odessa could predict highs of at least 95°F on

   40 days of the year.

2. A survey shows that 67 percent of peanut-butter lovers prefer chunky-style. Out of 850 people surveyed, how many can be predicted to say they prefer chunky-style peanut butter?

   Use the proportion to solve. Round to the nearest whole number.

   $\frac{67}{100} = \frac{x}{850}$ $x =$ _____

   _____ people can be expected to say they prefer chunky-style peanut butter.

3. A football player forces at least 1 turnover in 27.5 percent of the games he plays. If the player plays in 57 games, in how many games can he predict he will force a turnover? Show your work.

   Use the proportion to solve. Round to the nearest whole number.

   $\frac{27.5}{100} = \frac{x}{57}$ $x =$ _____

   He can expect to force a turnover in _____ games.

4. Sandy says she splits her time on her homework as follows: 45 percent on math, 20 percent on science, 18 percent on social studies, and 17 percent on language arts.
   a. If Sandy spends 100 hours on homework over a month, predict how much time she spend on each subject.

      Math: _____ Science: _____

      Social Studies: _____ Language Arts: _____

   b. If Sandy only spends 75 hours on homework over a month, predict how much time she spends on each subject to the nearest tenth of an hour.

      Math: _____ Science: _____

      Social Studies: _____ Language Arts: _____

Name _____ Date _____ Class _____

LESSON 12-4

Making Predictions with Experimental Probability

Reteach

When you have information about previous events, you can use that information to predict what will happen in the future.

If you can throw a basketball into the basket 3 out of 5 times, you can predict you will make 6 baskets in 10 tries. If you try 15 times, you will make 9 baskets. You can use a proportion or multiply to make predictions.

A. Use a proportion.

A survey found that 8 of 10 people chose apples as their favorite fruit. If you ask 100 people, how many can you predict will choose apples as their favorite fruit?

$\frac{8}{10} = \frac{x}{100}$ Write a proportion. *8 out of 10 is how many out of 100?*

$\frac{8}{10} = \frac{x}{100}$ (x 10) Since 10 times 10 is 100, multiply 8 times 10 to find the value of *x.*

$x = 80$

You can predict that 80 of the people will choose apples as their favorite fruit.

B. Multiply.

Eric's baseball coach calculated that Eric hits the ball 49 percent of the time. If Eric receives 300 pitches this season, how many times can Eric predict that he will hit the ball?

$0.49 \times 300 = x$

$147 = x$

Eric can predict that he will hit the ball 147 times.

Solve.

1. On average, 25 percent of the dogs who go to ABC Veterinarian need a rabies booster. If 120 dogs visit ABC Veterinarian, how many of them will likely need a rabies booster?

   Set up a proportion: $\frac{\_\_\_\_}{100} = \frac{x}{\_\_\_\_}$

   Solve for *x:* $x =$ _____

   _____ dogs will likely need a rabies booster.

2. About 90 percent of seventh graders prefer texting to emailing. In a sample of 550 seventh graders, how many do you predict will prefer texting?

   $0.9 \times 550 =$ _____

   _____ seventh graders will likely prefer texting.

Name _____ Date _____ Class _____

LESSON 12-4

Making Predictions with Experimental Probability

Reading Strategies: Use Models

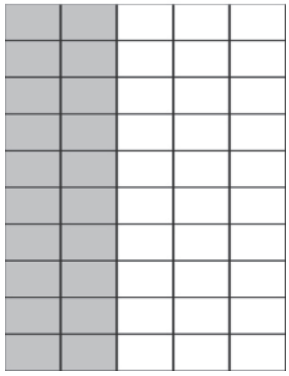
Predicting is making an educated guess about a future result.

You can use **experimental probability** to make a prediction.

Pat is able to flip a game disk into a cup 4 times in 10 tries. Out of 50 tries, how many flips will Pat predict she can make?

Use a grid to model Pat's successful flips: $\frac{4}{10}$.

Then expand your grid to 50 squares to predict the number of Pat's successful flips out of 50.



$$\frac{4}{10} \times 50 = \frac{200}{10} = 20$$

Pat will predict she can make 20 flips out of 50.

Solve. On a separate sheet of paper, create grids to model the situation and make your prediction.

1. A tire manufacturer checks 5 tires and finds that 1 of them has a leak. If they produced 20 tires, how many of them would be likely to have a leak?

_____

Solve. You may use a model if you like.

2. Will has calculated that he usually makes 60 percent of his attempted free throws. How many throws out of 15 should Will predict he can make?

_____

3. An ad for the elevated train line states that it is on time 96 percent of the time. The subway, which Bob has been taking, has been on time 75 times out of 83. Should Bob switch to the elevated train? Why or why not?

_____

_____

_____

Name _____ Date _____ Class _____

LESSON 12-4

Making Predictions with Experimental Probability

Success for English Learners

Problem

The Marino family plans a trip to Florida. They will go for 2 weeks.
They hope to have *at least* 10 out of 14 days when it does **not** rain.

Weather Report for July through September

JULY						
Su	Mo	Tu	We	Th	Fr	Sa
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

AUGUST						
Su	Mo	Tu	We	Th	Fr	Sa
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

SEPTEMBER						
Su	Mo	Tu	We	Th	Fr	Sa
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

All but 19 days are **not** rainy

92 days

19 days are rainy

Should the family go to Florida during those 3 months?

Write a proportion.

$$\frac{\text{rainy days}}{\text{total days}} = \frac{\text{predicted rainy days}}{\text{total vacation days}}$$

$$\frac{19}{92} = \frac{x}{14}$$

$x \approx 2.89$, or about 3 **rainy** days

14 vacation days − 3 rainy days
= **11 days that are not rainy**

The family should go!

Use the information above to answer the questions.

1. What if there were 32 rainy days? Would the family go to Florida? Explain.

_____

2. What if there were 10 rainy days in July and August? Would the family go to Florida? Explain.

_____

Name _____ Date _____ Class _____

MODULE 12

Experimental Probability

Challenge

Shlomo is the manager of a toy company. He needs to select a factory to produce toys his company will sell. His company cannot sell defective toys. The probability that each factory produces defective toys and the maximum production for each factory are shown below.

Factory	Probability of Producing a Defective Toy	Maximum Daily Production
A	$\frac{2}{49}$	3,000
B	$\frac{17}{99}$	3,300
C	$\frac{13}{70}$	2,900
D	$\frac{11}{83}$	3,200

Solve. Show your work.

1. Calculate the expected number of defective toys produced daily in each factory. Which one produces the fewest defective toys each day?

2. Shlomo's company would like to sell at least 2,750 toys each day. Which factory should the company select to produce the toys?

3. Shlomo's company decides to produce toys at Factory A and Factory C. It costs $2.49 to manufacture each toy at Factory A and $1.89 to manufacture each toy at Factory C. The toys sell for $29.99 each. How much profit will the toy company generate in one day?

Name _____ Date _____ Class _____

LESSON 13-1

Theoretical Probability of Simple Events

Practice and Problem Solving: A/B

Find the probability for each event.

1. tossing a number cube numbered from 1 to 6 and getting an even number that is greater than or equal to 2

_____

2. tossing a number cube numbered from 1 to 6 and getting an odd number that is less than or equal to 3

_____

3. randomly selecting a seventh grader from a school that has 250 sixth graders, 225 seventh graders, and 275 eighth graders

_____

4. without looking, **not** picking a red hat from a box that holds 20 red hats, 30 blue hats, 15 green hats, and 25 white hats

_____

Match each event to its likelihood.

5. rolling a number greater than 6 on a number cube labeled 1 through 6 _____ A. likely
6. flipping a coin and getting heads _____ B. unlikely
7. drawing a red or blue marble from a bag of red marbles and blue marbles _____ C. as likely as not
8. spinning a number less than 3 on a spinner with 8 equal sections labeled 1 through 8 _____ D. impossible
9. rolling a number less than 6 on a number cube labeled 1 through 6 _____ E. certain

Use the information to find probabilities in 10–13.

At a school health fair, individual pieces of fruit are placed in paper bags and distributed to students randomly. There are 20 apples, 15 apricots, 25 bananas, 25 pears, and 30 peaches.

10. the probability of getting an apple _____
11. the probability of **not** getting a pear _____
12. the probability of **not** getting an apple _____
13. the probability of getting an orange _____

Name _____ Date _____ Class _____

LESSON 13-1

Theoretical Probability of Simple Events

Practice and Problem Solving: C

Use the information below to answer 1–3.

Three students are playing a video game. Each player is randomly assigned a character from a collection of characters that includes 5 blue, 6 green, and 3 red characters. After each character is picked, it is not replaced in the collection.

1. What is the probability that the first player does **not** get a blue character?

_____

2. The first player gets a blue character. What is the probability that the second player also gets a blue character?

_____

3. Both the first and second players get blue characters. What is the probability that the third player does **not** get a blue character?

_____

Fill in the blank.

4. $P = 0.4$

   Total outcomes: 50

   Number of events: _____

5. Number of events: 75

   $P = 0.3$

   Total outcomes: _____

Use the information below to answer 6–9.

On its first day, a neighborhood pet show includes 5 rabbits, 7 cats, 8 dogs, and 4 hamsters. Each pet has its own petting station. Children who wish to pet the animals are randomly assigned to a station.

6. How many cats would need to be added on the second day to make the probability of picking a cat from the group at least one half?

_____

7. Assume that the cats in question 6 were added on the second day. What is the probability of picking a dog from the new group?

_____

8. On the third day, no more animals were added. What is the probability of picking a rabbit or a hamster on the third day of the show?

_____

9. What is the probability of **not** picking a goldfish on the third day of the show? Explain.

_____

Name _____ Date _____ Class _____

LESSON 13-1

Theoretical Probability of Simple Events

Practice and Problem Solving: D

Solve each problem. The first one is done for you.

1. The kitchen-tile installer has 20 green, 14 beige, and 16 white tiles in a box. What is the probability of picking a beige tile from the box without looking?

$$\frac{14}{20 + 14 + 16} = \frac{14}{50} = \frac{7}{25}$$

2. There are 25 spools each of blue, green, red, white, and yellow thread in the sewing basket. Without looking, what is the probability of picking a spool of blue thread from the basket?

_____

Find the probability. The first one is done for you.

3. A gardener has a bag of flower seeds. Half of the seeds are roses, one fourth are gardenias, and one fourth are irises.

P(gardenias)

$$\frac{1}{4}$$

P(not gardenias)

$$1 - \frac{1}{4} = \frac{3}{4}$$

4. The traffic-control monitor on the freeway shows 200 vehicles per minute passing the camera in 5 minutes. Of those vehicles, on average, 125 have one passenger, 60 have four or fewer passengers, and 15 have more than four passengers.

P(vehicle with more than four people) _____

P(vehicle with four or fewer people) _____

Use the information below to complete the table. The first row is done for you.

Tina has 3 quarters, 1 dime, and 6 nickels in her pocket. Find the probability of randomly drawing each of the following coins.

	Probability		
	Fraction	**Decimal**	**Percent**
5. quarter	$\frac{3}{10}$	**0.3**	**30%**
6. dime			
7. nickel			

Name _____ Date _____ Class _____

LESSON 13-1

Theoretical Probability of Simple Events

Reteach

The probability, P, of an event is a ratio.
It can be written as a fraction, decimal, or percent.

$$P(\text{probability of an event}) = \frac{\text{the number of outcomes of an event}}{\text{the total number of all events}}$$

Example 1

There are 20 red apples and green apples in a bag. The probability of randomly picking a red apple is 0.4. How many red apples are in the bag? How many green apples?

Total number of events $\longrightarrow$ 2

Probability, P: $0.4 = \frac{\text{number of red apples}}{20}$

So:

number of red apples $= 0.4 \times 20 = 8$

number of green apples $= 20 - 8 = 12$

There are 8 red apples and 12 green apples.

Example 2

A bag contains 1 red marble, 2 blue marbles, and 3 green marbles.

The probability of picking a red marble is $\frac{1}{6}$.

To find the probability of **not** picking a red marble, subtract the probability of picking a red marble from 1.

$$P = 1 - \frac{1}{6} = \frac{5}{6}$$

The probability of not picking a red marble from the bag is $\frac{5}{6}$.

Solve.

1. A model builder has 30 pieces of balsa wood in a box. Four pieces are 15 inches long, 10 pieces are 12 inches long, and the rest are 8 inches long. What is the probability the builder will pull an 8-inch piece from the box without looking?

   _____

2. There are 30 bottles of fruit juice in a cooler. Some are orange juice, others are cranberry juice, and the rest are other juices. The probability of randomly grabbing one of the other juices is 0.6. How many bottles of orange juice and cranberry juice are in the cooler?

   _____

3. There are 13 dimes and 7 pennies in a cup.

   a. What is the probability of drawing a penny out without looking?

   _____

   b. What is the probability of **not** drawing a penny? _____

4. If P(event A) $= 0.25$, what is P(**not** event A)? _____

5. If P(**not** event B) $= 0.95$, what is P(event B)? _____

Name _____ Date _____ Class _____

LESSON 13-1

Theoretical Probability of Simple Events

Reading Strategies: Building Vocabulary

The study of probability introduces new words and words used in ways with which you may not be familiar.

- probability ← the likelihood of an event occurring
- event ← an outcome of a calculation or an experiment
- outcome ← the result of an action or a calculation
- experimental probability ← based on experimental data; outcomes may not be equally likely to occur
- theoretical probability ← based on equally-likely outcomes

Several of these terms are combined in the definition of probability, *P*, of Event A occurring.

$$P(\text{event A}) = \frac{\text{number of event A outcomes}}{\text{number of all outcomes}}$$

Event A is an outcome of a calculation or an experiment.

In each situation, identify the outcomes and event. Find the probability.

1. A coin is tossed. What is the probability that a head will occur?

   a. Outcomes: _____

   b. Event: _____

   c. Probability of the event: _____

2. A softball team has a catcher, a pitcher, 4 infielders, and 3 outfielders. One player is chosen at random. What is the probability that the player is an outfielder?

   a. Outcomes: _____

   b. Event: _____

   c. Probability of the event: _____

Label the items.

3. A cube numbered 1 to 6 is rolled. What is the probability of a 4 being rolled?

   1, 2, 3, 4, 5, 6 ← a. _____

   4 ← b. _____

   $\frac{1}{6}$ ← c. _____

Name _____ Date _____ Class _____

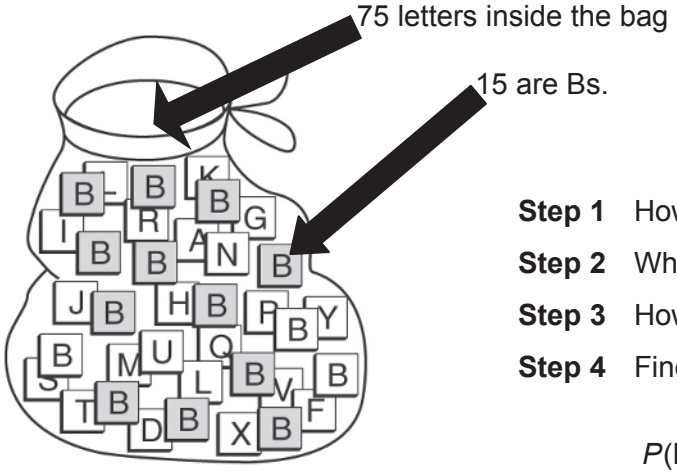
LESSON 13-1

Theoretical Probability of Simple Events

Success for English Learners

Problem 1

What is the probability of picking a B from the bag?



Step 1 How many possible outcomes? 75

Step 2 What event do you want? B

Step 3 How many of that event? 15

Step 4 Find the probability.

$$P(\text{B}) = \frac{\text{number of Bs}}{\text{number of tiles}} = \frac{15}{75} = \frac{1}{5}$$

Problem 2

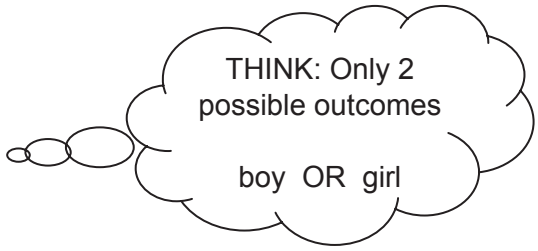
Math class has 25 students. 14 are boys.

How many are girls?

25 students − 14 boys = 11 girls

What is the probability of choosing a girl at random?

$$P(\text{girl}) = \frac{\text{event (girl)}}{\text{outcomes (boys + girls)}} = \frac{11}{25}$$



What is the probability of choosing a boy at random?

So, $P(\text{boy}) = 1 - P(\text{girl})$

$$P(\text{boy}) = 1 - \frac{11}{25} = \frac{14}{25}$$

Find the probability.

1. 12 dogs and 6 cats being given away

   $P(\text{cat}) =$ _____

2. 8 pencils and 5 pens in a bag

   $P(\text{pen}) =$ _____

Name _____ Date _____ Class _____

LESSON 13-2

Theoretical Probability of Compound Events

Practice and Problem Solving: A/B

Use the table of probabilities to answer questions 1–3.

	Burrito	Taco	Wrap
Cheese	$P = \frac{1}{9}$	$P = \frac{1}{9}$	$P = \frac{1}{9}$
Salsa	$P = \frac{1}{9}$	$P = \frac{1}{9}$	$P = \frac{1}{9}$
Veggie	$P = \frac{1}{9}$	$P = \frac{1}{9}$	$P = \frac{1}{9}$

1. List the members of the sample space that include a taco. Use parentheses.

   _____

2. List the members of the sample space that include cheese. Use parentheses.

   _____

3. What is the probability of choosing a burrito with cheese and a taco or a wrap with salsa? Explain.

   _____

   _____

Use the information below to answer questions 4–6.

A basket of 40 pairs of pliers at a discount hardware store includes 5 pairs of 6-inch pliers. A second basket contains 20 hammers, including 3 large hammers.

4. What is the probability of drawing a 6-inch pair of pliers from the first basket without looking? _____

5. What is the probability of **not** drawing a large hammer from the second basket without looking? _____

6. What is the probability of drawing a pair of 6-inch pliers and **not** drawing a large hammer? _____

7. What is the probability of drawing a pair of 6-inch pliers from the second basket? Explain. _____

Name _____ Date _____ Class _____

LESSON 13-2

Theoretical Probability of Compound Events

Practice and Problem Solving: C

The table below lists 3 brands of outdoor lights and 2 colors of lighting. It also identifies some of the probabilities of picking one brand and one color at random. Use the table to answer 1–5. (Hint: The probability in each cell is the product of two probabilities, one for the brand and one for the color.)

	Brand X	Brand Y	Brand Z
blue	0.18		0.3
white	0.12	0.08	

1. What is the probability of picking blue lighting or white lighting?

_____

2. How can you find the probability of picking blue lighting or the probability of picking white lighting made by Brand X?

_____

_____

3. What is the probability of picking Brand X, Y, or Z?

_____

4. What is the probability of picking blue lighting made by Brand Y?

_____

5. What is the probability of picking white lighting made by Brand Z?

_____

Solve.

6. Based on earlier expeditions to a dig site, a geologist expects to find igneous, metamorphic, and sedimentary rocks in the percentages by weight of 25%, 60%, and 15%, respectively. The rock sizes to be found and their percentages of appearance are pebbles (60%), small rocks (20%), medium rocks (15%), and boulders (5%).

   a. What is the most likely combination of rock type and weight the geologist will find? Use compound probability to prove your answer.

   _____

   b. Compute the probabilities for finding the four weights of igneous rock.

   _____

Name _____ Date _____ Class _____

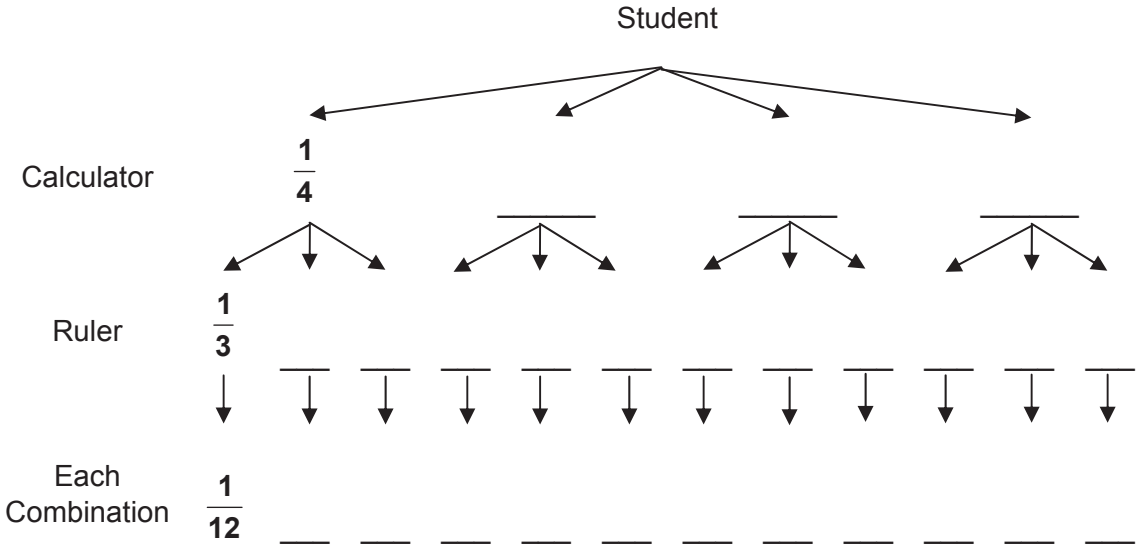
LESSON 13-2

Theoretical Probability of Compound Events

Practice and Problem Solving: D

Solve.

1. Each student receives one of 4 calculator models and one of 3 types of ruler. Fill in the tree diagram to show the probabilities of receiving each type of calculator and ruler. The first one in each row is done for you.



Use the tree diagram to complete Exercises 2–4.

2. What is the probability of receiving each calculator? _____

3. What is the probability of receiving each ruler? _____

4. What is the probability of receiving a certain combination of calculator and ruler? Show how this probability is calculated.

_____

Solve. The first one is done for you.

5. Two students are playing a game with a quarter and a spinner that is divided into equal sixths, with the sections numbered 1 to 6. Each player tosses the coin and spins the spinner.

   a. How many outcomes are possible for the coin toss? List them.

   two: (heads, tails)

   b. How many outcomes are possible for the spin? List them.

   _____

   c. How many outcomes are possible for the toss and spin? List them.

   _____

Name _____ Date _____ Class _____

LESSON 13-2

Theoretical Probability of Compound Events

Reteach

Compound probability is the likelihood of two or more events occurring.

1. To identify the sample space, use a list, tree diagram, or table. If order does not matter, cross out repeated combinations that differ only by order.
2. Count the number of outcomes in the desired event.
3. Divide by the total number of possible outcomes.

A student spins the spinner and rolls a number cube What is the probability that she will randomly spin a 1 and roll a number less than 4?

1. Identify the sample space.
2. Count the number of desired outcomes: 3.
3. Divide by the total number possibilities: 18.

Probability (1 and < 4) $= \frac{3}{18} = \frac{1}{6}$

	1	2	3	4	5	6
1	1-1	1-2	1-3	1-4	1-5	1-6
2	2-1	2-2	2-3	2-4	2-5	2-6
3	3-1	3-2	3-3	3-4	3-5	3-6

At a party, sandwiches are served on 5 types of bread: multi-grain, pita, rye, sourdough, and whole wheat. Sam and Ellen each randomly grab a sandwich. What is the probability that Ellen gets a sandwich on pita or rye and Sam gets a sandwich on multi-grain or sourdough?

1. The table shows the sample space. Draw an X in each cell in which Ellen gets a sandwich on pita or rye.
2. Draw a circle in each cell in which Sam gets a sandwich on multi-grain or sourdough.
3. Count the number of possibilities that have both an oval and a rectangle.

_____

4. Divide the number you counted in Step 4 by the total number of possibilities in the sample space.

_____

This is the probability that Ellen gets a pita or a rye sandwich *and* that Sam gets a multi-grain or a sourdough sandwich.

		Ellen				
		M	P	R	S	W
Sam	M					
	P					
	R					
	S					
	W					

Name _____ Date _____ Class _____

LESSON 13-2

Theoretical Probability of Compound Events

Reading Strategies: Choose a Strategy

Probability is a measure of how likely an event is to occur. To find probability, you must identify the number of possible outcomes.

- A **simple** *event* is an event with a single outcome.
- A **compound event** combines two or more simple events.

To find the number of possible outcomes for a compound event, you must find all the combinations of each of the outcomes of the simple events.

To keep track of the combinations of outcomes, you can create an organized list, table, or tree diagram.

Consider the following guidelines when choosing an organizer.

- A tree diagram will have a branch for each choice of each event. When there are more than three branches or three events, a tree diagram may not be the best choice.

	Organized List	Tree Diagram	Table
2 events	✓	✓	✘
3 events	✓	✓	✘
More than 3 events	✘	✘	✓

- When an event includes computation, a table can provide both a system for doing the mental math and a place to store the results.
- No matter which organizer you choose, if the possible outcomes involve long words, consider using a code for each choice.

Identify the number of events and choices in each situation. Tell which method you would choose to find all possible outcomes.

1. During an early-morning power outage, Sara must get dressed in the dark. Her clothing options include black or blue pants, a white or yellow shirt, and a solid red or a striped scarf.

_____

2. Hector can go to the movies with either Eddie or Miguel. He will see either a comedy or a drama.

_____

3. Ben rolls two six-sided number cubes. If the product of the numbers is an even number, he gets 5 points. If the sum of the numbers is an even number, he gets 2 points.

_____

Name _____ Date _____ Class _____

LESSON 13-2

Theoretical Probability of Compound Events

Success for English Learners

Problem 1

Sal's pizza sells 6 toppings.

extra cheese (C)	onion (O)
green olives (GO)	peppers (P)
mushrooms (M)	tomato (T)

Find the total number of 2-topping combinations.

	C	GO	M	O	P	T
C	~~C-C~~	~~C-GO~~	~~C-M~~	~~C-O~~	~~C-P~~	~~C-T~~
GO	GO-C	~~GO-GO~~	~~GO-M~~	~~GO-O~~	~~GO-P~~	~~GO-T~~
M	M-C	M-GO	~~M-M~~	~~M-O~~	~~M-P~~	~~M-T~~
O	O-C	O-GO	O-M	~~O-O~~	~~O-P~~	~~O-T~~
P	P-C	P-GO	P-M	P-O	~~P-P~~	~~P-T~~
T	T-C	T-GO	T-M	T-O	T-P	~~T-T~~

The pizza must have 2 different toppings. Cross out doubles.

Order does not matter. Cross out duplicates.

There are 15 unique combinations of 2-topping pizzas.

Look at the highlighted cell for T-GO above.

$$P(\text{tomato} + \text{green olive}) = \frac{1}{15}$$

1 combo out of 15 is tomato + green olive.

1. Why are more than half of the combinations crossed out?

_____

_____

2. What pattern do you see in the table?

_____

_____

3. What other ways could you have used to find the combinations?

_____

Name _____ Date _____ Class _____

LESSON 13-3

Making Predictions with Theoretical Probability

Practice and Problem Solving: A/B

In each odd-numbered question, find the theoretical probability. Then use that probability to make a prediction in the even-numbered question that follows it.

1. Martin flips a coin. What is the probability that the coin will land on heads?

   _____

2. Martin flips the coin 64 times. How many times can Martin expect the coin to land on heads?

   _____

3. A spinner is divided into five equal sections labeled 1 to 5. What is the probability that the spinner will land on 3?

   _____

4. If the spinner is spun 60 times, how many times can you expect the spinner to land on 3?

   _____

5. Harriet rolls a number cube. What is the probability that the number cube will land on 3 or 4?

   _____

6. If Harriet rolls the number cube 39 times, how many times can she expect to roll a 3 or 4?

   _____

7. A bag contains 6 red and 10 black marbles. If you pick a marble from the bag, what is the probability that the marble will be black?

   _____

8. If you pick a marble, record its color, and return it to the bag 200 times, how many times can you expect to pick a black marble?

   _____

Make a prediction based on the theoretical probability.

9. Gill rolls a number cube 78 times. How many times can he expect to roll an odd number greater than 1?

   _____

10. Jenna flips two pennies 105 times. How many times can she expect both coins to come up heads?

    _____

11. A shoebox holds a number of disks of the same size. There are 5 red, 6 white, and 7 blue disks. You pick out a disk, record its color, and return it to the box. If you repeat this process 250 times, how many times can you expect to pick either a red or white disk?

    _____

12. Ron draws 16 cards from a deck of 52 cards. The deck is made up of cards of four different colors—red, blue, yellow, and green. How many of the cards drawn can Ron expect to be green?

    _____

Name _____ Date _____ Class _____

LESSON 13-3

Making Predictions with Theoretical Probability

Practice and Problem Solving: C

1. Kamila has two number cubes each labeled 1 to 6. She is going to conduct an experiment by tossing both cubes a total of 150 times. She will find the sum of the two numbers in each roll.

   a. How many possible outcomes are there? _____

   b. What is the probability of tossing a sum of 6? _____

   c. How many times should Kamila toss a sum of 7? _____

   d. How many times should Kamila toss a sum of 10 or greater?

   _____

2. Eric has two number cubes each labeled 1 to 6. Eric is going to conduct an experiment by tossing the cubes a total of 180 times. He will find the product of the two numbers in each roll.

   a. How many possible outcomes are there? _____

   b. How many times should Eric toss a product of 12? _____

   c. How many times should Eric toss a product greater than 20?

   _____

   d. How many times should Eric toss a product less than 10?

   _____

3. Natalie has two number pyramids each labeled 1 to 4. Natalie is going to conduct an experiment by tossing both pyramids a total of 96 times. She will find the difference of each pair of numbers rolled by subtracting the lesser number from the greater number.

   a. How many possible outcomes are there? _____

   b. How many times should Natalie toss a difference of 1? _____

   c. How many times should Natalie toss a difference of 0? _____

Name _____ Date _____ Class _____

LESSON 13-3

Making Predictions with Theoretical Probability

Practice and Problem Solving D

Find the probability of each event. The first one is done for you.

1. Arjan flips a quarter. What is the probability of the quarter landing tails up?

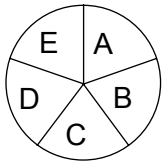
   $\frac{1}{2}$

2. Stephanie rolls a number cube that has sides numbered from 1 to 6. What is the probability of the cube landing on either 2 or 5?

   _____

3. What is the probability of spinning this spinner and having it land on B?

   _____



4. Jonathan has a bag that has 2 red marbles and 3 blue marbles inside of it. If you were to pick one marble from the bag without looking, what is the probability of picking a red marble?

   _____

Make a prediction based on a theoretical probability. Show your work. The first one is done for you.

5. The probability of flipping a coin and having it land on heads is $\frac{1}{2}$. If a coin is tossed 4 times, how many times can you expect it to land on heads?

   $\frac{1}{2} \times 4 = \frac{1}{2} \times \frac{4}{1} = \frac{4}{2} = 2$

6. A spinner is divided into 4 equal sections. The probability of landing on A is $\frac{1}{4}$. Norma spins the spinner 16 times. How many times can she expect the spinner to land on A?

   _____

7. The probability of a number cube landing on 4 is $\frac{1}{6}$. If a number cube is tossed 12 times, how many times can it be expected to land on 4?

   _____

8. The probability of picking a blue pen from a cup of pens is $\frac{1}{3}$. Tim picks one pen from the cup without looking, records the color, and puts the pen back. He does this 15 times. How many times can he expect to pick a blue pen?

   _____

Name _____ Date _____ Class _____

LESSON 13-3

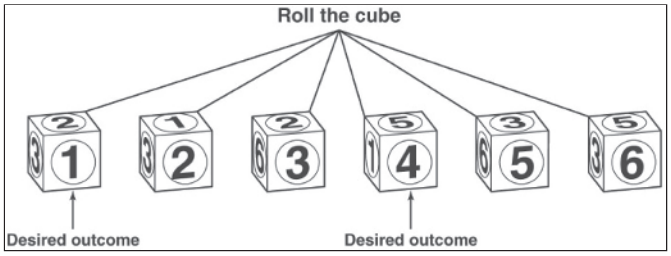
Making Predictions with Theoretical Probability

Reteach

Predictions are thoughtful guesses about what will happen.
You can create an "outcome tree" to keep track of outcomes.

Sally is going to roll a number cube 21 times.
She wants to know how many times she can expect to roll a 1 or 4.

There are a total of 6 **outcomes**.
Of these, *two* outcomes (1 and 4) are desirable.



Use probability to predict the number of times Sally would roll a 1 or 4.

$$P(1 \text{ or } 4) = \frac{\text{number of desirable outcomes}}{\text{number of possible outcomes}} = \frac{2}{6} = \frac{1}{3}$$

Set up a proportion relating the probability to the number of tries.

$$\frac{1}{3} = \frac{x}{21}$$

$3x = 21$ Cross-multiply.

$x = 7$ Simplify.

In 21 tries, Sally can expect to roll seven 1s or 4s.

For each odd-numbered question, find the theoretical probability. Use that probability to make a prediction in the even-numbered question that follows it.

1. Sandra flips a coin. What is the probability that the coin will land on tails?

   _____

2. Sandra flips the coin 20 times. How many times can Sandra expect the coin to land on tails?

   _____

3. A spinner is divided into four equal sections labeled 1 to 4. What is the probability that the spinner will land on 2?

   _____

4. If the spinner is spun 80 times, how often can you expect it to land on 2?

   _____

Name _____ Date _____ Class _____

LESSON 13-3

Making Predictions with Theoretical Probability

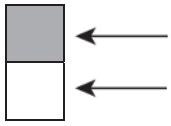
Reading Strategies: Use a Model

Predicting is making a thoughtful guess about a future result. You can use theoretical probability to make a prediction.

Al flips a coin 28 times. How many times can he expect to flip heads?

Make a bar model to help make a prediction.

Step 1 Find the theoretical probability of the coin landing on heads.

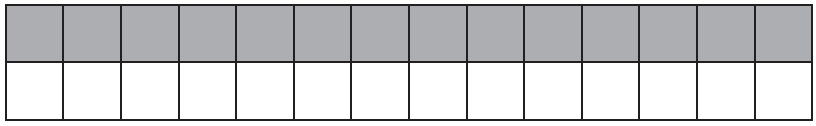


The shaded part represents heads.

The white part represents tails.

The probability of the coin landing on heads is $\frac{1}{2}$.

Step 2 Extend the model to show 28 tries.



$$\frac{1}{2} \times \frac{28}{1} = \frac{28}{2} = 14$$

Al can expect the coin to land on heads 14 times in 28 tries.

Use the probability to make a prediction.

1. Li rolls a number cube labeled 1 to 6 a total of 24 times. How many times can she expect to roll a 1?

_____

2. The theoretical probability of rolling a 1 or 2 is $\frac{1}{3}$. Out of 15 rolls, how many can you expect to be a 1 or 2?

_____

3. The theoretical probability of spinning green on a spinner is $\frac{1}{4}$. How many spins in 32 tries can you expect to land on green?

_____

4. The theoretical probability of drawing a red marble is $\frac{1}{9}$. How many red marbles can you expect to get in 72 draws?

_____

Name _____ Date _____ Class _____

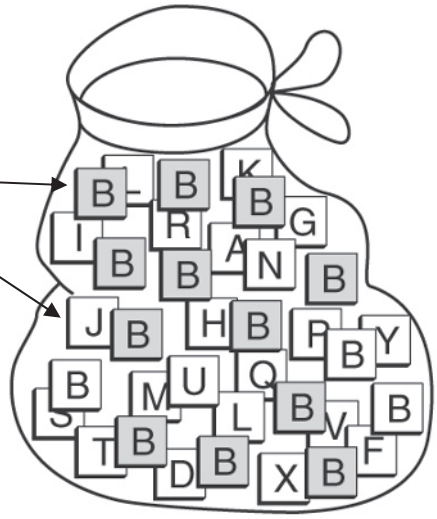
LESSON 13-3

Making Predictions with Theoretical Probability

Success for English Learners

Problem 1

What is the theoretical probability of choosing a B from the bag?



$$\textbf{Theoretical probability} = \frac{\text{number of desirable outcomes}}{\text{number of possible outcomes}}$$

$$= \frac{15}{75} = \frac{1}{5}$$

Problem 2

You pick a tile from the bag without looking.

You put it back. You pick again.

If you pick 30 times, how many times should you get a B?

Step 1 Write a proportion. $\frac{1}{5} = \frac{x}{30}$

Step 2 Cross-multiply. $5 \cdot x = 1 \cdot 30$

$5x = 30$

Step 3 Divide each side by 5. $\frac{5x}{5} = \frac{30}{5}$

$x = 6$

In 30 picks, you can expect to get 6 Bs.

1. Explain the difference between theoretical probability and experimental probability.

_____

_____

2. Explain how to use the theoretical probability to make a prediction.

_____

_____

3. Write your own word problem about using theoretical probability to make a prediction.

_____

_____

Name _____ Date _____ Class _____

LESSON 13-4

Using Technology to Conduct a Simulation

Practice and Problem Solving: A/B

Answer the questions below.

1. A marine biologist has historical records to show that the chance of finding shrimp in a catch of ocean animals is 20 percent. The simulation below models the experimental probability of finding shrimp in at least one of the next 5 catches. The numbers 1 and 2 represent catches with shrimp. The numbers 3–10 represent catches without shrimp.

   a. What does the marine biologist do?

   _____

   _____

   b. Here is the table the marine biologist created. Fill in the missing data.

Trial	Numbers Generated	Shrimp Caught	Trial	Numbers Generated	Shrimp Caught
1	7, 3, 2, 7, 10		6	8, 4, 7, 6, 5	
2	2, 4, 5, 3, 10		7	6, 10, 1, 7, 6	
3	9, 9, 7, 6, 6		8	7, 9, 8, 3, 8	
4	7, 9, 6, 6, 4		9	1, 4, 4, 8, 9	
5	10, 6, 4, 6, 4		10	7, 8, 9, 5, 3	

2. According to the simulation above, what is the experimental probability that shrimp will be caught in at least one of the next 5 catches?

   _____

3. At a television game show, prizes are placed under 10 percent of the seats in the studio audience. What is the experimental probability that you have to reserve exactly 4 seats before you win a prize?

   a. Describe a model to use for this simulation.

   _____

   _____

   _____

   b. Give an example of a trial that would result in winning a prize for exactly 4 seats.

   _____

Name _____ Date _____ Class _____

LESSON 13-4

Using Technology to Conduct a Simulation

Practice and Problem Solving: C

Answer the questions below.

1. During the regular season, a soccer team has a 30 percent chance of scoring more than 4 goals in a match. Use a calculator and the table to find the experimental probability that the team will score exactly 4 goals in a match.

Trial	Numbers Generated	Result
1		
2		
3		
4		
5		

2. What is the experimental probability based on these 5 trials?

_____

3. What do you think will happen to the *total* experimental probability if 5 more trials are run?

_____

4. Complete the table below for 5 more trials.

Trial	Numbers Generated	Result
6		
7		
8		
9		
10		

5. What is the experimental probability based on all 10 trials?

_____

Name _____ Date _____ Class _____

LESSON 13-4

Using Technology to Conduct a Simulation

Practice and Problem Solving: D

Use the information below to complete Exercises 1–3. The first parts of 1 and 2 are done for you.

A pizza parlor puts coupons in 25 percent of its pizza boxes. Answer the questions to find the experimental probability that a customer would need to buy exactly 4 pizzas before finding a coupon.

1. Choose a model.

   a. What is the probability of finding a coupon?

   $$25\% = \frac{25}{100} = \frac{1}{4}$$

   b. If you use the whole numbers 1–4 to represent getting or not getting a winning number, what would the winning number(s) be?

   _____

   c. What would the non-winning number(s) be?

   _____

2. Use your calculator to generate some random numbers for 10 trials. Remember, you are looking for exactly 4 pizzas, at least 1 of which has a coupon. Two trials are done for you. Fill in the rest of the table with your randomly generated numbers.

Trial	Numbers Generated	Pizzas Bought	Trial	Numbers Generated	Pizzas Bought
1	**3, 1, 1, 3**	4	6		
2	**3, 2, 4, 2**	4	7		
3			8		
4			9		
5			10		

   Why is Trial 1 a winner but Trial 2 is not? (Hint: Is there any limit on how many of the 4 boxes in a trial can have a coupon?)

   _____

3. Find the experimental probability of needing to buy exactly 4 pizzas before finding a coupon in the 10 trials.

   The experimental probability = ____ trials ÷ 10 trials = ____.

Name _____ Date _____ Class _____

LESSON 13-4

Using Technology to Conduct a Simulation

Reteach

Use a graphing calculator to help you conduct a probability simulation.

There is a 20 percent possibility of rain during the week of the school fair. What is the experimental probability that it will rain on at least one of the days of the festival, Monday through Friday?

Step 1 Choose a model.

Probability of rain: $20\% = \frac{20}{100} = \frac{1}{5}$

Use whole numbers 1–5 for the days.

Rain: 1 No rain: 2–5

Step 2 Generate random numbers from 1 to 5 until you get a 1.

Example: 1, 2, 2, 5, 2

This trial counts as an outcome that it will rain on at least one of the days of a week.

Step 3 Perform multiple trials by repeating Step 2:

Trial	Numbers Generated	Rain	Trial	Numbers Generated	Rain
1	1, 2, 2, 5, 2	1	6	1, 4, 5, 5, 3	1
2	5, 2, 2, 2, 3	0	7	3, 4, 5, 2, 2	0
3	5, 2, 3, 1, 5	1	8	4, 1, 2, 2, 2	1
4	3, 2, 3, 2, 2	0	9	2, 2, 2, 4, 2	0
5	3, 2, 2, 2, 2	0	10	2, 2, 4, 3, 3	0

Step 4 In 10 trials, the experimental probability that it will rain on 1 of the school days is 4 out of 10 or 40 percent, 0.4, or $\frac{2}{5}$ (two-fifths).

Find the experimental probability. Draw a table on a separate sheet of paper and use 10 trials.

1. An event has 5 outcomes. Each outcome: 50-50 chance or more.

_____

_____

2. An event has a 40 percent probability. Each outcome: exactly 3-in-5 chance.

_____

_____

Name _____ Date _____ Class _____

LESSON 13-4

Using Technology to Conduct a Simulation

Reading Strategies: Read a Table

A table that contains data from a simulation includes several types of information that must be interpreted.

The *Theoretical* Probability of an Event

Suppose there is a 1-in-6 chance that an event will occur. This can be represented with the numbers 1 through 6.

1 ⟶ The event occurs.

2–6 ⟶ The event does *not* occur.

These numbers are found in the columns labeled "Numbers Generated."

Trial	Numbers Generated	Result	Trial	Numbers Generated	Result
1	**1**, 4, 4, 5, 6, **1**	2	6	2, 5, 2, 2, 5, 2	0
2	6, 5, 6, **1**, **1**, 5	2	7	**1**, **1**, 6, **1**, 5, **1**	4
3	2, 4, **1**, 3, 5, 6	1	8	**1**, **1**, 5, **1**, 6, 5	3
4	2, 6, **1**, 3, **1**, 4	2	9	6, 6, 3, **1**, 3, 5	1
5	2, **1**, 4, 2, **1**, **1**	3	10	3, 6, 5, **1**, 6, 6	1

The *Experimental* Probability of an Event

Next, suppose you are asked to find the probability that the event occurs no more than 2 times in 10 trials. The event is a "1". In which trials does a "1" occur once or twice? Those outcomes are in bold above.

- To calculate the experimental probability, count the results 1 or 2 in the "Result" columns. There are 6 results of 1 or 2.
- The experimental probability is 6 out of 10, 0.6, 60 percent, or $\frac{3}{5}$.

Give the theoretical probability. Then, give the numbers for the random-number generation. Finally, find the experimental probability.

1. The chance of drawing a blue pen is 25 percent. What is the experimental probability for exactly 2 chances out of 10?

_____

_____

2. The chance of snow during the month of January is 7 out of 8. What is the experimental probability for at most 1 chance out of 10?

_____

_____

Name _____ Date _____ Class _____

LESSON 13-4

Using Technology to Conduct a Simulation

Success for English Learners

Problem

The six sides of a cube are numbered 1 to 6.

What is the chance of rolling a 5?

1 out of 6. That is about 17%.

Method 1

To find the **experimental probability,** you could roll the cube 60 times and record the results.

Method 2

There is another way to find the **experimental probability.** You could generate random numbers from 1 to 6.

- 5 is your desired outcome.
- So, 1, 2, 3, 4, and 6 are the other possible outcomes.
- Now, run 10 trials. Randomly generate 6 numbers in each trial.
- Record the results of your 10 trials in the chart. The first trial has been done for you.

Trial	Numbers	Desired Outcome?	Trial	Numbers	Desired Outcome?
1	**4, 3, 2, 2, 5**	**1**	6		
2			7		
3			8		
4			9		
5			10		

1. Use the data in your simulation. What is the experimental probability of rolling a 5?

_____

2. Look at your data. What is the experimental probability of rolling an odd number (the outcomes 1, 3, 5)?

_____

3. Choose your own outcome. Use the data in the chart to find its experimental probability. What is the experimental probability of your chosen outcome?

_____

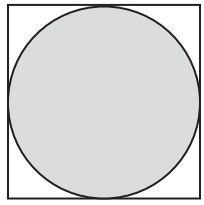
Name _____ Date _____ Class _____

MODULE 13

Theoretical Probability and Simulations

Challenge

1. Julia is shooting arrows at a target. The shape of the target is a square with a circle inscribed, as shown at right.

   Julia shoots an arrow and randomly hits a point within the target. What is the probability that the arrow lands inside the circle? Show your work. (Hint: *Area of a circle* $= \pi r^2$)

2. Tobias used a simulation to predict the number of rainy days in his town in a given week. In the simulation a "0" or "1" represents a rainy day. The numbers "2" through "9" represent sunny days. The results of the simulation are shown in the table below.

Trial	Simulation	Trial	Simulation
1	1, 2, 3, 2, 1, 0, 9	6	6, 5, 3, 9, 9, 8, 3
2	7, 8, 1, 9, 8, 5, 4	7	4, 3, 5, 6, 9, 2, 3
3	4, 5, 9, 9, 0, 2, 4	8	5, 5, 1, 8, 9, 0, 0
4	4, 3, 5, 2, 6, 9, 3	9	4, 5, 2, 8, 9, 0, 3
5	5, 5, 3, 2, 5, 1, 9	10	5, 0, 0, 9, 1, 8, 1

   Tobias claims that the probability of two or more rainy days in a week is greater than the probability of no rainy days in a week.

   a. According to the simulation model is Tobias correct? Explain.

   b. Use the simulation to calculate the probability of 0, 1, 2, 3, 4, 5, 6, and 7 rainy days in a week. Show your work. Which number of rainy days is most likely?

UNIT 6: Probability

MODULE 12 Experimental Probability

LESSON 12-1

Practice and Problem Solving: A/B

1. certain; 1
2. as likely as not; $\frac{1}{2}$
3. impossible; 0
4. $\frac{2}{3}$
5. $\frac{4}{5}$
6. $\frac{1}{2}$
7. No, 6 of the 9 cards involve forward moves. The probability of moving backward is $\frac{1}{3}$.
8. No; Only two cards will let him win. The probability that he will not win on his next turn is $\frac{7}{9}$.

Practice and Problem Solving: C

1. $\frac{4}{5}$
2. $\frac{4}{11}$
3. $\frac{3}{8}$
4. $\frac{2}{3}$
5. $\frac{1}{2}$
6. There were 8 cans in the cabinet, including 1 chicken noodle. Mother added 2 cans of chicken noodle soup and 5 cans of vegetable soup. So, there are 15 cans of soup, 3 of which are chicken noodle.
7. Answers will vary. Sample answer: The spinner is marked with numbers 1, 2, 3, 3, 4, 5, 5, 5. What is the probability that the spinner will not land on 5? $\left(\frac{5}{8}\right)$.

Practice and Problem Solving: D

1. A
2. C
3. B
4. E
5. D
6. $\frac{7}{9}$
7. $\frac{5}{6}$
8. as likely as not; Since he gets up by 7:15 about half the time, he will ride his bicycle about half the time. The probability is about $\frac{1}{2}$, or as likely as not.
9. likely; The probability of choosing a short-sleeved shirt is $\frac{4}{5}$, or likely.

Reteach

1. unlikely; $\frac{1}{24}$
2. as likely as not; $\frac{1}{2}$
3. impossible; 0

Reading Strategies

1. unlikely
2. impossible
3. certain

4.

Possible Outcomes	Desired Outcomes: 6	Desired Outcomes: Factor of 4	Desired Outcomes: Greater than 0
0	no	no	no
1	no	yes	yes
2	no	yes	yes
3	no	no	yes
4	no	yes	yes
5	no	no	yes
Results	0 out of 6	3 out of 6	5 out of 6
Probability	impossible	as likely as not	likely

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1. as likely as not; Sample answer: because there are 3 even numbers and 3 numbers that are not even
2. impossible; There are no purple marbles in the bag.

LESSON 12-2

Practice and Problem Solving: A/B

1. $\frac{11}{15}$
2. $\frac{7}{20}$
3. $\frac{2}{7}$
4. a. $\frac{99}{130}$

   b. $\frac{31}{130}$
5. a. $\frac{5}{8}$, 0.625, 62.5%

   b. $\frac{3}{8}$, 0.375, 37.5%

Practice and Problem Solving: C

1. a. $\frac{1}{150}$

   b. 14

2. a. $\frac{9}{200}$

   b. 270
3. a. $\frac{24}{25}$

   b. 400
4. a. $\frac{13}{8000}$

   b. Yes. The percent of defective spark plugs is 0.1625%, which is less than 2%.
5. a. $\frac{23}{300}$

   b. No. The percent of defective switches is 7.67%, which is greater than 1.5%.

Practice and Problem Solving: D

1. a. 9

   b. 15

   c. $\frac{9}{15} = \frac{3}{5}$
2. a. 40

   b. 48

   c. $\frac{40}{48} = \frac{5}{6}$
3. a. 36

   b. 132

   c. $\frac{36}{132} = \frac{3}{11}$

   d. $\frac{96}{132} = \frac{8}{11}$

Reteach

1. a. 12

   b. 15

   c. $\frac{12}{15} = \frac{4}{5}$
2. a. 9

   b. 14

   c. $\frac{9}{14}$
3. $P(\text{catch}) = \frac{4}{5}$; $P(\text{no catch}) = 1 - \frac{4}{5} = \frac{1}{5}$

Reading Strategies

1. 3; Sample: There are more 3's than any other number, so the probability that you will land on 3 is would be greater than the probability for the other numbers.
2. 1; Sample: There is only one 1, so the probability that you will on 1 is lower than the probability you will land on the other numbers.
3. Sample: No, I predicted the cube would land on 1 the least number of times.
4. Sample: No, I predicted the cube would land on 3 most often.

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1. a. 28

   b. 40

   c. $\frac{28}{40} = \frac{7}{10}$

2. $\frac{18}{52} = \frac{9}{26}$; $1 - \frac{9}{26} = \frac{17}{26}$
3. Sample answer: Elena tossed a coin 30 times. It landed on heads 18 times. What is the experimental probability the coin will land on heads on the next toss? $\left(\frac{18}{30} = \frac{3}{5}\right)$

LESSON 12-3

Practice and Problem Solving: A/B

1. $\frac{62}{354} = \frac{31}{177}$
2. $\frac{39}{160}$
3. $\frac{23}{137}$
4. $\frac{170}{190} = \frac{17}{19}$

Practice and Problem Solving: C

1. a. 50;

   b. $\frac{182}{250} = \frac{91}{125}$

2. Sample answer: You could use a spinner with 3 equal sections for the individual, pair, and team. You could use notecards for the artistry points, and a number cube for the precision points.
3. Sample answer: Tossing two number cubes to advance around a board game.
4. Sample answer: Boys and girls being assigned to either a science class or a reading class when the number of boys and girls is not equal.

Practice and Problem Solving: D

1. a. 32

   b. 100

   c. $\frac{32}{100} = \frac{8}{25}$

2. $\frac{8}{50} = \frac{4}{25}$
3. $\frac{45}{200} = \frac{9}{40}$

Reteach

1. 200
2. $\frac{19}{200}$
3. $\frac{85}{200} = \frac{17}{40}$
4. $\frac{136}{200} = \frac{17}{25}$

Reading Strategies

1.

Section	Heads	Tails
1	3	4
2	2	3
3	5	3

2. $\frac{3}{20}$
3. $\frac{1}{10}$
4. $\frac{9}{10}$
5. $\frac{1}{2}$

Success for English Learners

1. a. 5

   b. $\frac{5}{50} = \frac{1}{10}$

2. a. $4 + 3 + 6 + 4 + 4 + 5 = 26$

b. $\frac{26}{50} = \frac{13}{25}$

c. $1 - \frac{13}{25} = \frac{12}{25}$

LESSON 12-4

Practice and Problem Solving: A/B

1. 140 times
2. 135 serves
3. 64 days
4. 330 people
5. 298 times
6. 49 shots
7. in Classes 1 and 3, because the percents preferring digital were 80% and 81%

Practice and Problem Solving: C

1. Yes, they should keep their plans. The location is likely to provide over 9 days without rain.
2. The train is more reliable. The bus is on-time 87.5% of the time, while the train is on-time 90% of the time.
3. No. It is likely to snow heavily more than two of the days.
4. a. DEF provides more reliable service. They are late only 13% of the time, while ABC is late more than 14% of the time.

   b. DEF did better than its average on Thursday and Friday, with delays of 9% and 10%.

Practice and Problem Solving: D

1. 40; 40
2. 570; 570
3. 15.675; 16
4. a. Math: 45 h; Science: 20 h; Social Studies: 18 h; Language Arts: 17 h

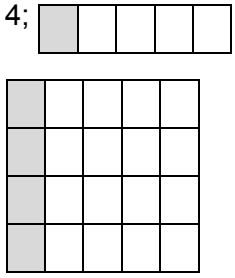
   b. Math: 33.8 h; Science: 15 h; Social Studies: 13.5 h; Language Arts: 12.8 h

Reteach

1. $\frac{25}{100} = \frac{x}{120}$; 30; 30
2. 495; 495

Reading Strategies

1. 4;



2. 9
3. Yes. The subway has been on time about 90% of the time. The elevated train is on time about 96% of the time.

Success for English Learners

1. No; $\frac{32}{91} = \frac{x}{14}$; $x = 4.9$, or about 5 days;
   $14 - 5 = 9$ days
2. Yes; $\frac{10}{62} = \frac{x}{14}$; $x = 2.3$, or about 2 days;
   $14 - 2 = 12$ days

MODULE 12 Challenge

1. The expected daily number of defective toys produced in each factory is calculated by multiplying the probability of producing a defective toy by the total production in each factory.

   Factory A: $\frac{2}{49} \times 3{,}000 \approx 122$

   Factory B: $\frac{17}{99} \times 3{,}300 \approx 567$

   Factory C: $\frac{13}{70} \times 2{,}900 \approx 539$

   Factory D: $\frac{11}{83} \times 3{,}200 \approx 424$

   Factory A produces the least defective toys.
2. Shlomo can select Factory A or Factory D.
   Factory A produces $3{,}000 - 122 = 2{,}878$ toys that can be sold.
   Factory D produces $3{,}200 - 424 = 2{,}776$ toys that can be sold.
3. Factory A produces $3{,}000 - 122 = 2{,}878$ toys that can be sold.
   Factory C produces $2{,}900 - 539 = 2{,}361$ toys that can be sold.
   The two factories produce $2{,}878 + 2{,}361 = 5{,}239$ toys that can be sold in one day.
   The total revenue produced by the factory

is $5{,}239 \times \$29.99 = \$157{,}117.61$.
Each day Factory A spends $3{,}000 \times \$2.39 = \$7{,}170$ to produce toys.
Each day Factory C spends $2{,}900 \times \$1.89 = \$5{,}481$ to produce toys.
The total expenses in Factory A and Factory C are $\$7{,}170 + \$5{,}481 = \$12{,}651$.
The profit earned in one day is $\$157{,}117.61 - \$12{,}651 = \$144{,}466.61$.

MODULE 13 Theoretical Probability and Simulations

LESSON 13-1

Practice and Problem Solving: A/B

1. $\frac{1}{2}$
2. $\frac{1}{3}$
3. 0.3
4. $\frac{7}{9}$
5. D
6. C
7. E
8. B
9. A
10. $\frac{4}{23}$
11. $\frac{18}{23}$
12. $1 - \frac{4}{23} = \frac{19}{23}$
13. 0

Practice and Problem Solving: C

1. $\frac{9}{14}$
2. $\frac{4}{13}$
3. $\frac{3}{4}$
4. 20
5. 250
6. 10 cats
7. $\frac{4}{17}$
8. $\frac{9}{34}$
9. $\frac{34}{34}$ or 1. Since there are no goldfish in the show, it is certain that one will not be picked.

Practice and Problem Solving: D

1. $\frac{7}{25}$
2. $\frac{1}{5}$
3. $\frac{1}{4}; \frac{3}{4}$
4. $\frac{3}{40}; \frac{37}{40}$
5. $\frac{3}{10}$; 0.3; 30%
6. $\frac{1}{10}$; 0.1; 10%
7. $\frac{6}{10}$ or $\frac{3}{5}$; 0.6; 60%

Reteach

1. $\frac{8}{15}$
2. 12 bottles of orange juice and cranberry juice
3. a. $\frac{7}{20}$

   b. $\frac{13}{20}$
4. 0.75
5. 0.05

Reading Strategies

1. a. heads or tails

   b. heads

   c. 0.5 or $\frac{1}{2}$
2. a. any of the 9 players

b. an outfielder

c. $\frac{3}{9}$ or $\frac{1}{3}$

3. a. outcomes

b. event

c. theoretical probability

Success for English Learners

1. $\frac{6}{18}$ or $\frac{1}{3}$

2. $\frac{5}{13}$

LESSON 13-2

Practice and Problem Solving: A/B

1. (Taco, Cheese), (Taco, Salsa), (Taco, Veggie)
2. (Burrito, Cheese), (Taco, Cheese), (Wrap, Cheese)
3. $P(\text{Burrito/Cheese}) = \frac{1}{9}$; $P(\text{Taco or Wrap with salsa}) = \frac{2}{9}$;

   $P(\text{Burrito/Cheese and Taco or Wrap with Salsa}) = \frac{1}{9} \times \frac{2}{9} = \frac{2}{81}$, since these are independent events.
4. $\frac{1}{8}$
5. $1 - \frac{3}{20} = \frac{17}{20}$
6. $P = \frac{1}{8} \times \frac{17}{20} = \frac{17}{160}$, since these are independent events.
7. $P = 0$. There are no pliers in the second basket.

Practice and Problem Solving: C

1. $P(\text{blue}) + P(\text{white}) = P(\text{blue or white}) = 1$
2. Let B = blue and W = white. $P(X) \bullet P(B) = 0.18$; $P(X) \bullet P(W) = 0.12$; $0.18 \bullet P(W) = 0.12 \bullet P(B)$ and from Ex. 1, $P(B) + P(W) = 1$, which gives $P(B) = 0.6$ and $P(W) = 0.4$.
3. The values of $P(B)$ and $P(W)$ can be used with either row of brands X, Y, and Z to find those values by a process of elimination:

   $P(X) = 0.3$; $P(Y) = 0.2$; $P(Z) = 0.5$
4. $P(B) \bullet P(Y) = 0.6 \bullet 0.2 = 0.12$
5. $P(W) \bullet P(Z) = 0.4 \bullet 0.5 = 0.2$
6. a. $P(\text{metamorphic}) \bullet P(\text{pebbles}) = 0.6 \bullet 0.6 = 0.36$

   b. $P(\text{igneous}) = 0.25$, so pebbles: $(0.25)(0.6) = 0.15$; small rocks: $(0.25)(0.2) = 0.05$; medium rocks: $(0.25)(0.15) = 0.0375$; boulders: $(0.25)(0.05) = 0.0125$

Practice and Problem Solving: D

1. calculator: $\frac{1}{4}; \frac{1}{4}; \frac{1}{4}; \frac{1}{4}$; ruler: $\frac{1}{3}; \frac{1}{3}; \frac{1}{3}; \frac{1}{3}; \frac{1}{3}; \frac{1}{3}; \frac{1}{3}; \frac{1}{3}; \frac{1}{3}; \frac{1}{3}; \frac{1}{3}; \frac{1}{3}$

   each combination of calculator and ruler: $\frac{1}{12}; \frac{1}{12}; \frac{1}{12}; \frac{1}{12}; \frac{1}{12}; \frac{1}{12}; \frac{1}{12}; \frac{1}{12}; \frac{1}{12}; \frac{1}{12}; \frac{1}{12}; \frac{1}{12}$
2. $\frac{1}{4}$
3. $\frac{1}{3}$
4. $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$
5. a. two: (heads, tails)

   b. six: (1, 2, 3, 4, 5, 6)

   c. twelve: (H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6)

Reteach

1–2.

		Ellen				
		M	P	R	S	W
Sam	M	○	⊗	⊗	○	○
	P		×	×		
	R		×	×		
	S	○	⊗	⊗	○	○
	W		×	×		

3. 4 possibilities

4. $P = \frac{4}{25}$

Reading Strategies

1. There are 3 events: picking pants, shirts, and scarves; 2 pants × 2 shirts × 2 scarves give 8 choices. Answers will vary. Sample answer: Use a tree diagram.
2. There are two events: person, movie genre; 2 people × 2 movie genres give 4 choices. Answers will vary. Sample answer: Use a list.
3. There are more than three events: 36 products and 36 sums. For an even product, there are 27 choices; for an even sum, there are 18 choices. Use a table.

Success for English Learners

1. They are duplicates.
2. Sample answer: The “doubles” such as C-C ad GO-GO form a diagonal from upper left to lower right.
3. Sample answer: tree diagram

LESSON 13-3

Practice and Problem Solving: A/B

1. $\frac{1}{2}$
2. 32
3. $\frac{1}{5}$
4. 12
5. $\frac{1}{3}$
6. 13
7. $\frac{5}{8}$
8. 125
9. 26
10. about 26
11. about 153
12. 4

Practice and Problem Solving: C

1. a. 36
   b. $\frac{5}{36}$
   c. 25
   d. 25
2. a. 36
   b. 20
   c. 30
   d. 85
3. a. 16
   b. 36
   c. 24

Practice and Problem Solving: D

1. $\frac{1}{2}$
2. $\frac{1}{3}$
3. $\frac{1}{5}$
4. $\frac{2}{5}$
5. $\frac{1}{2} \times 4 = \frac{1}{2} \times \frac{4}{1} = \frac{4}{2} = 2$
6. $\frac{1}{4} \times 16 = \frac{1}{4} \times \frac{16}{1} = \frac{16}{4} = 4$
7. $\frac{1}{6} \times 12 = \frac{1}{6} \times \frac{12}{1} = \frac{12}{6} = 2$
8. $\frac{1}{3} \times 15 = \frac{1}{3} \times \frac{15}{1} = \frac{15}{3} = 5$

Reteach

1. $\frac{1}{2}$
2. 10
3. $\frac{1}{4}$
4. 20

Reading Strategies

1. 4
2. 5
3. 8
4. 8

Success for English Learners

1. Theoretical probability is based on what should happen. Experimental probability is based on what has already happened.
2. To make a prediction, multiply the theoretical probability times the number of trials.
3. Answers may vary. Sample answer: Max rolls a number cube labeled 1–6 a total of 60 times. How many times can he expect the cube to land on 6? (10)

LESSON 13-4

Practice and Problem Solving: A/B

1. a. He or she runs multiple trials with 5 random numbers between 1 and 10 in each.

   b.

Trial	Numbers Generated	Shrimp Caught	Trial	Numbers Generated	Shrimp Caught
1	7, 3, 2, 7, 10	1	6	8, 4, 7, 6, 5	0
2	2, 4, 5, 3, 10	1	7	6, 10, 1, 7, 6	1
3	9, 9, 7, 6, 6	0	8	7, 9, 8, 3, 8	0
4	7, 9, 6, 6, 4	0	9	1, 4, 4, 8, 9	1
5	10, 6, 4, 6, 4	0	10	7, 8, 9, 5, 3	0

2. $\frac{4}{10}$ or 0.4

3. a. Let "1" represent seats with a prize and numbers 2 – 10 seats without a prize. Run multiple trials with the numbers 1 – 10 until a "1" appears. Record the number of seats reserved with each trial until the "1" appears.

   b. Answers will vary, but a "1" has to appear in the list.

Practice and Problem Solving: C

1. Results will vary, but model should use randomly generated numbers 1–10. Since the chance of making more than 4 goals is 30%, the chance of making 4 goals or less is 70% or 7 out of 10, so generate 7 numbers for each trial. Count the number of trials in which 4 appears, and divide it by the number of trials (5) to find the experimental probability. Sample answer:

Trial	Numbers Generated	Result
1	5, 9, 2, 1, 1, 5, 7	0
2	1, 8, 5, 10, 5, 8, 3	0
3	4, 6, 6, 8, 8, 7, 6	1
4	5, 7, 9, 3, 9, 10, 6	0
5	6, 7, 9, 9, 2, 4, 3	1

2. The experimental probability for 5 trials of a trial containing a 4 is $\frac{2}{5}$, or 0.4.
3. Answers will vary. Sample answer: It will be the same because the chance of getting a 4 is the same.
4. Results will vary. Sample answer:

Trial	Numbers Generated	Result
6	8, 9, 10, 9, 1, 6, 3	0
7	6, 5, 5, 8, 5, 7, 10	0
8	5, 7, 8, 10, 6, 4, 9	1
9	7, 7, 6, 1, 9, 1, 9	0
10	6, 8, 7, 7, 2, 4, 9	1

5. The experimental probability for 10 trials of a trial containing a 4 is $\frac{4}{10}$, or 0.4.

Practice and Problem Solving: D

1. a. $\frac{1}{4}$

   b. Answers will vary. Sample answer: 1

   c. Answers will vary. Sample answer: 2, 3, and 4

2. Results will vary. Sample answer:

Trial	Numbers Generated	Pizzas Bought	Trial	Numbers Generated	Pizzas Bought
1	3, 1, 1, 3	4	6	4, 2, 4, 3	4
2	3, 2, 4, 2	4	7	1, 1, 1, 2	4
3	2, 4, 3, 3	4	8	3, 4, 1, 4	4
4	3, 4, 2, 1	4	9	3, 2, 3, 4	4
5	2, 3, 1, 2	4	10	2, 3, 2, 2	4

Trial 1 is a winner since it has at least one 1. Trial 2 is not a winner, because it does not have a 1.

3. 5; 0.5 or $\frac{1}{2}$

Reteach

1. Results will vary. Sample answer:

Trial	Numbers Generated	Result	Trial	Numbers Generated	Result
1	1, 1, 1, 1, 1	5	6	1, 0, 1, 0, 0	2
2	0, 0, 1, 1, 1	3	7	1, 1, 0, 1, 1	4
3	1, 0, 1, 0, 1	3	8	1, 1, 0, 0, 1	3
4	0, 0, 1, 0, 0	1	9	0, 1, 1, 0, 0	2
5	1, 0, 0, 0, 0	1	10	0, 1, 0, 0, 1	2

The experimental probability is 5 out 10, 0.5, 50 percent, or one half or more that an outcome has a 50–50 chance or greater of occurring.

2. Results will vary. Sample answer: Let 1 and 2 represent the probability that an event occurs; let 3–5 be the probability that it does not occur.

Trial	Numbers Generated	Result	Trial	Numbers Generated	Result
1	4, 4, 3, 4, 4	0	6	3, 2, 1, 5, 3	2
2	3, 5, 2, 4, 2	1	7	2, 1, 3, 4, 2	3
3	2, 5, 5, 4, 3	1	8	2, 2, 1, 5, 3	3
4	3, 3, 4, 4, 1	1	9	2, 3, 2, 4, 1	3
5	2, 2, 1, 4, 1	4	10	2, 5, 5, 1, 3	1

The experimental probability is 3 out of 10, 0.3, 30 percent, or three tenths that an outcome has a 3 in 5 chance of occurring.

Reading Strategies

1. 1 out of 4; use the numbers 1–4 for randomization with 1 being the favorable outcome. Experimental probability results will vary, but only the outcome of 1 will be counted as a favorable result when it occurs exactly twice out of 10 randomizations of the numbers 1–4, e.g. **1**, 2, 4, 2, **1**, 3, 4, 2, 2, 4
2. 7 out of 8; use the numbers 1–8 for randomization with 1–7 being favorable outcomes. Experimental probability results will vary, but only one of the outcomes 1–7 will be counted as a favorable result out of 10 randomizations of the numbers 1–8, e.g. **6**, **5**, **4**, **6**, **3**, 8, **1**, **5**, **3**, **7**

Success for English Learners

1. Answers will vary. Results or outcomes of 5 should be counted. Experimental probability should be near 17%.
2. Answers will vary. Results or outcomes of 1, 3, and 5 should be counted. Experimental probability should be near 50%.
3. Choices will vary. Some possibilities include the number 3, numbers less than 4, and numbers divisible by 3.

MODULE 13 Challenge

1. The probability that the arrow will land inside the circle is equal to the area of the circle divided by the area of the square. Let the side of the square have length x. The area of the square is then $x(x) = x^2$. The diameter of the circle is x, since the circle is inscribed in the square. The radius of the circle is half the length of the diameter, or $\frac{x}{2}$.
   The area of the circle is given by the formula $A = \pi r^2; \pi\left(\frac{x}{2}\right)^2 = \frac{\pi x^2}{4}$.
   The probability of the arrow landing inside the circle equals $\frac{\frac{\pi x^2}{4}}{x^2} = \frac{\pi}{4} \approx 0.785$.
2. a. Tobias is not correct. According to the simulation the probability of two or more days of rain per week equals 0.3 (Trials 1, 8, and 10 are weeks in which there were two or more rainy days). The probability of no rainy days in a week is 0.3 (Trials 4, 6, and 7 produced no rainy days). The probability of no rainy days is the same as the probability of two or more rainy days.
   b. The probability of 0 rainy days is 0.3 (Trials 4, 6, 7).
   The probability of 1 rainy day is 0.4 (Trials 2, 3, 5 and 9).
   The probability of 2 rainy days is 0.
   The probability of 3 rainy days is 0.2 (Trials 1 and 8).
   The probability of 4 rainy days is 0.1 (Trial 10).
   The probability of 5, 6 or 7 rainy days is 0.
   One rainy day per week is most likely.