

# Expressions and Equations

MODULE

6



## ESSENTIAL QUESTION

How can you use algebraic expressions and equations to solve real-world problems?

LESSON 6.1

**Algebraic Expressions**

LESSON 6.2

**One-Step Equations with Rational Coefficients**

LESSON 6.3

**Writing Two-Step Equations**

LESSON 6.4

**Solving Two-Step Equations**



### Real-World Video

When you take a taxi, you will be charged an initial fee plus a charge per mile. To describe situations like this, you can write a two-step equation.

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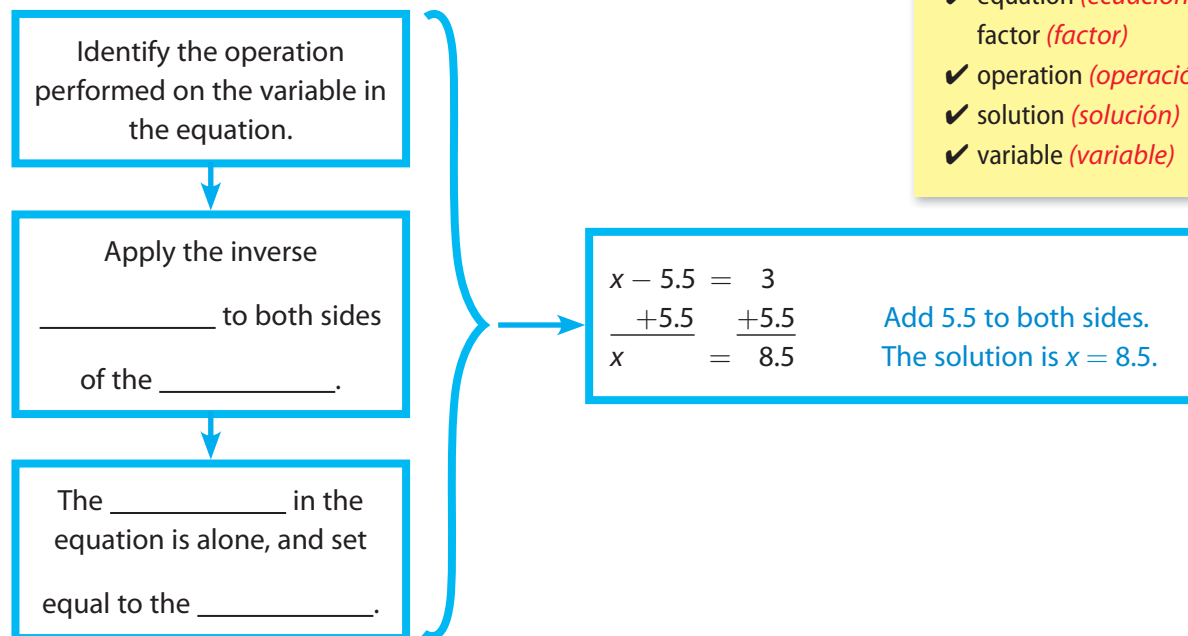
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# Reading Start-Up

## Visualize Vocabulary

Use the ✓ words to complete the graphic. You may put more than one word in each box.



## Vocabulary

### Review Words

- algebraic expression (*expresión algebraica*)
- Distributive Property (*Propiedad distributiva*)
- ✓ equation (*ecuación*)
- factor (*factor*)
- ✓ operation (*operación*)
- ✓ solution (*solución*)
- ✓ variable (*variable*)

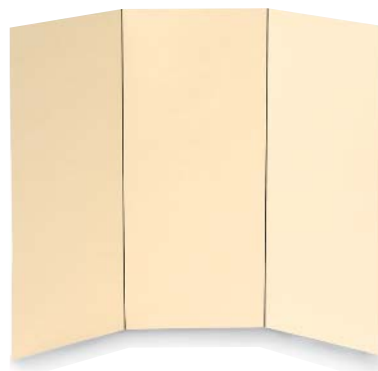
## Understand Vocabulary

Complete the sentences using the review words.

1. A(n) \_\_\_\_\_ contains at least one variable.
2. A mathematical sentence that shows that two expressions are equivalent is called a(n) \_\_\_\_\_.

## Active Reading

**Tri-Fold** Before beginning the module, create a tri-fold to help you learn the concepts and vocabulary in this module. Fold the paper into three sections. Label the columns "What I Know," "What I Need to Know," and "What I Learned." Complete the first two columns before you read. After studying the module, complete the third column.



# Are YOU Ready?

Complete these exercises to review skills you will need for this chapter.



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## Words for Operations

**EXAMPLE** the difference of 2 and  $b$

$$2 - b$$

*Difference means subtraction.*

the product of  $-8$  and a number

$$(-8)x \text{ or } -8x$$

*Product means multiplication.*

*Let  $x$  represent the unknown number.*

**Write an algebraic expression for each word expression.**

1. the sum of 5 and a number  $x$  \_\_\_\_\_ 2. 11 decreased by  $n$  \_\_\_\_\_

3. the quotient of  $-9$  and  $y$  \_\_\_\_\_ 4. twice a number, minus 13 \_\_\_\_\_

## Evaluate Expressions

**EXAMPLE** Evaluate  $3x - 5$  for  $x = -2$ .

$$\begin{aligned} 3x - 5 &= 3(-2) - 5 \\ &= -6 - 5 \\ &= -11 \end{aligned}$$

*Substitute the given value of  $x$  for  $x$ .  
Multiply.  
Subtract.*

**Evaluate each expression for the given value of  $x$ .**

5.  $2x + 3$  for  $x = 3$  \_\_\_\_\_ 6.  $-4x + 7$  for  $x = -1$  \_\_\_\_\_ 7.  $1.5x - 2.5$  for  $x = 3$  \_\_\_\_\_

8.  $0.4x + 6.1$  for  $x = -5$  \_\_\_\_\_ 9.  $\frac{2}{3}x - 12$  for  $x = 18$  \_\_\_\_\_ 10.  $-\frac{5}{8}x + 10$  for  $x = -8$  \_\_\_\_\_

## Operations with Fractions

**EXAMPLE**  $\frac{2}{5} \div \frac{7}{10}$

$$\begin{aligned} \frac{2}{5} \div \frac{7}{10} &= \frac{2}{5} \times \frac{10}{7} \\ &= \frac{2}{\cancel{5}^1} \times \frac{10^2}{7} \\ &= \frac{4}{7} \end{aligned}$$

*Multiply by the reciprocal of the divisor.  
Divide by the common factors.*

*Simplify.*

**Divide.**

11.  $\frac{1}{2} \div \frac{1}{4}$  \_\_\_\_\_ 12.  $\frac{3}{8} \div \frac{13}{16}$  \_\_\_\_\_ 13.  $\frac{2}{5} \div \frac{14}{15}$  \_\_\_\_\_ 14.  $\frac{4}{9} \div \frac{16}{27}$  \_\_\_\_\_

# Are YOU Ready? (cont'd)

Complete these exercises to review skills you will need for this module.

## Words for Operations

15. Delia has two more than three times the number of model cars that Orlando has. If Orlando has  $c$  model cars, which algebraic expressions represent the number of model cars Delia has? Select all that apply.

(A)  $3(c + 2)$     (B)  $3c + 2$     (C)  $3c - 2$     (D)  $2 - 3c$     (E)  $2 + 3c$

## Evaluate Expressions

16. Trent and Rita each did some hiking. The expression  $2.5x - 1.5$  represents the number of miles each hiked this week. In the expression,  $x$  represents the number of miles each hiked last week. Trent hiked 4 miles last week and Rita hiked 3 miles last week.
- How many miles did Trent hike this week? Show your work.
  - How many miles did Rita hike this week? Show your work. Last week, Rita hiked 1 mile less than Trent did. Did she also hike 1 less mile than Trent did this week?

## Operations with Fractions

17. How are the quotients  $\frac{3}{4} \div \frac{2}{3}$  and  $\frac{2}{3} \div \frac{3}{4}$  related? Explain your thinking.



# 6.1 Algebraic Expressions

Students will add, subtract, factor, and multiply algebraic expressions.



## ESSENTIAL QUESTION

How do you add, subtract, factor, and multiply algebraic expressions?

## Adding and Subtracting Expressions

You can use the properties of addition along with the Distributive Property to add and subtract algebraic expressions.

### EXAMPLE 1



Jill and Kyle get paid per project. Jill is paid a project fee of \$25 plus \$10 per hour. Kyle is paid a project fee of \$18 plus \$14 per hour. Write an expression to represent how much a company will pay to hire both to work the same number of hours on a project.

#### STEP 1

Write expressions for how much the company will pay each person. Let  $h$  represent the number of hours they will work on the project.

$$\text{Jill: } \$25 + \$10h$$

$$\text{Kyle: } \$18 + \$14h$$

Fee + Hourly rate  $\times$  Hours

Fee + Hourly rate  $\times$  Hours

#### STEP 2

Add the expressions to represent the amount the company will pay to hire both.

$$25 + 10h + 18 + 14h \quad \text{Combine their pay.}$$

$$= 25 + 18 + 10h + 14h \quad \text{Use the Commutative Property.}$$

$$= 43 + 24h \quad \text{Combine like terms.}$$

The company will pay  $43 + 24h$  dollars to hire both Jill and Kyle.

### Reflect

- Critical Thinking** What can you read directly from the expression  $43 + 24h$  that you cannot read directly from the equivalent expression  $25 + 10h + 18 + 14h$ ?

### YOUR TURN

Simplify each expression.

$$2. \left(3x + \frac{1}{2}\right) + \left(7x - 4\frac{1}{2}\right)$$

$$3. (-0.25x - 3) - (1.5x + 1.4)$$



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## Using the Distributive Property

You can use the Distributive Property to remove the parentheses from an algebraic expression like  $3(x + 5)$ . Sometimes this is called “simplifying” or “expanding” the expression. Multiply the quantity in front of parentheses by each term within parentheses:  $3(x + 5) = 3 \cdot x + 3 \cdot 5 = 3x + 15$ .

### EXAMPLE 2



Marc is selling tickets for a high school band concert. The band gets to keep 25% of the money he collects from ticket sales to put toward new uniforms. Write an expression to represent how much the band gets to keep.

Let  $a$  represent the number of adult tickets he sells.

Let  $y$  represent the number of youth tickets he sells.



The expression  $16.60a + 12.20y$  represents the amount of money Marc collects from ticket sales.

Write 25% as a decimal: 0.25

Write an expression to represent 25% of the money he collects:

$$0.25 \times (16.60a + 12.20y)$$

25% of adult ticket sales and youth ticket sales

Use the Distributive Property to simplify the expression.

$$0.25(16.60a) + 0.25(12.20y) = 4.15a + 3.05y$$

### Reflect

4. **Analyze Relationships** Instead of using the Distributive Property to expand  $0.25 \times (16.60a + 12.20y)$ , could you have first found the sum  $16.60a + 12.20y$ ? Explain.

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### Math Talk

#### Mathematical Processes

How much does the band get to keep if Marc sells 20 adult tickets and 40 youth tickets?



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### YOUR TURN

Simplify each expression.

5.  $7(9k + 6m)$

6.  $0.2(3b - 15c)$

7.  $\frac{2}{3}(6e + 9f - 21g)$

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## EXPLORE ACTIVITY

# Factoring Expressions

A factor is a number that is multiplied by another number to get a product. To **factor** is to write a number or an algebraic expression as a product.

**Factor  $4x + 8$ .**

- A** Model the expression with algebra tiles.

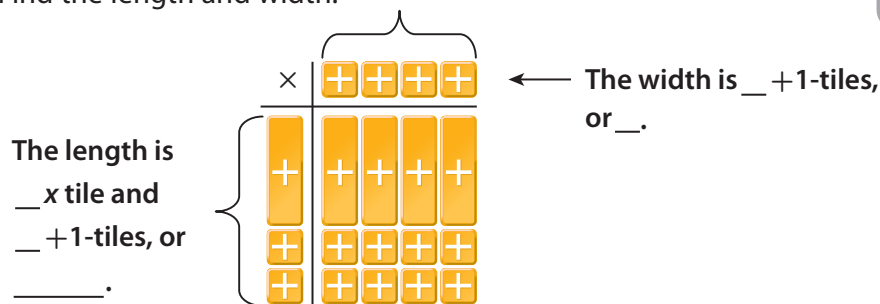
Use \_\_\_\_\_ positive  $x$  tiles and \_\_\_\_\_  $+1$ -tiles.



- B** Arrange the tiles to form a rectangle. The total area represents  $4x + 8$ .



- C** Since the length multiplied by the width equals the area, the length and the width of the rectangle are the factors of  $4x + 8$ . Find the length and width.



- D** Use the expressions for the length and width of the rectangle to write the area of the rectangle,  $4x + 8$ , in factored form. \_\_\_\_\_

## Reflect

- 8. Communicate Mathematical Ideas** How could you use the Distributive Property to check your factoring?

\_\_\_\_\_

## YOUR TURN

Factor each expression.

**9.**  $2x + 2$

\_\_\_\_\_

**10.**  $3x + 9$

\_\_\_\_\_

**11.**  $5x + 15$

\_\_\_\_\_

**12.**  $4x + 16$

\_\_\_\_\_



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## Guided Practice

1. The manager of a summer camp has 14 baseballs and 23 tennis balls. The manager buys some boxes of baseballs with 12 baseballs to a box and an equal number of boxes of tennis balls with 16 tennis balls to a box. Write an expression to represent the total number of balls.

(Example 1)

- STEP 1** Write expressions for the total number of baseballs and tennis balls. Let  $n$  represent the number of boxes of each type.

baseballs: \_\_\_\_\_ + (\_\_\_\_\_)  $n$       tennis balls: \_\_\_\_\_ + (\_\_\_\_\_)  $n$

- STEP 2** Find an expression for the total number of balls.

\_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_      *Combine the two expressions.*

\_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_      *Use the Commutative Property.*

\_\_\_\_\_ + \_\_\_\_\_      *Combine like terms.*

○ So, the total number of baseballs and tennis balls is \_\_\_\_\_ + \_\_\_\_\_.

2. Use the expression you found above to find the total number of baseballs and tennis balls if the manager bought 9 boxes of each type. (Example 1) \_\_\_\_\_

**Use the Distributive Property to expand each expression. (Example 2)**

3.  $0.5(12m - 22n)$

$0.5(12m - 22n) = 0.5(\text{_____}) - 0.5(\text{_____})$       *Distribute 0.5 to both terms in parentheses.*

$= \text{_____} - \text{_____}$       *Multiply.*

4.  $\frac{2}{3}(18x + 6z)$

$\frac{2}{3}(\text{_____}) + \frac{2}{3}(\text{_____}) = \text{_____} + \text{_____}$

**Factor each expression. (Example 3)**

5.  $2x + 12$

6.  $12x + 24$

7.  $7x + 35$

### ESSENTIAL QUESTION CHECK-IN

8. What is the relationship between multiplying and factoring?

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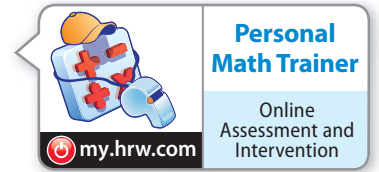
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# 6.1 Independent Practice



**Write and simplify an expression for each situation.**

9. A company rents out 15 food booths and 20 game booths at the county fair. The fee for a food booth is \$100 plus \$5 per day. The fee for a game booth is \$50 plus \$7 per day. The fair lasts for  $d$  days, and all the booths are rented for the entire time. Write and simplify an expression for the amount in dollars that the company is paid.


\_\_\_\_\_

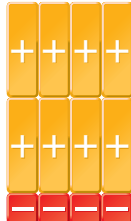
10. A rug maker is using a pattern that is a rectangle with a length of 96 inches and a width of 60 inches. The rug maker wants to increase each dimension by a different amount. Let  $\ell$  and  $w$  be the increases in inches of the length and width. Write and simplify an expression for the perimeter of the new pattern.

\_\_\_\_\_



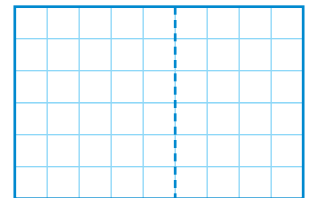
**In 11–12, identify the two factors that were multiplied together to form the array of tiles. Then identify the product of the two factors.**

11.  \_\_\_\_\_

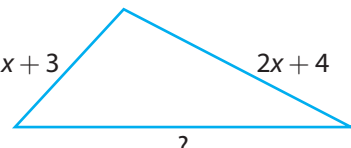
12.  \_\_\_\_\_

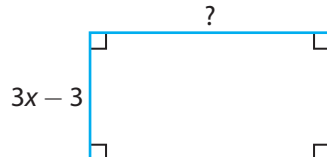
13. Explain how the figure illustrates that  $6(9) = 6(5) + 6(4)$ .

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

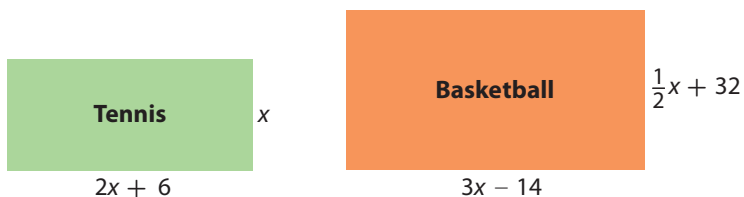


**In 14–15, the perimeter of the figure is given. Find the length of the indicated side.**

14.   
Perimeter =  $6x$  \_\_\_\_\_

15.   
Perimeter =  $10x + 6$  \_\_\_\_\_

- 16. Persevere in Problem Solving** The figures show the dimensions of a tennis court and a basketball court given in terms of the width  $x$  in feet of the tennis court.



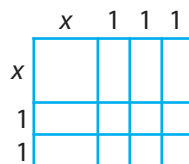
- Write an expression for the perimeter of each court. \_\_\_\_\_
- Write an expression that describes how much greater the perimeter of the basketball court is than the perimeter of the tennis court. \_\_\_\_\_
- Suppose the tennis court is 36 feet wide. Find all dimensions of the two courts. \_\_\_\_\_



**FOCUS ON HIGHER ORDER THINKING**

- 17. Draw Conclusions** Use the figure to find the product  $(x + 3)(x + 2)$ . (*Hint: Find the area of each small square or rectangle, then add.*)

$(x + 3)(x + 2) =$  \_\_\_\_\_



- 18. Communicate Mathematical Ideas** Desmond claims that the product shown at the right illustrates the Distributive Property. Do you agree? Explain why or why not.

$$\begin{array}{r} 58 \\ \times 23 \\ \hline 174 \\ 1160 \\ \hline 1,334 \end{array}$$

- 19. Justify Reasoning** Describe two different ways that you could find the product  $8 \times 997$  using mental math. Find the product and explain why your methods work.

Work Area

# One-Step Equations with Rational Coefficients

Students will use one-step equations with rational coefficients to solve problems.



## ESSENTIAL QUESTION

How do you use one-step equations with rational coefficients to solve problems?

## One-Step Equations

You have written and solved one-step equations involving whole numbers. Now you will learn to work with equations containing negative numbers.



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### EXAMPLE 1



Use inverse operations to solve each equation.

**A**  $x + 3.2 = -8.5$

$$x + 3.2 = -8.5$$

$$\begin{array}{r} -3.2 \quad -3.2 \\ x + 3.2 = -8.5 \\ \hline x = -11.7 \end{array}$$

Subtract 3.2 from both sides.

**B**  $-\frac{2}{3} + y = 8$

$$-\frac{2}{3} + y = 8$$

$$\begin{array}{r} +\frac{2}{3} \quad +\frac{2}{3} \\ -\frac{2}{3} + y = 8 \\ \hline y = 8\frac{2}{3} \end{array}$$

Add  $\frac{2}{3}$  to both sides.

**C**  $30 = -0.5a$

$$30 = -0.5a$$

$$\begin{array}{r} 30 = -0.5a \\ -0.5 \quad -0.5 \\ \hline -60 = a \end{array}$$

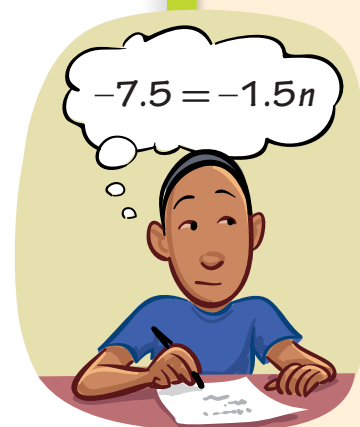
Divide both sides by  $-0.5$ .

**D**  $-\frac{q}{3.5} = 9.2$

$$-\frac{q}{3.5} (-3.5) = 9.2 (-3.5)$$

$$q = -32.2$$

Multiply both sides by  $-3.5$ .



### YOUR TURN

Use inverse operations to solve each equation.

1.  $4.9 + z = -9$

\_\_\_\_\_

2.  $r - 17.1 = -4.8$

\_\_\_\_\_

3.  $-3c = 36$

\_\_\_\_\_



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## Writing and Solving One-Step Addition and Subtraction Equations

Negative numbers often appear in real-world situations. For example, elevations below sea level are represented by negative numbers. When you increase your elevation, you are moving in a positive direction. When you decrease your elevation, you are moving in a negative direction.

### EXAMPLE 2



A scuba diver is exploring at an elevation of  $-12.2$  meters. As the diver rises to the surface, she plans to stop and rest briefly at a reef that has an elevation of  $-4.55$  meters. Find the vertical distance that the diver will travel.

#### STEP 1

Write an equation. Let  $x$  represent the vertical distance between her initial elevation and the elevation of the reef.

$$-12.2 + x = -4.55$$

#### STEP 2

Solve the equation using an inverse operation.

$$-12.2 + x = -4.55$$

$$\begin{array}{r} +12.2 \\ -12.2 + x = -4.55 \\ \hline x = 7.65 \end{array}$$

Add 12.2 to both sides.

The diver will travel a vertical distance of 7.65 meters.

### Reflect

4. **Make a Prediction** Explain how you know whether the diver is moving in a positive or a negative direction before you solve the equation.

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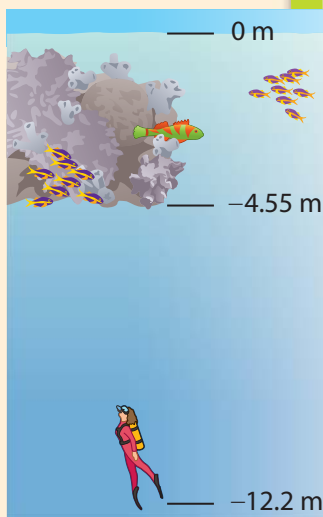
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### YOUR TURN

5. An airplane descends 1.5 miles to an elevation of 5.25 miles. Find the elevation of the plane before its descent.

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# Writing and Solving One-Step Multiplication and Division Problems

Temperatures can be both positive and negative, and they can increase or decrease during a given period of time. A decrease in temperature is represented by a negative number. An increase in temperature is represented by a positive number.



## EXAMPLE 3



Between the hours of 10 P.M. and 6 A.M., the temperature decreases an average of  $\frac{3}{4}$  of a degree per hour. How many minutes will it take for the temperature to decrease by  $5^{\circ}\text{F}$ ?

### STEP 1

Write an equation. Let  $x$  represent the number of hours it takes for the temperature to decrease by  $5^{\circ}\text{F}$ .

$$-\frac{3}{4}x = -5$$

### STEP 2

Solve the equation using an inverse operation.

$$-\frac{3}{4}x = -5$$

$$-\frac{4}{3}\left(-\frac{3}{4}x\right) = -\frac{4}{3}(-5) \quad \text{Multiply both sides by } -\frac{4}{3}.$$

$$x = \frac{20}{3}$$

### STEP 3

Convert the number of hours to minutes.

$$\frac{20}{3} \text{ hours} \times \frac{60 \text{ minutes}}{1 \text{ hour}} = 400 \text{ minutes}$$

It takes 400 minutes for the temperature to decrease by  $5^{\circ}\text{F}$ .

## Math Talk

### Mathematical Processes

Why is multiplying by  $-\frac{4}{3}$  the inverse of multiplying by  $-\frac{3}{4}$ ?

## YOUR TURN

6. The value of a share of stock decreases in value at a rate of  $\$1.20$  per hour during the first 3.5 hours of trading. Write and solve an equation to find the decrease in the value of the share of stock during that time.

7. After a power failure, the temperature in a freezer increased at an average rate of  $2.5^{\circ}\text{F}$  per hour. The total increase was  $7.5^{\circ}\text{F}$ . Write and solve an equation to find the number of hours until the power was restored.



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## Guided Practice

The table shows the average temperature in Barrow, Alaska, for three months during one year.

Month	Average Temperature (°F)
January	-13.4
June	34.0
November	-1.7

1. How many degrees warmer is the average temperature in November than in January? (Examples 1 and 2)

**STEP 1** Write an equation. Let  $x$  represent \_\_\_\_\_

\_\_\_\_\_.

$x + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$ , or  $x - \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

**STEP 2** Solve the equation. Show your work.

The average temperature in November  
is \_\_\_\_\_ warmer.

2. Suppose that during one period of extreme cold, the average daily temperature decreased  $1\frac{1}{2}$  °F each day. How many days did it take for the temperature to decrease by 9 °F? (Examples 1 and 3)

**STEP 1** Write an equation. Let  $x$  represent \_\_\_\_\_

\_\_\_\_\_.

\_\_\_\_\_  $x =$  \_\_\_\_\_

**STEP 2** Solve the equation. Show your work.

It took \_\_\_\_\_ days for the  
temperature to decrease by 9 °F.

Use inverse operations to solve each equation. (Example 1)

3.  $-2x = 34$

\_\_\_\_\_

4.  $y - 3.5 = -2.1$

\_\_\_\_\_

5.  $\frac{2}{3}z = -6$

\_\_\_\_\_



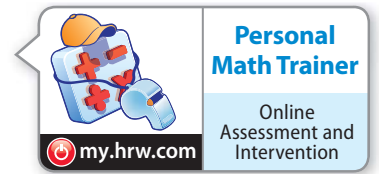
### ESSENTIAL QUESTION CHECK-IN

6. How does writing an equation help you solve a problem?

\_\_\_\_\_

\_\_\_\_\_

## 6.2 Independent Practice



The table shows the elevation in feet at the peaks of several mountains. Use the table for 7–9.

Mountain	Elevation (feet)
Mt. McKinley	20,321.5
K2	28,251.31
Tupungato	22,309.71
Dom	14,911.42

7. Mt. Everest is 8,707.37 feet higher than Mt. McKinley. What is the elevation of Mt. Everest?  
\_\_\_\_\_
8. Liam descended from the summit of K2 to an elevation of 23,201.06 feet. How many feet did Liam descend? What was his change in elevation?  
\_\_\_\_\_
9. K2 is 11,194.21 feet higher than Mt. Kenya. Write and solve an equation to find the elevation of Mt. Kenya.  
\_\_\_\_\_  
\_\_\_\_\_
10. A hot air balloon begins its descent at a rate of  $22\frac{1}{2}$  feet per minute. How long will it take for the balloon's elevation to change by  $-315$  feet?  
\_\_\_\_\_
11. During another part of its flight, the balloon in Exercise 10 had a change in elevation of  $-901$  feet in 34 minutes. What was its rate of descent?  
\_\_\_\_\_

The table shows the average temperatures in several states from January through March. Use the table for 12–14.



State	Average Temperature ( $^{\circ}\text{C}$ )
Florida	18.1
Minnesota	$-2.5$
Montana	$-0.7$
Texas	12.5

12. Write and solve an equation to find how much warmer Montana's average 3-month temperature is than Minnesota's.  
\_\_\_\_\_
13. How much warmer is Florida's average 3-month temperature than Montana's?  
\_\_\_\_\_
14. How would the average temperature in Texas have to change to match the average temperature in Florida?  
\_\_\_\_\_
15. A football team has a net yardage of  $-26\frac{1}{3}$  yards on a series of plays. The team needs a net yardage of 10 yards to get a first down. How many yards do they have to get on their next play to get a first down?  
\_\_\_\_\_

16. A diver begins at sea level and descends vertically at a rate of  $2\frac{1}{2}$  feet per second. How long does the diver take to reach  $-15.6$  feet? \_\_\_\_\_

17. **Analyze Relationships** In Exercise 16, what is the relationship between the rate at which the diver descends, the elevation he reaches, and the time it takes to reach that elevation?  
\_\_\_\_\_  
\_\_\_\_\_

18. **Check for Reasonableness** Jane withdrew money from her savings account in each of 5 months. The average amount she withdrew per month was \$45.50. How much did she withdraw in all during the 5 months? Show that your answer is reasonable.  
\_\_\_\_\_  
\_\_\_\_\_



**FOCUS ON HIGHER ORDER THINKING**

19. **Justify Reasoning** Consider the two problems below. Which values in the problems are represented by negative numbers? Explain why.

(1) A diver below sea level ascends 25 feet to a reef at  $-35.5$  feet. What was the elevation of the diver before she ascended to the reef?

(2) A plane descends 1.5 miles to an elevation of 3.75 miles. What was the elevation of the plane before its descent?  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

20. **Analyze Relationships** How is solving  $-4x = -4.8$  different from solving  $-\frac{1}{4}x = -4.8$ ? How are the solutions related?  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

21. **Communicate Mathematical Ideas** Flynn opens a savings account. In one 3-month period, he makes deposits of \$75.50 and \$55.25. He makes withdrawals of \$25.15 and \$18.65. His balance at the end of the 3-month period is \$210.85. Explain how you can find his initial deposit amount.  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

Work Area

# LESSON 6.3

## Writing Two-Step Equations

7.3.6.3

Students will write a two-step equation.



### ESSENTIAL QUESTION

How do you write a two-step equation?

### EXPLORE ACTIVITY

## Modeling Two-Step Equations

You can use algebra tiles to model two-step equations.

Use algebra tiles to model  $3x - 4 = 5$ .

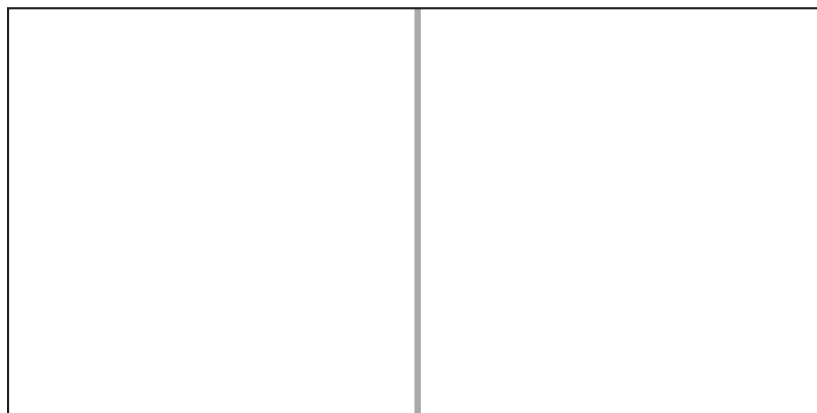
**A** How can you model the left side of the equation?

---

**B** How can you model the right side of the equation?

---

**C** Use algebra tiles or draw them to model the equation on the mat.



### KEY

= positive variable

= negative variable

= 1    = -1

### Math Talk

#### Mathematical Processes

Why is the mat divided into two equal halves with a line?

### Reflect

- What If?** How would you change the algebra tile model in the Explore Activity to model  $-3x + 4 = 5$ ?

---



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## Writing Two-Step Equations

You can write two-step equations to represent real-world problems by translating the words of the problems into numbers, variables, and operations.

### EXAMPLE 1



A one-year membership to Metro Gym costs \$460. There is a fee of \$40 when you join, and the rest is paid monthly. Write an equation to represent the situation that can help members find how much they pay per month.



#### STEP 1

Identify what you are trying to find. This will be the variable in the equation.

Let  $m$  represent the amount of money members pay per month.

#### STEP 2

Identify important information in the problem that can be used to help write an equation.

one-time joining fee: \$40  
fee charged for 1 year:  $12 \cdot m$   
total cost for the year: \$460

Convert 1 year into 12 months to find how much members pay per month.

#### STEP 3

Use words in the problem to tie the information together and write an equation.

One-time joining fee	plus	12	times	monthly cost	equals	\$460
↓	↓	↓	↓	↓	↓	↓
\$40	+	12	·	$m$	=	\$460

The equation  $40 + 12m = 460$  can help members find out their monthly fee.

### Reflect

2. **Multiple Representations** Why would this equation for finding the monthly fee be difficult to model with algebra tiles?

---

3. Can you rewrite the equation in the form  $52m = 460$ ? Explain.

---

---




## YOUR TURN

4. Billy has a gift card with a \$150 balance. He buys several video games that cost \$35 each. After the purchases, his gift card balance is \$45. Write an equation to help find out how many video games Billy bought.
- \_\_\_\_\_



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## Writing a Verbal Description of a Two-Step Equation

You can also write a verbal description to fit a two-step equation.



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### EXAMPLE 2



Write a corresponding real-world problem to represent  $5x + 50 = 120$ .

**STEP 1** Analyze what each part of the equation means mathematically.

$x$  is the solution of the problem, the quantity you are looking for.

$5x$  means that, for a reason given in the problem, the quantity you are looking for is multiplied by 5.

$+ 50$  means that, for a reason given in the problem, 50 is added to  $5x$ .

$= 120$  means that after multiplying the solution  $x$  by 5 and adding 50 to it, the result is 120.

**STEP 2** Think of some different situations in which a quantity  $x$  might be multiplied by 5.

You have  $x$  number of books, each weighing 5 pounds, and you want to know their total weight.

You save \$5 each week for  $x$  weeks and want to know the total amount you have saved.

**STEP 3** Build on the situation and adjust it to create a verbal description that takes all of the information of the equation into account.

- A publisher ships a package of  $x$  number of books each weighing 5 pounds, plus a second package weighing 50 pounds. The total weight of both packages is 120 pounds. How many books are being shipped?
- Leon receives a birthday gift of \$50 from his parents and decides to save it. Each week he adds \$5 to his savings. How many weeks will it take for him to save \$120?

### My Notes

## YOUR TURN

5. Write a real-world problem that can be represented by  $10x + 40 = 100$ .

---



---

## Guided Practice

Draw algebra tiles to model the given two-step equation. (Explore Activity)

1.  $2x + 5 = 7$

--	--

2.  $-3 = 5 - 4x$

--	--

3. A group of adults plus one child attend a movie at Cineplex 15. Tickets cost \$9 for adults and \$6 for children. The total cost for the movie is \$78.

Write an equation to find the number of adults in the group. (Example 1)

---

4. Break down the equation  $2x + 10 = 16$  to analyze each part. (Example 2)

$x$  is \_\_\_\_\_ of the problem.

$2x$  is the quantity you are looking for \_\_\_\_\_.

$+ 10$  means 10 is \_\_\_\_\_.  $= 16$  means the \_\_\_\_\_ is 16.

5. Write a corresponding real-world problem to represent  $2x - 125 = 400$ .

(Example 2) \_\_\_\_\_

---



## ESSENTIAL QUESTION CHECK-IN

6. Describe the steps you would follow to write a two-step equation you can use to solve a real-world problem.

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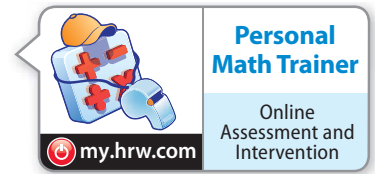


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## 6.3 Independent Practice



7. Describe how to model  $-3x + 7 = 28$  with algebra tiles.  
 \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_
8. Val rented a bicycle while she was on vacation. She paid a flat rental fee of \$55.00, plus \$8.50 each day. The total cost was \$123. Write an equation you can use to find the number of days she rented the bicycle.  
 \_\_\_\_\_
9. A restaurant sells a coffee refill mug for \$6.75. Each refill costs \$1.25. Last month Keith spent \$31.75 on a mug and refills. Write an equation you can use to find the number of refills that Keith bought.  
 \_\_\_\_\_
10. A gym holds one 60-minute exercise class on Saturdays and several 45-minute classes during the week. Last week all of the classes lasted a total of 285 minutes. Write an equation you can use to find the number of weekday classes.  
 \_\_\_\_\_
11. **Multiple Representations** There are 172 South American animals in the Springdale Zoo. That is 45 more than half the number of African animals in the zoo. Write an equation you could use to find  $n$ , the number of African animals in the zoo.  
 \_\_\_\_\_
12. A school bought \$548 in basketball equipment and uniforms costing \$29.50 each. The total cost was \$2,023. Write an equation you can use to find the number of uniforms the school purchased.  
 \_\_\_\_\_
13. **Financial Literacy** Heather has \$500 in her savings account. She withdraws \$20 per week for gas. Write an equation Heather can use to see how many weeks it will take her to have a balance of \$220.  
 \_\_\_\_\_
14. **Critique Reasoning** For  $9x + 25 = 88$ , Deena wrote the situation "I bought some shirts at the store for \$9 each and received a \$25 discount. My total bill was \$88. How many shirts did I buy?"
  - a. What mistake did Deena make?  
 \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_
  - b. Rewrite the equation to match Deena's situation.  
 \_\_\_\_\_  
 \_\_\_\_\_
  - c. How could you rewrite the situation to make it fit the equation?  
 \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_

- 15. Multistep** Sandy charges each family that she babysits a flat fee of \$10 for the night and an extra \$5 per child. Kimmi charges \$25 per night, no matter how many children a family has.
- Write a two-step equation that would compare what the two girls charge and find when their fees are the same. \_\_\_\_\_
  - How many children must a family have for Sandy and Kimmi to charge the same amount? \_\_\_\_\_
  - The Sanderson family has five children. Which babysitter should they choose if they wish to save some money on babysitting, and why?  
\_\_\_\_\_  
\_\_\_\_\_

**FOCUS ON HIGHER ORDER THINKING**

- 16. Analyze Relationships** Each student wrote a two-step equation. Peter wrote the equation  $4x - 2 = 10$ , and Andres wrote the equation  $16x - 8 = 40$ . The teacher looked at their equations and asked them to compare them. Describe one way in which the equations are similar.  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_
- 17. What's the Error?** Damon has 5 dimes and some nickels in his pocket, worth a total of \$1.20. To find the number of nickels Damon has, a student wrote the equation  $5n + 50 = 1.20$ . Find the error in the student's equation.  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_
- 18. Represent Real-World Problems** Write a real-world problem you could answer by solving the equation  $-8x + 60 = 28$ .  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

# LESSON 6.4 Solving Two-Step Equations

7.3.6.4

Students will solve a two-step equation.



## ESSENTIAL QUESTION

How do you solve a two-step equation?

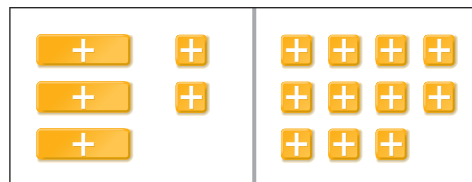
## Modeling and Solving Two-Step Equations

You can solve two-step equations using algebra tiles.

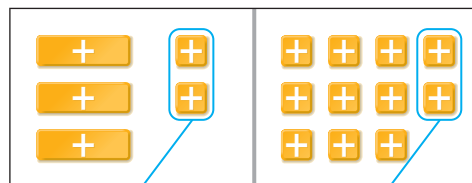
### EXAMPLE 1

Use algebra tiles to model and solve  $3n + 2 = 11$ .

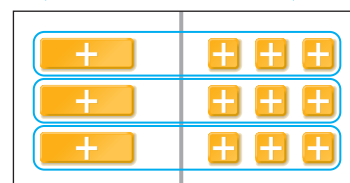
**STEP 1** Model the equation.



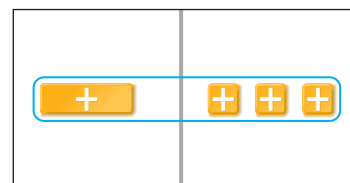
**STEP 2** Remove 2 +1-tiles from each side of the mat.



**STEP 3** Divide each side into 3 equal groups.



**STEP 4** The solution is  $n = 3$ .



Since there are +1-tiles on both sides of the equation, you can remove, or subtract, 2 +1-tiles from each side to help isolate the variable.



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### YOUR TURN

Use algebra tiles to model and solve each equation.

- $2x + 5 = 11$  \_\_\_\_\_
- $3n - 1 = 8$  \_\_\_\_\_
- $2a - 3 = -5$  \_\_\_\_\_
- $-4y + 2 = -2$  \_\_\_\_\_



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## Solving Two-Step Equations

You can use inverse operations to solve equations with more than one operation.

### EXAMPLE 2



A dog sled driver added more gear to the sled, doubling its weight. This felt too heavy, so the driver removed 20 pounds to reach the final weight of 180 pounds. Write and solve an equation to find the sled's original weight.

**STEP 1** Write an equation. Let  $w$  represent the original weight of the sled.

$$2w - 20 = 180.$$

**STEP 2** Solve the equation.

$$2w - 20 = 180$$

$$\quad + 20 \quad + 20 \quad \text{Add 20 to both sides.}$$

$$2w = 200$$

$$\frac{2w}{2} = \frac{200}{2} \quad \text{Divide both sides by 2.}$$

$$w = 100$$

○ The sled's original weight was 100 pounds.

### Reflect

5. **Analyze Relationships** Describe how you could find the original weight of the sled using only arithmetic. Compare this method with the method shown in Example 2.

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### YOUR TURN

Solve each problem by writing and solving an equation.

6. The Wilsons have triplets and another child who is ten years old. The sum of the ages of their children is 37. How old are the triplets?
- 
7. Five less than the quotient of a number and 4 is 15. What is the number?
- 



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# Two-Step Equations with Negative Numbers

Many real-world quantities such as altitude or temperature involve negative numbers. You solve equations with negative numbers just as you did equations with positive numbers.



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## EXAMPLE 3



- A** To convert a temperature from degrees Fahrenheit to degrees Celsius, first subtract 32. Then multiply the result by  $\frac{5}{9}$ . An outdoor thermometer showed a temperature of  $-10^\circ\text{C}$ . What was the temperature in degrees Fahrenheit?

**STEP 1** Write an equation. Let  $x$  represent the temperature in degrees Fahrenheit.

$$-10 = \frac{5}{9}(x - 32)$$

**STEP 2** Solve the equation.

$$\frac{9}{5}(-10) = \frac{9}{5}\left(\frac{5}{9}(x - 32)\right) \quad \text{Multiply both sides by } \frac{9}{5}.$$

$$-18 = x - 32$$

$$\begin{array}{r} + 32 \\ + 32 \end{array}$$

Add 32 to both sides.

$$14 = x$$

- The temperature was 14 degrees Fahrenheit.

- B** An airplane flies at an altitude of 38,000 feet. As it nears the airport, the plane begins to descend at a rate of 600 feet per minute. At this rate, how many minutes will the plane take to descend to 18,800 feet?

**STEP 1** Write an equation. Let  $m$  represent the number of minutes.

$$38,000 - 600m = 18,800$$

**STEP 2** Solve the equation. Start by isolating the term that contains the variable.

$$38,000 - 600m = 18,800$$

$$\begin{array}{r} - 38,000 \\ - 38,000 \end{array}$$

$$-600m = -19,200$$

Subtract 38,000 from both sides.

$$\begin{array}{r} -600m \\ -600 \end{array} = \begin{array}{r} -19,200 \\ -600 \end{array}$$

Divide both sides by  $-600$ .

$$m = 32$$

- The plane will take 32 minutes to descend to 18,800 feet.

## Math Talk

### Mathematical Processes

How can you check the solution?



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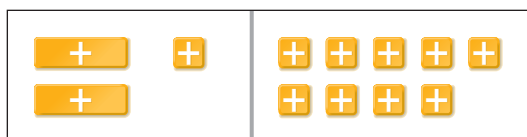
## YOUR TURN

Solve each problem by writing and solving an equation.

8. What is the temperature in degrees Fahrenheit of a freezer kept at  $-20^{\circ}\text{C}$ ?  
\_\_\_\_\_
9. Jenny earned 92 of a possible 120 points on a test. She lost 4 points for each incorrect answer. How many incorrect answers did she have?  
\_\_\_\_\_

## Guided Practice

The equation  $2x + 1 = 9$  is modeled below. (Example 1)



1. To solve the equation with algebra tiles, first remove \_\_\_\_\_.  
Then divide each side into \_\_\_\_\_.
2. The solution is  $x =$  \_\_\_\_\_.

Solve each problem by writing and solving an equation.

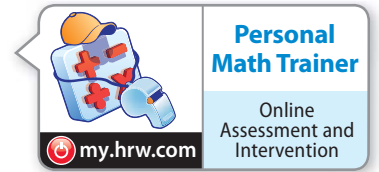
3. A rectangular picture frame has a perimeter of 58 inches. The height of the frame is 18 inches. What is the width of the frame? (Example 2)  
\_\_\_\_\_
4. A school store has 1200 pencils in stock, and sells an average of 25 pencils per day. The manager reorders when the number of pencils in stock is 500. In how many days will the manager have to reorder? (Example 3)  
\_\_\_\_\_



## ESSENTIAL QUESTION CHECK-IN

5. How can you decide which operations to use to solve a two-step equation?

## 6.4 Independent Practice



**Solve.**

**6.**  $9s + 3 = 57$

\_\_\_\_\_

**7.**  $4d + 6 = 42$

\_\_\_\_\_

**8.**  $-3y + 12 = -48$

\_\_\_\_\_

**9.**  $\frac{k}{2} + 9 = 30$

\_\_\_\_\_

**10.**  $\frac{g}{3} - 7 = 15$

\_\_\_\_\_

**11.**  $\frac{z}{5} + 3 = -35$

\_\_\_\_\_

**12.**  $-9h - 15 = 93$

\_\_\_\_\_

**13.**  $-3(n + 5) = 12$

\_\_\_\_\_

**14.**  $-17 + \frac{b}{8} = 13$

\_\_\_\_\_

**15.**  $7(c - 12) = -21$

\_\_\_\_\_

**16.**  $-3 + \frac{p}{7} = -5$

\_\_\_\_\_

**17.**  $46 = -6t - 8$

\_\_\_\_\_

- 18.** After making a deposit, Puja had \$264 in her savings account. She noticed that if she added \$26 to the amount originally in the account and doubled the sum, she would get the new amount. How much did she originally have in the account?

\_\_\_\_\_

- 19.** The current temperature in Smalltown is 20 °F. This is 6 degrees less than twice the temperature that it was six hours ago. What was the temperature in Smalltown six hours ago?

\_\_\_\_\_

- 20.** One reading at an Arctic research station showed that the temperature was  $-35^{\circ}\text{C}$ . What is this temperature in degrees Fahrenheit?

\_\_\_\_\_

- 21.** Artaud noticed that if he takes the opposite of his age and adds 40, he gets the number 28. How old is Artaud?

\_\_\_\_\_

- 22.** Sven has 11 more than twice as many customers as when he started selling newspapers. He now has 73 customers. How many did he have when he started?

\_\_\_\_\_

- 23.** Paula bought a ski jacket on sale for \$6 less than half its original price. She paid \$88 for the jacket. What was the original price?

\_\_\_\_\_

- 24.** The McIntosh family went apple picking. They picked a total of 115 apples. The family ate a total of 8 apples each day. After how many days did they have 19 apples left?

\_\_\_\_\_

**Use a calculator to solve each equation.**

**25.**  $-5.5x + 0.56 = -1.64$

\_\_\_\_\_

**26.**  $-4.2x + 31.5 = -65.1$

\_\_\_\_\_

**27.**  $\frac{k}{5.2} + 81.9 = 47.2$

\_\_\_\_\_

28. Write a two-step equation that involves multiplication and subtraction, includes a negative coefficient, and has a solution of  $x = 7$ .

---

29. Write a two-step equation involving division and addition that has a solution of  $x = -25$

---

30. **Explain the Error** A student's solution to the equation  $3x + 2 = 15$  is shown. Describe and correct the error that the student made.

$$3x + 2 = 15 \quad \text{Divide both sides by 3.}$$

$$x + 2 = 5 \quad \text{Subtract 2 from both sides.}$$

$$x = 3$$

---

---

31. **Multiple Representations** Explain how you could use the work backward problem-solving strategy to solve the equation  $\frac{x}{4} - 6 = 2$ .

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**FOCUS ON HIGHER ORDER THINKING**

32. **Reason Abstractly** The formula  $F = 1.8C + 32$  allows you to find the Fahrenheit ( $F$ ) temperature for a given Celsius ( $C$ ) temperature. Solve the equation for  $C$  to produce a formula for finding the Celsius temperature for a given Fahrenheit temperature.

---

33. **Reason Abstractly** The equation  $P = 2(\ell + w)$  can be used to find the perimeter  $P$  of a rectangle with length  $\ell$  and width  $w$ . Solve the equation for  $w$  to produce a formula for finding the width of a rectangle given its perimeter and length.

---

34. **Reason Abstractly** Solve the equation  $ax + b = c$  for  $x$ .

---



# Ready to Go On?



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## 6.1 Algebraic Expressions

1. The Science Club went on a two-day field trip. The first day the members paid \$60 for transportation plus \$15 per ticket to the planetarium. The second day they paid \$95 for transportation plus \$12 per ticket to the geology museum. Write an expression to represent the total cost for two days for the  $n$  members of the club. \_\_\_\_\_

## 6.2 One-Step Equations with Rational Coefficients

Solve.

- |                                |   |
|--------------------------------|---|
| 2. $h + 9.7 = -9.7$ _____      | 3. $-\frac{3}{4} + p = \frac{1}{2}$ _____ |
| 4. $-15 = -0.2k$ _____         | 5. $\frac{y}{-3} = \frac{1}{6}$ _____     |
| 6. $-\frac{2}{3}m = -12$ _____ | 7. $2.4 = -\frac{t}{4.5}$ _____           |

## 6.3 Writing Two-Step Equations

8. Jerry started doing sit-ups every day. The first day he did 15 sit-ups. Every day after that he did 2 more sit-ups than he had done the previous day. Today Jerry did 33 sit-ups. Write an equation that could be solved to find the number of days Jerry has been doing sit-ups, not counting the first day.
- \_\_\_\_\_

## 6.4 Solving Two-Step Equations

Solve.

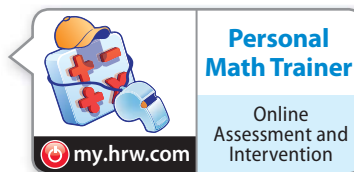
- |                                    |                                   |
|------------------------------------|-----------------------------------|
| 9. $5n + 8 = 43$ _____             | 10. $\frac{y}{6} - 7 = 4$ _____   |
| 11. $8w - 15 = 57$ _____           | 12. $\frac{g}{3} + 11 = 25$ _____ |
| 13. $\frac{f}{5} - 22 = -25$ _____ | 14. $-4p + 19 = 11$ _____         |



### ESSENTIAL QUESTION

15. How can you use two-step equations to represent and solve real-world problems?
- \_\_\_\_\_
- \_\_\_\_\_

## Assessment Readiness



## Selected Response

- A taxi cab costs \$1.50 for the first mile and \$0.75 for each additional mile. Which equation could be solved to find how many miles you can travel in a taxi for \$10, given that  $x$  is the number of additional miles?
  - $1.5x + 0.75 = 10$
  - $0.75x + 1.5 = 10$
  - $1.5x - 0.75 = 10$
  - $0.75x - 1.5 = 10$
- Which is the solution of  $\frac{t}{2.5} = -5.2$ ?
  - $t = -13$
  - $t = -2.08$
  - $t = 2.08$
  - $t = 13$
- Which expression is equivalent to  $5x - 30$ ?
  - $5(x - 30)$
  - $5(x - 6)$
  - $5x(x - 6)$
  - $x(5 - 30)$
- In a science experiment, the temperature of a substance is changed from  $42^\circ\text{F}$  to  $-54^\circ\text{F}$  at an average rate of  $-12$  degrees per hour. Over how many hours does the change take place?
  - $-8$  hours
  - $\frac{1}{8}$  hour
  - 1 hour
  - 8 hours

- Which statement best represents the distance on a number line between  $-14$  and  $-5$ ?
  - $-14 - (-5)$
  - $-14 + (-5)$
  - $-5 - (-14)$
  - $-5 + (-14)$
- Which cereal costs the most per ounce?
  - \$4.92 for 12 ounces
  - \$4.25 for 10 ounces
  - \$5.04 for 14 ounces
  - \$3.92 for 8 ounces

## Mini-Task

- Casey bought 9 tickets to a concert. The total charge was \$104, including a \$5 service charge.
  - Write an equation you can solve to find  $c$ , the cost of one ticket.  


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  - Explain how you could estimate the solution of your equation.  


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  - Solve the equation. How much did each ticket cost?  


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