

Scatter Plots

MODULE

14



ESSENTIAL QUESTION

How can you use scatter plots to solve real-world problems?

LESSON 14.1

Scatter Plots and Association


LESSON 14.2

Trend Lines and Predictions



Real-World Video

An anthropologist measures dinosaur bones. To estimate a dinosaur's height based on the length of a bone, he can make a scatter plot comparing bone length and height of several dinosaurs.

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Reading Start-Up

Visualize Vocabulary

Use the ✓ words to complete the right column of the chart.

Reviewing Slope	
Mathematical Representation	Review Word
$y = mx + b$	
y	
m	
x	
b	

Understand Vocabulary

Match the term on the left to the correct expression on the right.

- | | |
|---------------|---------------------------------------------------------------------------|
| 1. cluster | A. A data point that is very different from the rest of the data in a set |
| 2. outlier | B. A straight line that comes closest to the points on a scatter plot. |
| 3. trend line | C. A set of closely grouped data. |

Vocabulary

Review Words

- bivariate data (*datos bivariados*)
- data (*datos*)
- ✓ linear equation (*ecuación lineal*)
- ✓ slope (*pendiente*)
- ✓ slope-intercept form of an equation (*forma pendiente-intersección*)
- ✓ x-coordinate (*coordenada x*)
- ✓ y-coordinate (*coordenada y*)
- ✓ y-intercept (*intersección con el eje y*)

Preview Words

- cluster (*agrupación*)
- outlier (*valor extremo*)
- scatter plot (*diagrama de dispersión*)
- trend line (*línea de tendencia*)

Active Reading

Two-Panel Flip Chart Create a two-panel flip chart, to help you understand the concepts in this module. Label each flap with the title of one of the lessons in the module. As you study each lesson, write important ideas under the appropriate flap. Include any sample problems or equations that will help you remember the concepts later when you look back at your notes.



Are YOU Ready?

Complete these exercises to review skills you will need for this module.



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Evaluate Expressions

EXAMPLE Evaluate $4x + 3$ for $x = 5$.

$$\begin{aligned}4x + 3 &= 4(5) + 3 \\ &= 20 + 3 \\ &= 23\end{aligned}$$

Substitute the given value for x .

Multiply.

Add.

Evaluate each expression for the given value of x .

1. $6x - 5$ for $x = 4$

2. $-2x + 7$ for $x = 2$

3. $5x - 6$ for $x = 3$

4. $0.5x + 8.4$ for $x = -1$

5. $\frac{3}{4}x - 9$ for $x = -20$

6. $1.4x + 3.5$ for $x = -4$

Solve Two-Step Equations

EXAMPLE $5x + 3 = -7$

$$\begin{aligned}\underline{-3} &= \underline{-3} \\ 5x &= -10 \\ \frac{5x}{5} &= \frac{-10}{5} \\ x &= -2\end{aligned}$$

Subtract 3 from both sides.

Divide both sides by 5.

Solve for x .

7. $3x + 4 = 10$

8. $5x - 11 = 34$

9. $-2x + 5 = -9$

10. $8x + 13 = -11$

11. $4x - 7 = -27$

12. $\frac{1}{2}x + 16 = 39$

13. $\frac{2}{3}x - 16 = 12$

14. $0.5x - 1.5 = -6.5$

Are YOU Ready? (cont'd)

Complete these exercises to review skills you will need for this module.

Evaluate Expressions

15. Show why the expression $-1.3x - 6.5$ is neither positive nor negative when $x = -5$.

16. Evaluate the expression $\frac{1}{2}x + 6.8$ for $x = -2, 0, 2,$ and 4 . Describe a pattern in the results.

Solve Two-Step Equations

17. Describe a plan using inverse operations to solve $\frac{1}{2}x + 8 = 42$. Then find the solution.

18. Caroline solved the equation below as shown. What was Caroline's error?

$$\begin{aligned}0.5x - 1.2 &= 8 \\0.5x &= 6.8 \\x &= 13.6\end{aligned}$$

LESSON 14.1 Scatter Plots and Association

8.6.14.1

Students will construct and interpret scatter plots.



ESSENTIAL QUESTION

How can you construct and interpret scatter plots?

EXPLORE ACTIVITY 1

Making a Scatter Plot

Recall that a set of bivariate data involves two variables. Bivariate data are used to explore the relationship between two variables. You can graph bivariate data on a *scatter plot*. A **scatter plot** is a graph with points plotted to show the relationship between two sets of data.

The final question on a math test reads, “How many hours did you spend studying for this test?” The teacher records the number of hours each student studied and the grade the student received on the test.

- A** Make a prediction about the relationship between the number of hours spent studying and test grades.

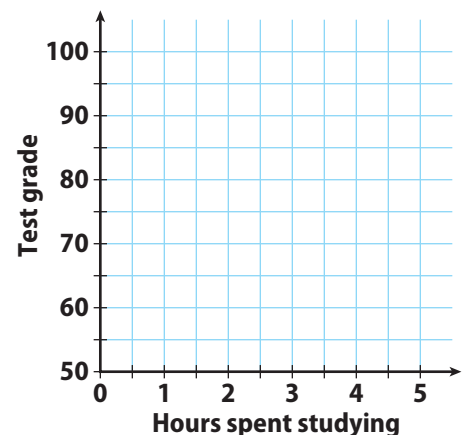
- B** Make a scatter plot. Graph hours spent studying as the independent variable and test grades as the dependent variable.

Reflect

1. What trend do you see in the data?

2. **Justify Reasoning** Do you think the grade associated with studying for 10 hours would follow this trend?

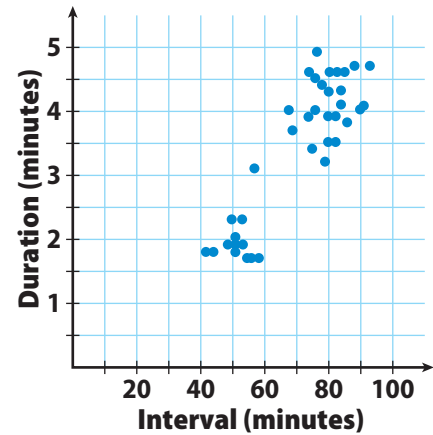
Hours Spent Studying	Test Grade
0	75
0.5	80
1	80
1	85
1.5	85
1.5	95
2	90
3	100
4	90



Interpreting Clusters and Outliers

A **cluster** is a set of closely grouped data. Data may cluster around a point or along a line. An **outlier** is a data point that is very different from the rest of the data in the set.

A scientist gathers information about the eruptions of Old Faithful, a geyser in Yellowstone National Park. She uses the data to create a scatter plot. The data show the length of time between eruptions (interval) and how long the eruption lasts (duration).



A Describe any clusters you see in the scatter plot.

B What do the clusters tell you about eruptions of Old Faithful?

C Describe any outliers you see in the scatter plot.

Math Talk

Mathematical Processes

If the point (20, 1) appeared on the scatter plot, would it be an outlier? Explain.

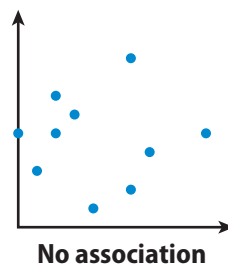
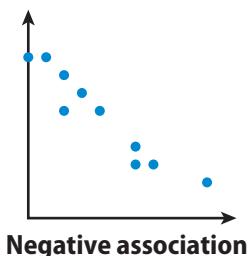
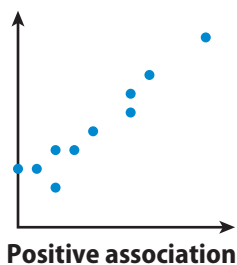
Reflect

3. Suppose the geyser erupts for 2.2 minutes after a 75-minute interval. Would this point lie in one of the clusters? Would it be an outlier? Explain your answer.

4. Suppose the geyser erupts after an 80-minute interval. Give a range of possible duration times for which the point on the scatter plot would not be considered an outlier. Explain your reasoning.

Determining Association

Association describes how sets of data are related. A *positive* association means that both data sets increase together. A *negative* association means that as one data set increases, the other decreases. *No* association means that there is no relationship between the two data sets.



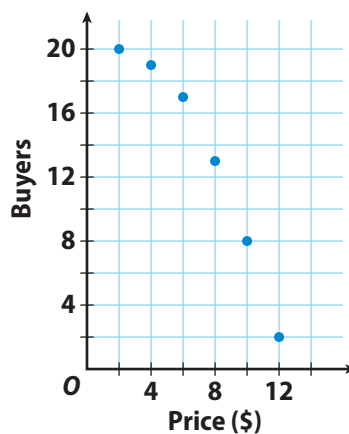
Data that show a positive or negative association and lie basically along a line exhibit a *linear* association. Data that show a positive or negative association but do not lie basically along a line exhibit a *nonlinear* association.

EXAMPLE 1



Susan asked 20 people if they would buy a new product she developed at each of several prices. The scatter plot shows how many of the 20 said “yes” at a given price. Describe the association between price and the number of buyers.

As price increases, the number of buyers decreases. So, there is a negative association. Because the data points do not lie along a line, the association is nonlinear.

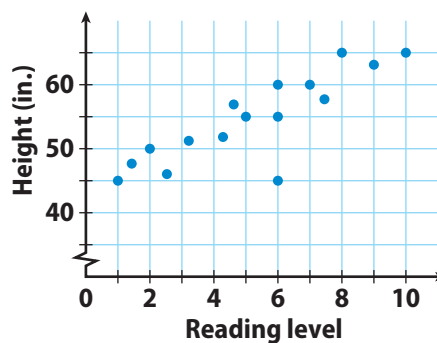


Reflect

5. **What If?** Based on the association shown in the scatter plot, what might happen if Susan increased the price to \$14?

YOUR TURN

6. The plot shows the reading level and height for 16 students in a district. Describe the association and give a possible reason for it.



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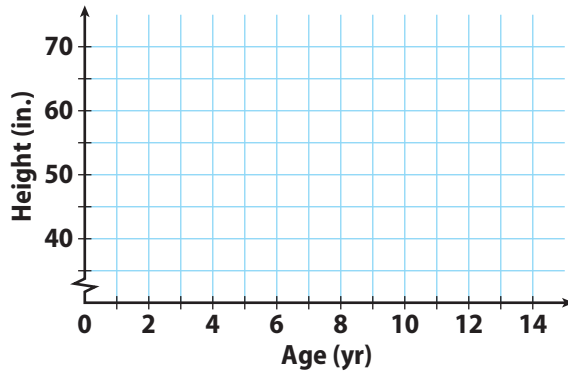
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Guided Practice

Bob recorded his height at different ages. The table below shows his data.

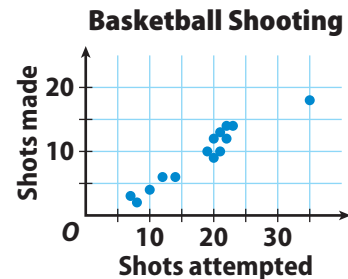
Age (years)	6	8	10	12	14
Height (inches)	45	50	55	61	63

1. Make a scatter plot of Bob's data. ([Explore Activity 1](#))



2. Describe the association between Bob's age and his height. Explain the association. ([Example 1](#))

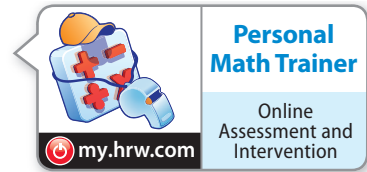
3. The scatter plot shows the basketball shooting results for 14 players. Describe any clusters you see in the scatter plot. Identify any outliers. ([Explore Activity 2](#))



ESSENTIAL QUESTION CHECK-IN

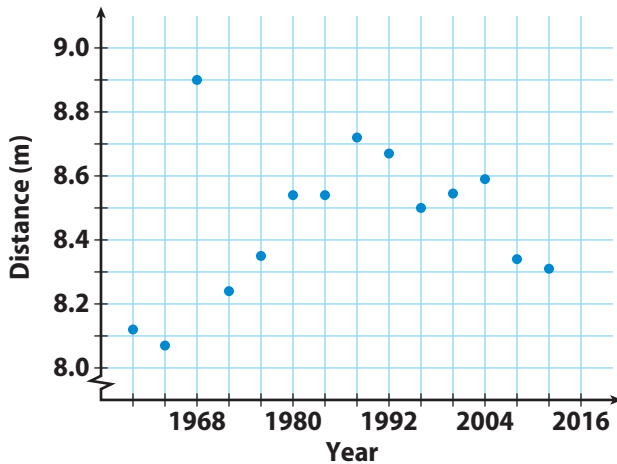
4. Explain how you can make a scatter plot from a set of bivariate data.

14.1 Independent Practice



Sports Use the scatter plot for 5–8.

Olympic Men’s Long Jump Winning Distances



5. Describe the association between the year and the distance jumped for the years 1960 to 1988.

6. Describe the association between the year and the distance jumped for the years after 1988.

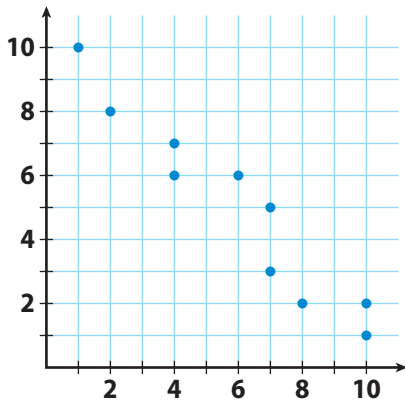
7. For the entire scatter plot, is the association between the year and the distance jumped linear or nonlinear?

8. Identify the outlier and interpret its meaning.

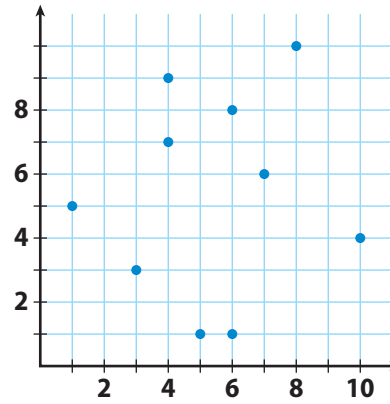
9. **Communicate Mathematical Ideas**
Compare a scatter plot that shows no association to one that shows negative association.

For 10–11, describe a set of real-world bivariate data that the given scatter plot could represent. Define the variable represented on each axis.

10.



11.





FOCUS ON HIGHER ORDER THINKING

12. **Multiple Representations** Describe what you might see in a table of bivariate data that would lead you to conclude that the scatter plot of the data would show a cluster.

13. **Justify Reasoning** Is it possible for a scatter plot to have a positive or negative association that is not linear? Explain.

14. **Critical Thinking** To try to increase profits, a theater owner increases the price of a ticket by \$25 every month. Describe what a scatter plot might look like if x represents the number of months and y represents the profits. Explain your reasoning.

Work Area

LESSON 14.2 Trend Lines and Predictions

8.6.14.2

Students will use a trend line to make a prediction from a scatter plot.



ESSENTIAL QUESTION

How can you use a trend line to make a prediction from a scatter plot?

EXPLORE ACTIVITY 1

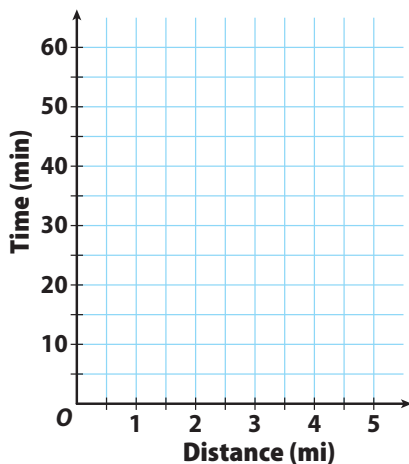
Drawing a Trend Line

When a scatter plot shows a linear association, you can use a line to model the relationship between the variables. A **trend line** is a straight line that comes closest to the points on a scatter plot.



Joyce is training for a 10K race. For some of her training runs, she records the distance she ran and how many minutes she ran.

- A** Make a scatter plot of Joyce's running data.



Distance (mi)	Time (min)
4	38
2	25
1	7
2	16
3	26
5	55
2	20
4	45
3	31

- B** To draw a trend line, use a straight edge to draw a line that has about the same number of points above and below it. Ignore any outliers.
- C** Use your trend line to predict how long it would take Joyce to run 4.5 miles.

Reflect

1. How well does your trend line fit the data? Explain.

EXPLORE ACTIVITY 1 (cont'd)

2. Do you think you can use a scatter plot that shows no association to make a prediction? Explain your answer.



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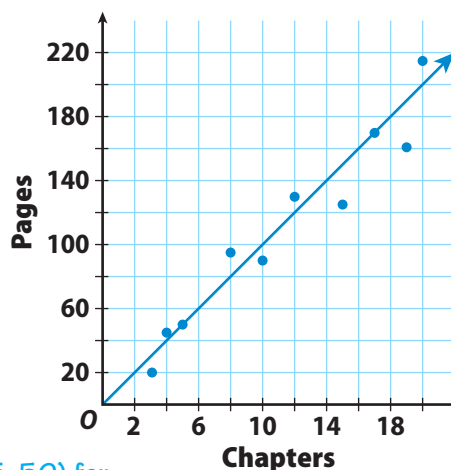
Finding the Equation of a Trend Line

You can use two points on a trend line to write an equation in slope-intercept form for the trend line.

EXAMPLE 1



The scatter plot and trend line show the relationship between the number of chapters and the total number of pages for several books. Write an equation for the trend line.



- STEP 1** Find the slope of the trend line. The line passes through points (5, 50) and (17, 170).

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Use the slope formula.}$$

$$m = \frac{170 - 50}{17 - 5} \quad \text{Substitute (5, 50) for } (x_1, y_1) \text{ and (17, 170) for } (x_2, y_2).$$

$$m = \frac{120}{12} = 10 \quad \text{Simplify.}$$

- STEP 2** Find the y-intercept of the trend line.

$$y = mx + b \quad \text{Slope-intercept form}$$

$$50 = 10 \cdot 5 + b \quad \text{Substitute 50 for } y, 10 \text{ for } m, \text{ and 5 for } x.$$

$$50 = 50 + b \quad \text{Simplify.}$$

$$50 - 50 = 50 - 50 + b \quad \text{Subtract 50 from both sides.}$$

$$0 = b \quad \text{Simplify.}$$

- STEP 3** Use your slope and y-intercept values to write the equation.

$$y = mx + b \quad \text{Slope-intercept form}$$

$$y = 10x + 0 \quad \text{Substitute 10 for } m \text{ and 0 for } y.$$

The equation for the trend line is $y = 10x$.

Math Talk

Mathematical Processes

Why are (5, 50) and (17, 170) the best points to use to draw the trend line?

Reflect

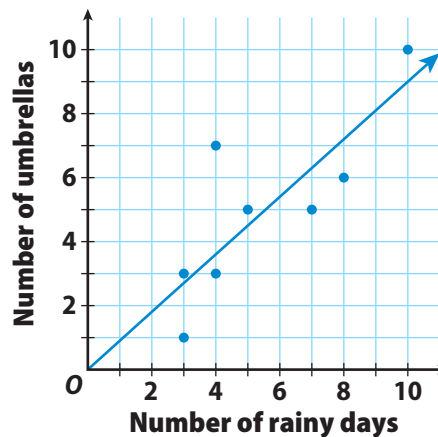
3. What type(s) of association does the scatter plot show?

4. What is the meaning of the slope in this situation?

5. What is the meaning of the y-intercept in this situation?

YOUR TURN

6. The scatter plot and trend line show the relationship between the number of rainy days in a month and the number of umbrellas sold each month. Write an equation for the trend line.



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EXPLORE ACTIVITY 2



Making Predictions

When you use a trend line or its equation to predict a value between data points that you already know, you *interpolate* the predicted value. When you make a prediction that is outside the data that you know, you *extrapolate* the predicted value.

Use the equation of the trend line in Example 1 to predict how many pages would be in a book with 26 chapters.

Is this prediction an example of interpolation or extrapolation? _____

$y =$ Write the equation for your trend line.

$y =$ Substitute the number of chapters for x .

$y =$ Simplify.

I predict that a book with 26 chapters will have _____ pages.

EXPLORE ACTIVITY 2 (cont'd)

Reflect

7. **Make a Prediction** Predict how many pages would be in a book with 14 chapters. Is this prediction an example of interpolation or extrapolation?

8. Do you think that extrapolation or interpolation is more accurate? Explain.

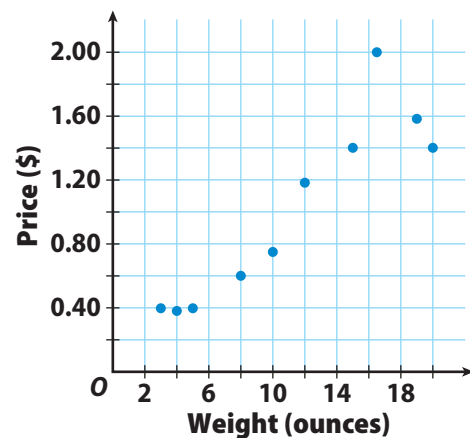
Guided Practice

Angela recorded the price of different weights of several bulk grains. She made a scatter plot of her data. Use the scatter plot for 1–4.

1. Draw a trend line for the scatter plot. (Explore Activity 1)
2. How do you know whether your trend line is a good fit for the data? (Explore Activity 1)

3. Write an equation for your trend line. (Example 1) _____
4. Use the equation for your trend line to interpolate the price of 7 ounces and extrapolate the price of 50 ounces.

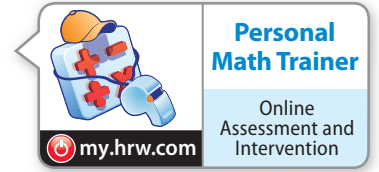
(Explore Activity 2) _____



ESSENTIAL QUESTION CHECK-IN

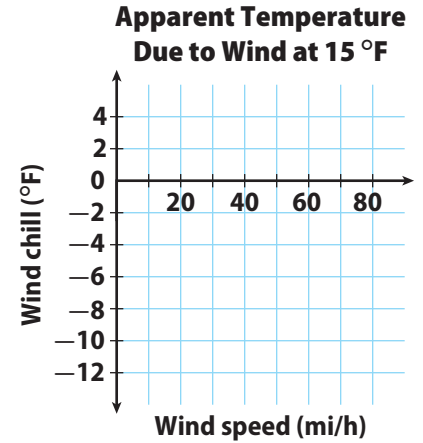
5. A trend line passes through two points on a scatter plot. How can you use the trend line to make a prediction between or outside the given data points?

14.2 Independent Practice



Use the data in the table for Exercises 6–10.

Apparent Temperature Due to Wind at 15 °F						
Wind speed (mi/h)	10	20	30	40	50	60
Wind chill (°F)	2.7	-2.3	-5.5	-7.9	-9.8	-11.4

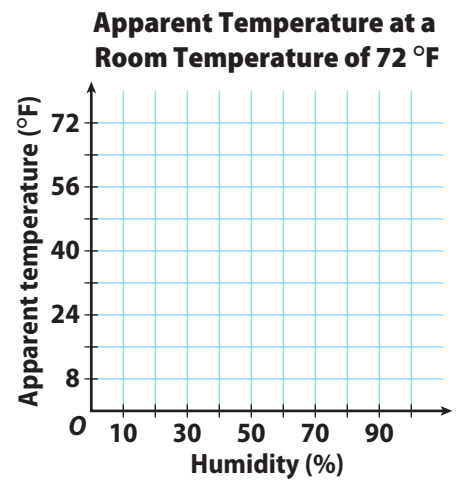


- Make a scatter plot of the data and draw a trend line.
- What type of association does the trend line show?

- Write an equation for your trend line. _____
- Make a Prediction** Use the trend line to predict the wind chill at these wind speeds.
 - 36 mi/h _____
 - 100 mi/h _____
- What is the meaning of the slope of the line?

Use the data in the table for Exercises 11–14.

Apparent Temperature Due to Humidity at a Room Temperature of 72 °F						
Humidity (%)	0	20	40	60	80	100
Apparent temperature (°F)	64	67	70	72	74	76



- Make a scatter plot of the data and draw a trend line.
- Write an equation for your trend line.

- Make a Prediction** Use the trend line to predict the apparent temperature at 70% humidity. _____
- What is the meaning of the y-intercept of the line?



15. Communicate Mathematical Ideas Is it possible to draw a trend line on a scatter plot that shows no association? Explain.

16. Critique Reasoning Sam drew a trend line that had about the same number of data points above it as below it, but did not pass through any data points. He then picked two data points to write the equation for the line. Is this a correct way to write the equation? Explain.

17. Marlene wanted to find a relationship between the areas and populations of counties in Texas. She plotted x (area in square miles) and y (population) for two counties on a scatter plot:

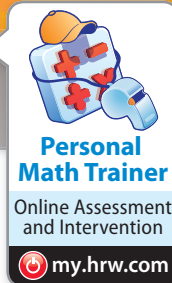
Kent County (903, 808) Edwards County (2118, 2002)

She concluded that the population of Texas counties is approximately equal to their area in square miles and drew a trend line through her points.

a. Critique Reasoning Do you agree with Marlene's method of creating a scatter plot and a trend line? Explain why or why not.

b. Counterexamples Harris County has an area of 1778 square miles and a population of about 4.3 million people. Dallas County has an area of 908 square miles and a population of about 2.5 million people. What does this data show about Marlene's conjecture that the population of Texas counties is approximately equal to their area?

Ready to Go On?

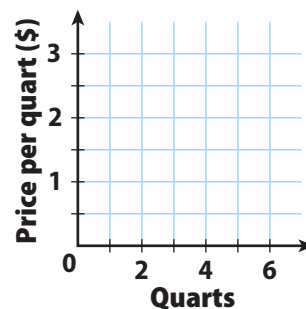


14.1 Scatter Plots and Association

An auto store is having a sale on motor oil. The chart shows the price per quart as the number of quarts purchased increases. Use the data for Exs. 1–2.

Number of quarts	1	2	3	4	5	6
Price per quart (\$)	2	1.50	1.25	1.10	1	0.95

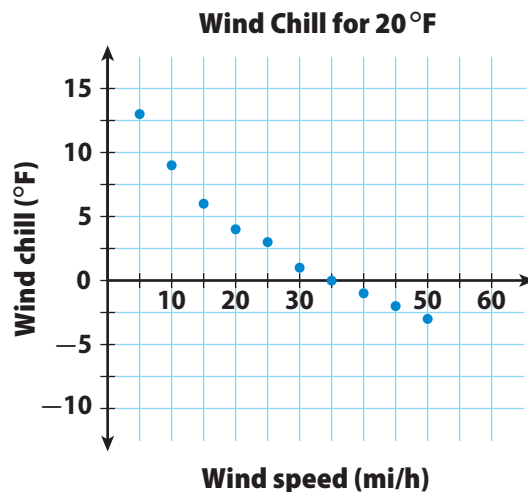
- Use the given data to make a scatter plot.
- Describe the association you see between the number of quarts purchased and the price per quart. Explain.



14.2 Trend Lines and Predictions

The scatter plot below shows data comparing wind speed and wind chill for an air temperature of 20°F. Use the scatter plot for Exs. 3–5.

- Draw a trend line for the scatter plot.
- Write an equation for your trend line.
- Use your equation to predict the wind chill to the nearest degree for a wind speed of 60 mi/h.



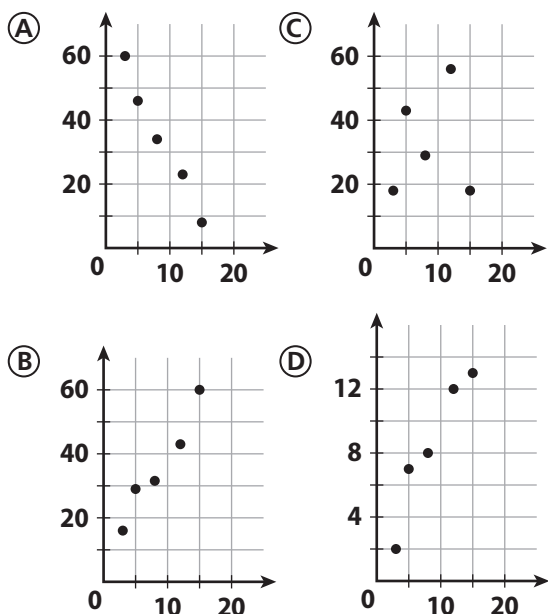
ESSENTIAL QUESTION

- How can you use scatter plots to solve real-world problems?



Selected Response

1. Which scatter plot could have a trend line whose equation is $y = 3x + 10$?

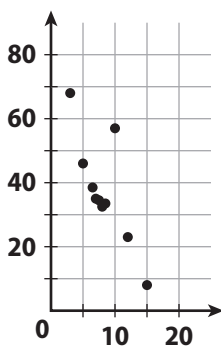


2. What type of association would you expect between a person's age and hair length?

- (A) linear (C) none
(B) negative (D) positive

3. Which is **not** shown on the scatter plot?

- (A) cluster
(B) negative association
(C) outlier
(D) positive association



4. A restaurant claims to have served 352,000,000 hamburgers. What is this number in scientific notation?

- (A) 3.52×10^6 (C) 35.2×10^7
(B) 3.52×10^8 (D) 352×10^6

5. Which equation describes the relationship between x and y in the table?

x	-8	-4	0	4	8
y	2	1	0	-1	-2

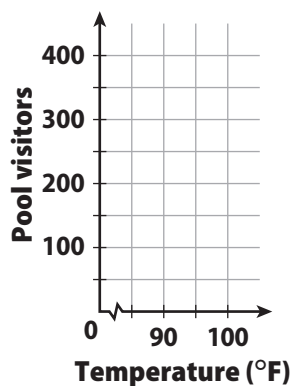
- (A) $y = -4x$ (C) $y = 4x$
(B) $y = -\frac{1}{4}x$ (D) $y = \frac{1}{4}x$

Mini-Task

6. Use the data in the table.

Temp (°F)	97	94	87	92	100	90
Pool visitors	370	315	205	135	365	240

- a. Make a scatterplot of the data.



- b. Which data point is an outlier?

- c. Predict the number of visitors on a day when the high temperature is 102°F.
