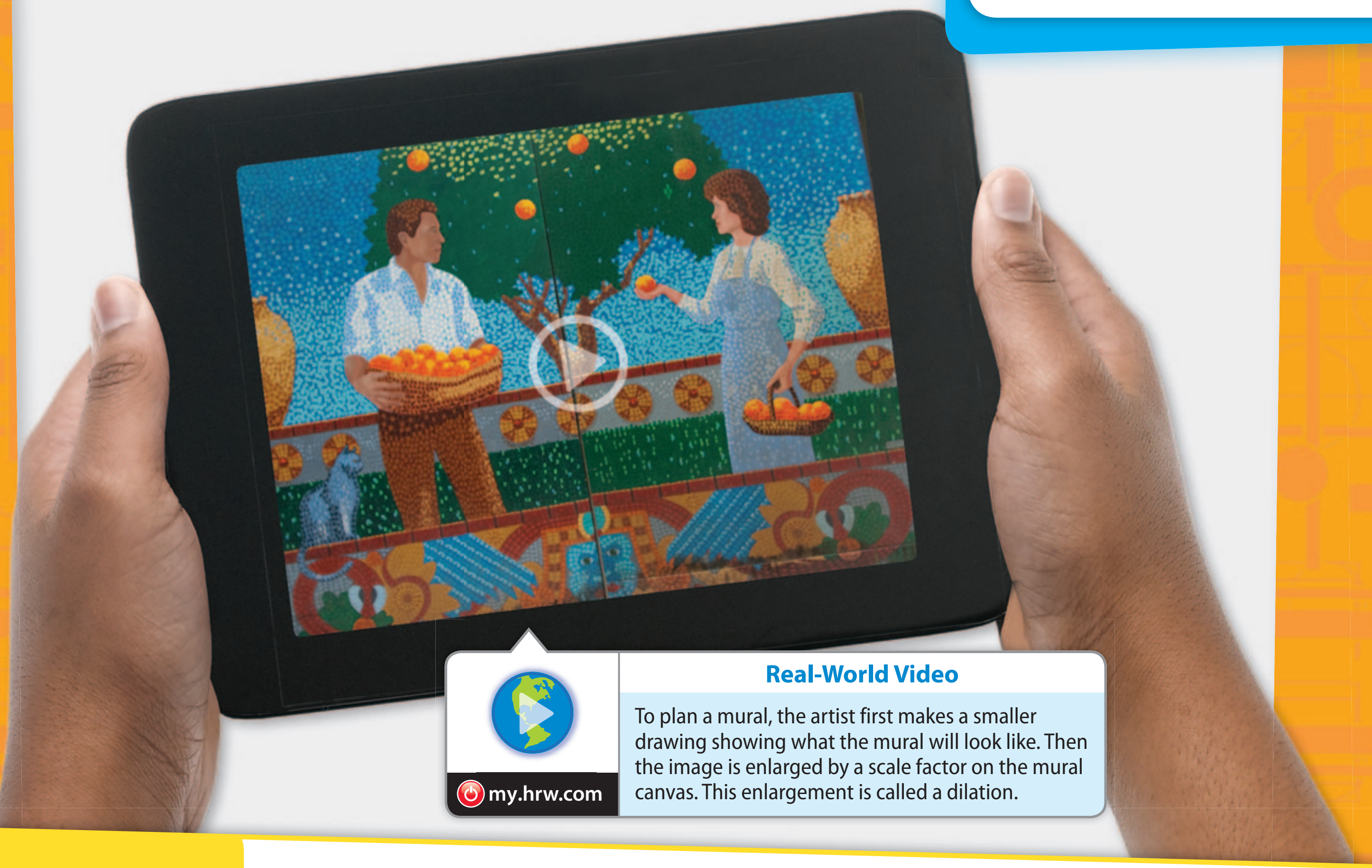
Transformations and Similarity

MODULE 10

ESSENTIAL QUESTION

How can you use dilations and similarity to solve real-world problems?



Real-World Video

To plan a mural, the artist first makes a smaller drawing showing what the mural will look like. Then the image is enlarged by a scale factor on the mural canvas. This enlargement is called a dilation.

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Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the graphic organizer. You will put one word in each rectangle.

The four regions on a coordinate plane.

The point where the axes intersect to form the coordinate plane.

Reviewing the Coordinate Plane

The horizontal axis of a coordinate plane.

The vertical axis of a coordinate plane.

Vocabulary

Review Words

- coordinate plane *(plano cartesiano)*
- image *(imagen)*
- ✔ origin *(origen)*
- preimage *(imagen original)*
- ✔ quadrants *(cuadrante)*
- ratio *(razón)*
- scale *(escala)*
- ✔ *x*-axis *(eje x)*
- ✔ *y*-axis *(eje y)*

Preview Words

- center of dilation *(centro de dilatación)*
- dilation *(dilatación)*
- enlargement *(agrandamiento)*
- reduction *(reducción)*
- scale factor *(factor de escala)*
- similar *(similar)*

Understand Vocabulary

Complete the sentences using the preview words.

1. A figure larger than the original, produced through dilation, is an ________________.

2. A figure smaller than the original, produced through dilation, is a ________________.

Active Reading

Key-Term Fold Before beginning the module, create a key-term fold to help you learn the vocabulary in this module. Write the highlighted vocabulary words on one side of the flap. Write the definition for each word on the other side of the flap. Use the key-term fold to quiz yourself on the definitions used in this module.

Are YOU Ready?

Complete these exercises to review skills you will need for this module.

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Simplify Ratios

EXAMPLE

$$\frac{35}{21} = \frac{35 \div 7}{21 \div 7} = \frac{5}{3}$$

To write a ratio in simplest form, find the greatest common factor of the numerator and denominator. Divide the numerator and denominator by the GCF.

Write each ratio in simplest form.

1. $\frac{6}{15}$ ________ **2.** $\frac{8}{20}$ ________ **3.** $\frac{30}{18}$ ________ **4.** $\frac{36}{30}$ ________

Multiply with Fractions and Decimals

EXAMPLE

$$2\tfrac{3}{5} \times 20 = \frac{13 \times 20}{5 \times 1} = \frac{13 \times \cancel{20}^{4}}{{}_{1}\cancel{5} \times 1} = 52$$

Write numbers as fractions and multiply.

Simplify.

$$\begin{array}{r} 68 \\ \times 4.5 \\ \hline 340 \\ +272 \\ \hline 306.0 \end{array}$$

Multiply as you would with whole numbers.

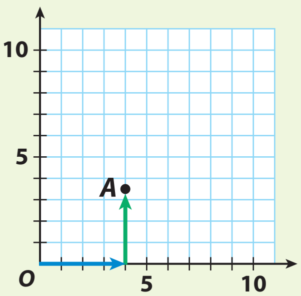
Place the decimal point in the answer based on the total number of decimal places in the two factors.

Multiply.

5. $60 \times \frac{25}{100}$ ________ **6.** 3.5×40 ________ **7.** 4.4×44 ________ **8.** $24 \times \frac{8}{9}$ ________

Graph Ordered Pairs (First Quadrant)

EXAMPLE



Graph the point $A(4, 3.5)$.
Start at the origin.
Move 4 units right.
Then move 3.5 units up.
Graph point $A(4, 3.5)$.

Graph each point on the coordinate grid above.

9. B (9, 0) **10.** C (2, 7) **11.** D (0, 4.5) **12.** E (6, 2.5)

Are YOU Ready? (cont'd)

Complete these exercises to review skills you will need for this module.

Simplify Ratios

13. a. Explain how to find the simplest form of a ratio.

b. Describe what these ratios have in common.

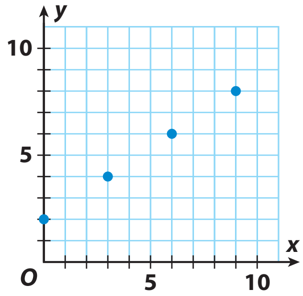
$\frac{75}{100}, \frac{24}{32}, \frac{15}{20}, \frac{9}{12}$

Multiply with Fractions and Decimals

14. To find $32 \times 1\frac{3}{4}$, Jose rewrote $1\frac{3}{4}$ as $\frac{6}{4}$ and multiplied to get 48. Find and correct Jose's error.

Graph Ordered Pairs (First Quadrant)

15. If the *x*- and *y*-coordinates of each of the points shown are doubled, what are the coordinates of the resulting points? Can they be graphed on the same grid? Explain.



16. Adam drew a graph by plotting the value of a car each year after purchase. The *x*-axis label was "Time (years)" and the *y*-axis label was "Value (thousands of dollars)." The car's value was $5,749 after 4 years. Explain how he graphed this point.

LESSON
10.1 Properties of Dilations

8.4.10.1

Students will describe the properties of dilations.

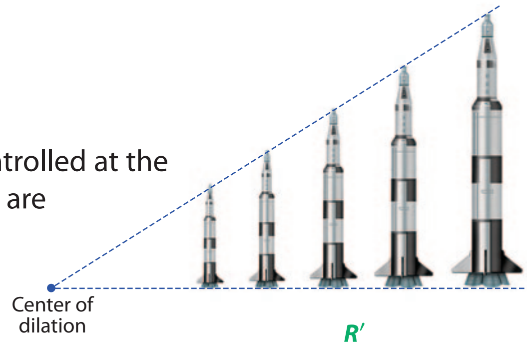
ESSENTIAL QUESTION

How do you describe the properties of dilations?

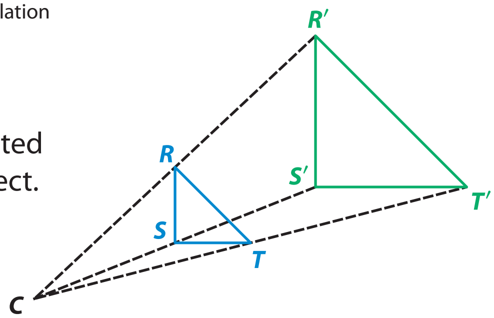
EXPLORE ACTIVITY 1 Real World

Exploring Dilations

The missions that placed 12 astronauts on the moon were controlled at the Johnson Space Center in Houston. The toy models at the right are scaled-down replicas of the Saturn V rocket that powered the moon flights. Each replica is a transformation called a **dilation**. Unlike the other transformations you have studied—translations, rotations, and reflections—dilations change the size (but not the shape) of a figure.



Every dilation has a fixed point called the **center of dilation** located where the lines connecting corresponding parts of figures intersect.



Triangle $R'S'T'$ is a dilation of triangle RST. Point C is the center of dilation.

A Use a ruler to measure segments $\overline{CR}$, $\overline{CR'}$, $\overline{CS}$, $\overline{CS'}$, $\overline{CT}$, and $\overline{CT'}$ to the nearest millimeter. Record the measurements and ratios in the table.

CR'	CR	$\frac{CR'}{CR}$	CS'	CS	$\frac{CS'}{CS}$	CT'	CT	$\frac{CT'}{CT}$

B Write a conjecture based on the ratios in the table.

____

C Measure and record the corresponding side lengths of the triangles.

$R'S'$	RS	$\frac{R'S'}{RS}$	$S'T'$	ST	$\frac{S'T'}{ST}$	$R'T'$	RT	$\frac{R'T'}{RT}$

D Write a conjecture based on the ratios in the table.

____

E Measure the corresponding angles and describe your results.

____

EXPLORE ACTIVITY 1 *(cont'd)*

Reflect

1. Two figures that have the same shape but different sizes are called *similar*. Are triangles *RST* and *R′S′T′* similar? Why or why not?

____

____

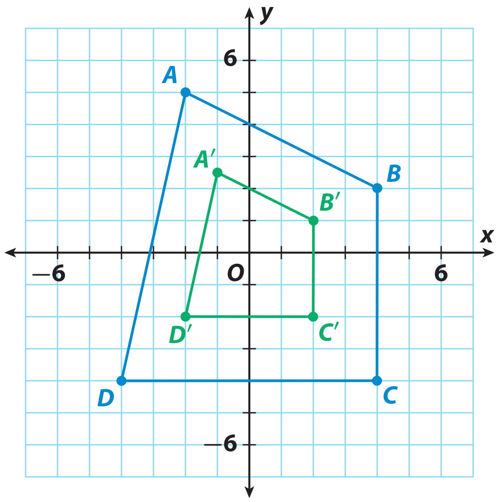
2. Compare the orientation of a figure with the orientation of its dilation.

____

EXPLORE ACTIVITY 2

Exploring Dilations on a Coordinate Plane

In this activity you will explore how the coordinates of a figure on a coordinate plane are affected by a dilation.



A Complete the table. Record the *x*- and *y*-coordinates of the points in the two figures and the ratios of the *x*-coordinates and the *y*-coordinates.

Vertex	*x*	*y*	Vertex	*x*	*y*	Ratio of *x*-coordinates ($A'B'C'D' \div ABCD$)	Ratio of *y*-coordinates ($A'B'C'D' \div ABCD$)
A′			*A*				
B′			*B*				
C′			*C*				
D′			*D*				

B Write a conjecture about the ratios of the coordinates of a dilation image to the coordinates of the original figure.

____

____

Reflect

3. In Explore Activity 1, triangle $R'S'T'$ was larger than triangle RST. How is the relationship between quadrilateral $A'B'C'D'$ and quadrilateral $ABCD$ different?

____

Math Talk
Mathematical Processes
How are dilations different from the other transformations you have learned about?

Finding a Scale Factor

As you have seen in the two activities, a dilation can produce a larger figure (an **enlargement**) or a smaller figure (a **reduction**). The **scale factor** describes how much the figure is enlarged or reduced. The scale factor is the ratio of a length of the image to the corresponding length on the original figure.

In Explore Activity 1, the side lengths of triangle $R'S'T'$ were twice the length of those of triangle RST, so the scale factor was 2. In Explore Activity 2, the side lengths of quadrilateral $A'B'C'D'$ were half those of quadrilateral $ABCD$, so the scale factor was 0.5.

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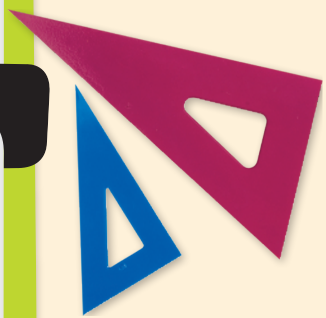
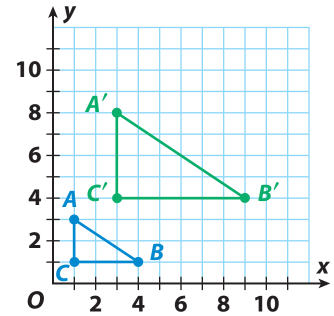


EXAMPLE 1 Real World

An art supply store sells several sizes of drawing triangles. All are dilations of a single basic triangle. The basic triangle and one of its dilations are shown on the grid. Find the scale factor of the dilation.



STEP 1 Use the coordinates to find the lengths of the sides of each triangle.

Triangle ABC: $AC = 2$ $CB = 3$

Triangle $A'B'C'$: $A'C' = 4$ $C'B' = 6$

Since the scale factor is the same for all corresponding sides, you can record just two pairs of side lengths. Use one pair as a check on the other.

STEP 2 Find the ratios of the corresponding sides.

$\frac{A'C'}{AC} = \frac{4}{2} = 2$ $\frac{C'B'}{CB} = \frac{6}{3} = 2$

The scale factor of the dilation is 2.

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Reflect

4. Is the dilation an enlargement or a reduction? How can you tell?

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YOUR TURN

5. Find the scale factor of the dilation.

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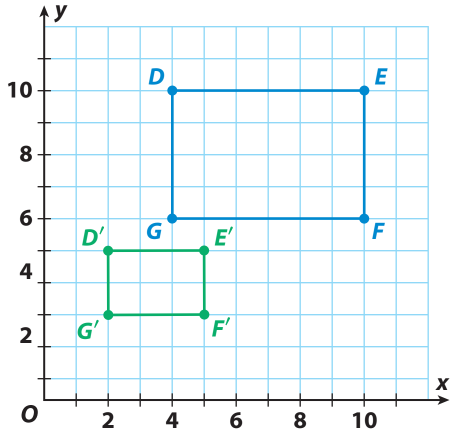
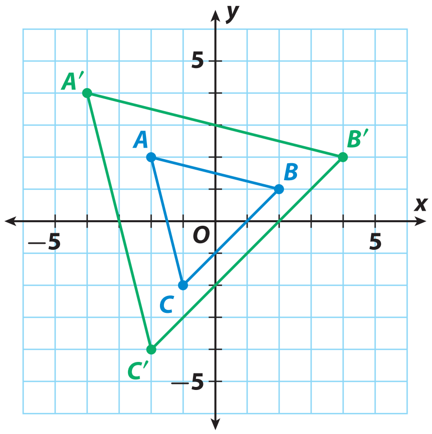


Math Talk
Mathematical Processes
Which scale factors lead to enlargements? Which scale factors lead to reductions?

Guided Practice

Use triangles *ABC* and *A′B′C′* for 1–5. (Explore Activities 1 and 2, Example 1)



1. For each pair of corresponding vertices, find the ratio of the *x*-coordinates and the ratio of the *y*-coordinates.

ratio of *x*-coordinates = ______________

ratio of *y*-coordinates = ______________

2. I know that triangle *A′B′C′* is a dilation of triangle *ABC* because the ratios of the corresponding *x*-coordinates are ______________ and the ratios of the corresponding *y*-coordinates are ______________.

3. The ratio of the lengths of the corresponding sides of triangle *A′B′C′* and triangle *ABC* equals ______________.

4. The corresponding angles of triangle *ABC* and triangle *A′B′C′* are ______________.

5. The scale factor of the dilation is ______________.

ESSENTIAL QUESTION CHECK-IN

6. How can you find the scale factor of a dilation?

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10.1 Independent Practice

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For 7–11, tell whether one figure is a dilation of the other or not. Explain your reasoning.

7. Quadrilateral *MNPQ* has side lengths of 15 mm, 24 mm, 21 mm, and 18 mm. Quadrilateral *M′N′P′Q′* has side lengths of 5 mm, 8 mm, 7 mm, and 4 mm.

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8. Triangle *RST* has angles measuring 38° and 75°. Triangle *R′S′T′* has angles measuring 67° and 38°. The sides are proportional.

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9. Two triangles, Triangle 1 and Triangle 2, are similar.

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10. Quadrilateral *MNPQ* is the same shape but a different size than quadrilateral *M′N′P′Q*.

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11. On a coordinate plane, triangle *UVW* has coordinates $U(20, -12)$, $V(8, 6)$, and $W(-24, -4)$. Triangle *U′V′W′* has coordinates $U'(15, -9)$, $V'(6, 4.5)$, and $W'(-18, -3)$.

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Complete the table by writing "same" or "changed" to compare the image with the original figure in the given transformation.

	Image Compared to Original Figure			
		Orientation	Size	Shape
12.	Translation			
13.	Reflection			
14.	Rotation			
15.	Dilation			

16. Describe the image of a dilation with a scale factor of 1.

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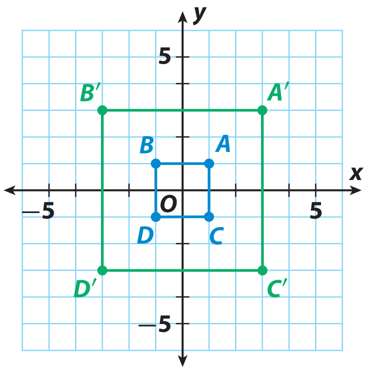
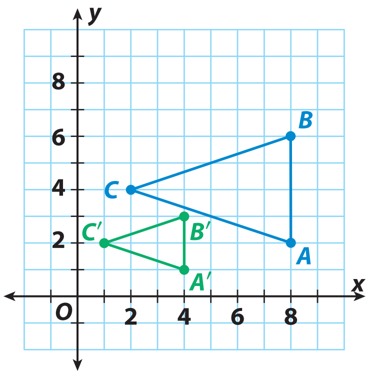
Identify the scale factor used in each dilation.

17.



18.



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H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

19. Critical Thinking Explain how you can find the center of dilation of a triangle and its dilation.

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20. Make a Conjecture

a. A square on the coordinate plane has vertices at (–2, 2), (2, 2), (2, –2), and (–2, –2). A dilation of the square has vertices at (–4, 4), (4, 4), (4, –4), and (–4, –4). Find the scale factor and the perimeter of each square.

____

____

b. A square on the coordinate plane has vertices at (–3, 3), (3, 3), (3, –3), and (–3, –3). A dilation of the square has vertices at (–6, 6), (6, 6), (6, –6), and (–6, –6). Find the scale factor and the perimeter of each square.

____

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c. Make a conjecture about the relationship of the scale factor to the perimeter of a square and its image.

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LESSON
10.2 Algebraic Representations of Dilations

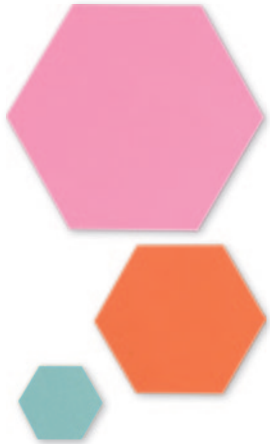
8.4.10.2

Students will describe the effect of a dilation on coordinates using an algebraic representation.

ESSENTIAL QUESTION

How can you describe the effect of a dilation on coordinates using an algebraic representation?

EXPLORE ACTIVITY 1



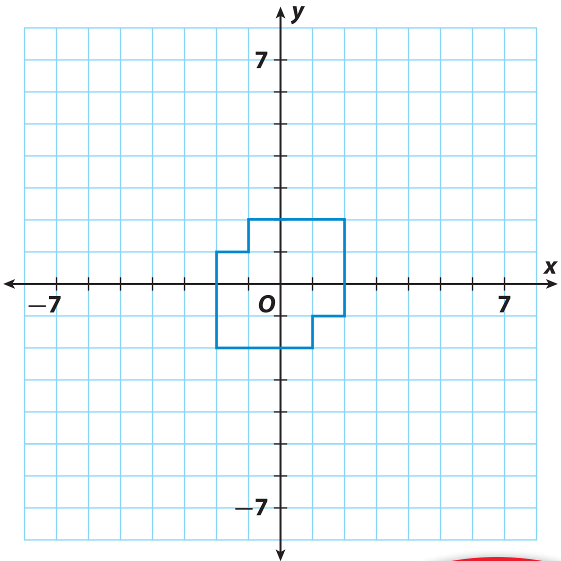
Graphing Enlargements

When a dilation in the coordinate plane has the origin as the center of dilation, you can find points on the dilated image by multiplying the x- and y-coordinates of the original figure by the scale factor. For scale factor k, the algebraic representation of the dilation is $(x, y) \rightarrow (kx, ky)$. For enlargements, $k > 1$.

The figure shown on the grid is the preimage. The center of dilation is the origin.

A List the coordinates of the vertices of the preimage in the first column of the table.

Preimage (x, y)	Image $(3x, 3y)$
(2, 2)	(6, 6)



B What is the scale factor for the dilation? ________

C Apply the dilation to the preimage and write the coordinates of the vertices of the image in the second column of the table.

D Sketch the image after the dilation on the coordinate grid.

Math Talk
Mathematical Processes

What effect would the dilation $(x, y) \rightarrow (4x, 4y)$ have on the radius of a circle?

EXPLORE ACTIVITY 1 *(cont'd)*

Reflect

1. How does the dilation affect the length of line segments?

____

____

2. How does the dilation affect angle measures?

____

EXPLORE ACTIVITY 2

Graphing Reductions

For scale factors between 0 and 1, the image is smaller than the preimage. This is called a reduction.

The arrow shown is the preimage. The center of dilation is the origin.

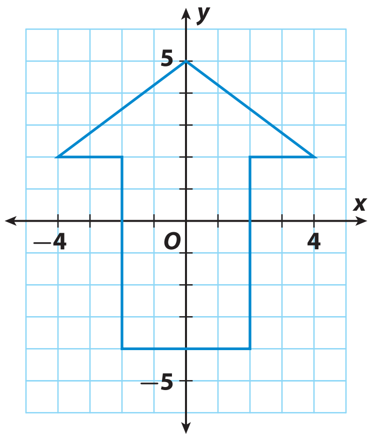
A List the coordinates of the vertices of the preimage in the first column of the table.

B What is the scale factor for the dilation? _______

C Apply the dilation to the preimage and write the coordinates of the vertices of the image in the second column of the table.

D Sketch the image after the dilation on the coordinate grid.

Preimage (x, y)	Image $\left(\frac{1}{2}x, \frac{1}{2}y\right)$



Reflect

3. How does the dilation affect the length of line segments?

____

4. How would a dilation with scale factor 1 affect the preimage?

____

Center of Dilation Outside the Image

The center of dilation can be inside *or* outside the original image and the dilated image. The center of dilation can be anywhere on the coordinate plane as long as the lines that connect each pair of corresponding vertices between the original and dilated image intersect at the center of dilation.

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EXAMPLE 1

Graph the image of $\triangle ABC$ after a dilation with the origin as its center and a scale factor of 3. What are the vertices of the image?

STEP 1 Multiply each coordinate of the vertices of $\triangle ABC$ by 3 to find the vertices of the dilated image.

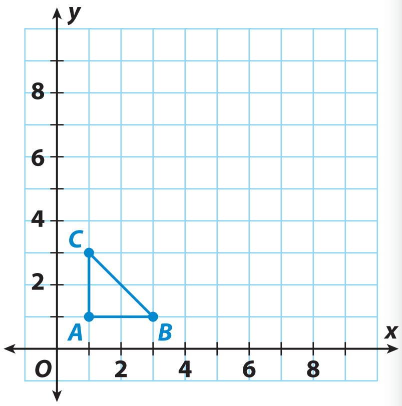
$\triangle ABC\ (x, y) \rightarrow (3x, 3y)\ \triangle A'B'C'$

$A(1, 1) \rightarrow A'(1 \cdot 3, 1 \cdot 3) \rightarrow A'(3, 3)$

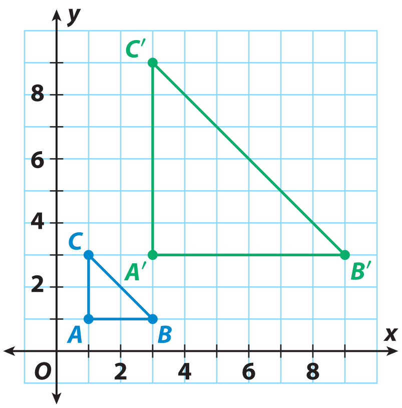
$B(3, 1) \rightarrow B'(3 \cdot 3, 1 \cdot 3) \rightarrow B'(9, 3)$

$C(1, 3) \rightarrow C'(1 \cdot 3, 3 \cdot 3) \rightarrow C'(3, 9)$

The vertices of the dilated image are $A'(3, 3)$, $B'(9, 3)$, and $C'(3, 9)$.



STEP 2 Graph the dilated image.



Math Talk
Mathematical Processes
Describe how you can check graphically that you have drawn the image triangle correctly.

YOUR TURN

5. Graph the image of $\triangle XYZ$ after a dilation with a scale factor of $\frac{1}{3}$ and the origin as its center. Then write an algebraic rule to describe the dilation.

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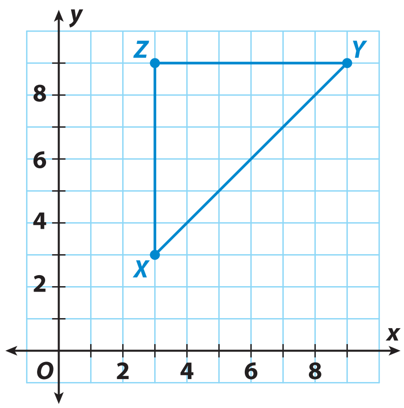

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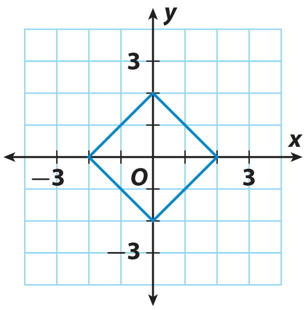
Guided Practice

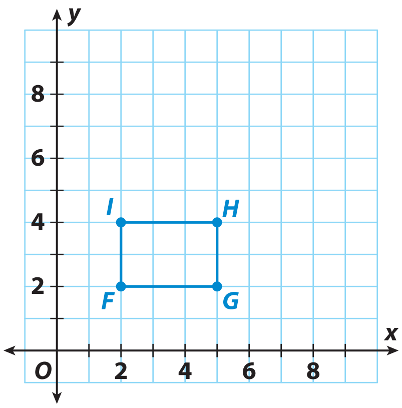
1. The grid shows a diamond-shaped preimage. Write the coordinates of the vertices of the preimage in the first column of the table. Then apply the dilation $(x, y) \rightarrow \left(\frac{3}{2}x, \frac{3}{2}y\right)$ and write the coordinates of the vertices of the image in the second column. Sketch the image of the figure after the dilation. (Explore Activities 1 and 2)

Preimage	Image
(2, 0)	(3, 0)



Graph the image of each figure after a dilation with the origin as its center and the given scale factor. Then write an algebraic rule to describe the dilation. (Example 1)

2. scale factor of 1.5



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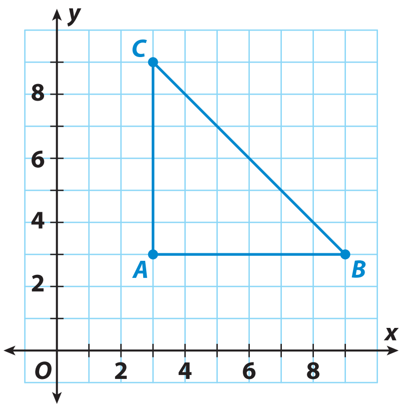
3. scale factor of $\frac{1}{3}$



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ESSENTIAL QUESTION CHECK-IN

4. A dilation of $(x, y) \rightarrow (kx, ky)$ when $0 < k < 1$ has what effect on the figure? What is the effect on the figure when $k > 1$?

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10.2 Independent Practice

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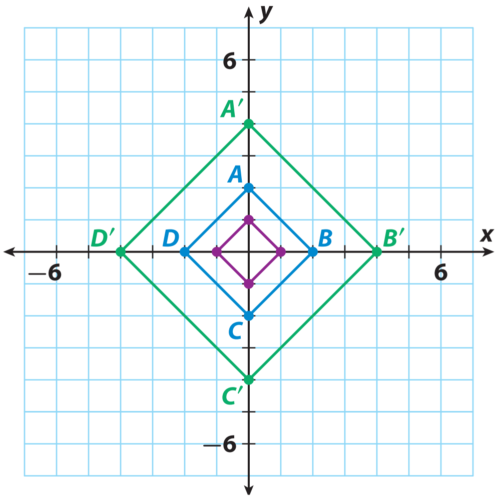
5. The blue square is the preimage. Write two algebraic representations, one for the dilation to the green square and one for the dilation to the purple square.



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6. **Critical Thinking** A triangle has vertices $A(-5, -4)$, $B(2, 6)$, and $C(4, -3)$. The center of dilation is the origin and $(x, y) \rightarrow (3x, 3y)$. What are the vertices of the dilated image?

____

____

7. **Critical Thinking** $M'N'O'P'$ has vertices at $M'(3, 4)$, $N'(6, 4)$, $O'(6, 7)$, and $P'(3, 7)$. The center of dilation is the origin. $MNOP$ has vertices at $M(4.5, 6)$, $N(9, 6)$, $O'(9, 10.5)$, and $P'(4.5, 10.5)$. What is the algebraic representation of this dilation?

____

8. **Critical Thinking** A dilation with center (0,0) and scale factor k is applied to a polygon. What dilation can you apply to the image to return it to the original preimage?

____

9. **Represent Real-World Problems** The blueprints for a new house are scaled so that $\frac{1}{4}$ inch equals 1 foot. The blueprint is the preimage and the house is the dilated image. The blueprints are plotted on a coordinate plane.

a. What is the scale factor in terms of inches to inches?

____

b. One inch on the blueprint represents how many inches in the actual house? How many feet?

____

c. Write the algebraic representation of the dilation from the blueprint to the house.

____

d. A rectangular room has coordinates $Q(2, 2)$, $R(7, 2)$, $S(7, 5)$, and $T(2, 5)$ on the blueprint. The homeowner wants this room to be 25% larger. What are the coordinates of the new room?

____

____

e. What are the dimensions of the new room, in inches, on the blueprint? What will the dimensions of the new room be, in feet, in the new house?

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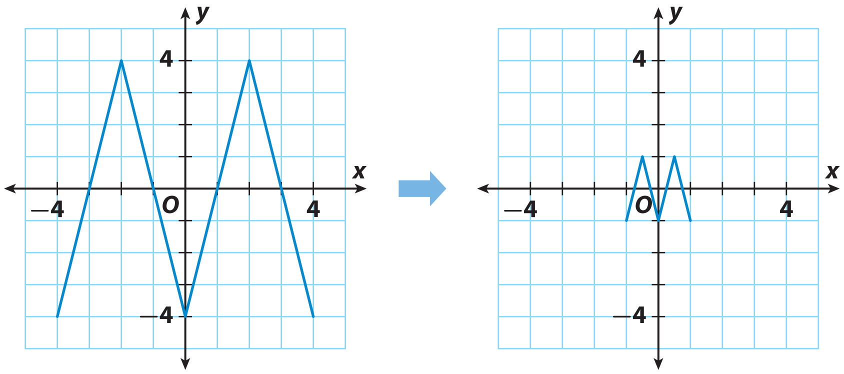
10. Write the algebraic representation of the dilation shown.



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H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

11. **Critique Reasoning** The set for a school play needs a replica of a historic building painted on a backdrop that is 20 feet long and 16 feet high. The actual building measures 400 feet long and 320 feet high. A stage crewmember writes $(x, y) \rightarrow \left(\frac{1}{12}x, \frac{1}{12}y\right)$ to represent the dilation. Is the crewmember's calculation correct if the painted replica is to cover the entire backdrop? Explain.

____

____

12. **Communicate Mathematical Ideas** Explain what each of these algebraic transformations does to a figure.

a. $(x, y) \rightarrow (y, -x)$ ______________________

b. $(x, y) \rightarrow (-x, -y)$ ______________________

c. $(x, y) \rightarrow (x, 2y)$ ______________________

____

d. $(x, y) \rightarrow \left(\frac{2}{3}x, y\right)$ ______________________

____

e. $(x, y) \rightarrow (0.5x, 1.5y)$ ______________________

____

13. **Communicate Mathematical Ideas** Triangle ABC has coordinates $A(1, 5)$, $B(-2, 1)$, and $C(-2, 4)$. Sketch triangle ABC and $A'B'C'$ for the dilation $(x, y) \rightarrow (-2x, -2y)$. What is the effect of a negative scale factor?

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LESSON

10.3 Similar Figures

8.4.10.3

Students will determine the connection between transformations and similar figures.

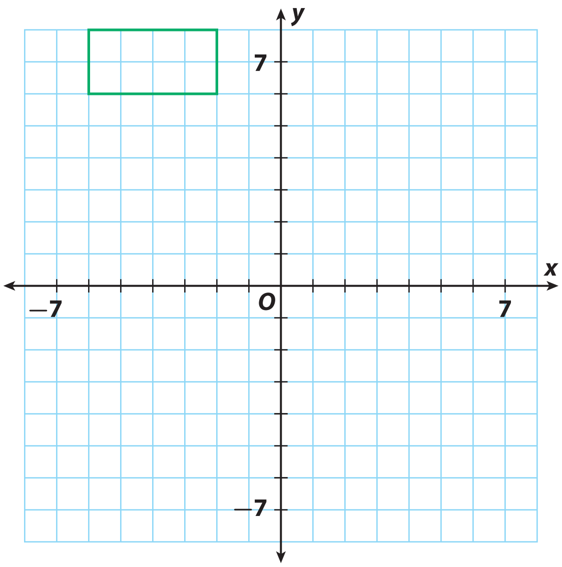
ESSENTIAL QUESTION

What is the connection between transformations and similar figures?

EXPLORE ACTIVITY

Combining Transformations with Dilations

When creating an animation, figures need to be translated, reflected, rotated, and sometimes dilated. As an example of this, apply the indicated sequence of transformations to the rectangle. Each transformation is applied to the image of the previous transformation, not to the original figure. Label each image with the letter of the transformation applied.



A $(x, y) \rightarrow (x + 7, y - 2)$

B $(x, y) \rightarrow (x, -y)$

C rotation 90° clockwise around the origin

D $(x, y) \rightarrow (x + 5, y + 3)$

E $(x, y) \rightarrow (3x, 3y)$

F List the coordinates of the vertices of rectangle *E*.

____

G Compare the following attributes of rectangle *E* to those of the original figure.

Shape	
Size	
Angle Measures	

EXPLORE ACTIVITY *(cont'd)*

Reflect

1. Which transformation represents the dilation? How can you tell?

____

____

2. A sequence of transformations containing a single dilation is applied to a figure. Are the original figure and its final image congruent? Explain.

____

____

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Similar Figures

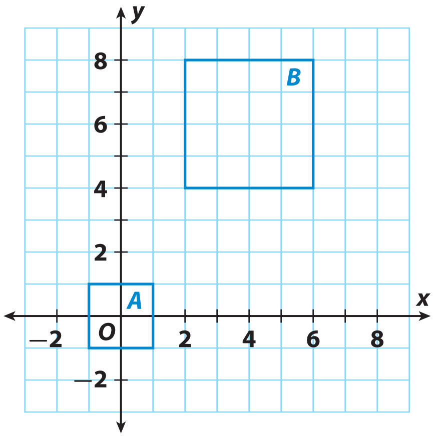
Two figures are **similar** if one can be obtained from the other by a sequence of translations, reflections, rotations, and dilations. Similar figures have the same shape but may be different sizes.

When you are told that two figures are similar, there must be a sequence of translations, reflections, rotations, and/or dilations that can transform one to the other.

EXAMPLE 1

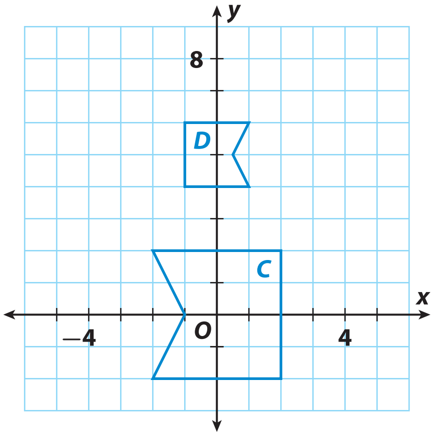
My Notes

A Identify a sequence of transformations that will transform figure *A* into figure *B*. Tell whether the figures are congruent. Tell whether they are similar.



Both figures are squares whose orientations are the same, so no reflection or rotation is needed. Figure *B* has sides twice as long as figure *A*, so a dilation with a scale factor of 2 is needed. Figure *B* is moved to the right and above figure *A*, so a translation is needed. A sequence of transformations that will accomplish this is a dilation by a scale factor of 2 centered at the origin followed by the translation $(x, y) \rightarrow (x + 4, y + 6)$. The figures are not congruent, but they are similar.

B Identify a sequence of transformations that will transform figure *C* into figure *D*. Include a reflection. Tell whether the figures are congruent. Tell whether they are similar.



The orientation of figure *D* is reversed from that of figure *C*, so a reflection over the *y*-axis is needed. Figure *D* has sides that are half as long as figure *C*, so a dilation with a scale factor of $\frac{1}{2}$ is needed. Figure *D* is moved above figure *C*, so a translation is needed. A sequence of transformations that will accomplish this is a dilation by a scale factor of $\frac{1}{2}$ centered at the origin, followed by the reflection $(x, y) \rightarrow (-x, y)$, followed by the translation $(x, y) \rightarrow (x, y + 5)$. The figures are not congruent, but they are similar.

Math Talk
Mathematical Processes
A figure and its image have different sizes and orientations. What do you know about the sequence of transformations that generated the image?

C Identify a sequence of transformations that will transform figure *C* into figure *D*. Include a rotation.

The orientation of figure *D* is reversed from that of figure *C*, so a rotation of 180° is needed. Figure *D* has sides that are half as long as figure *C*, so a dilation with a scale factor of $\frac{1}{2}$ is needed. Figure *D* is moved above figure *C*, so a translation is needed. A sequence of transformations that will accomplish this is a rotation of 180° about the origin, followed by a dilation by a scale factor of $\frac{1}{2}$ centered at the origin, followed by the translation $(x, y) \rightarrow (x, y + 5)$.

YOUR TURN

3. Look again at the Explore Activity. Start with the original figure. Create a new sequence of transformations that will yield figure *E*, the final image. Your transformations do not need to produce the images in the same order in which they originally appeared.

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Guided Practice

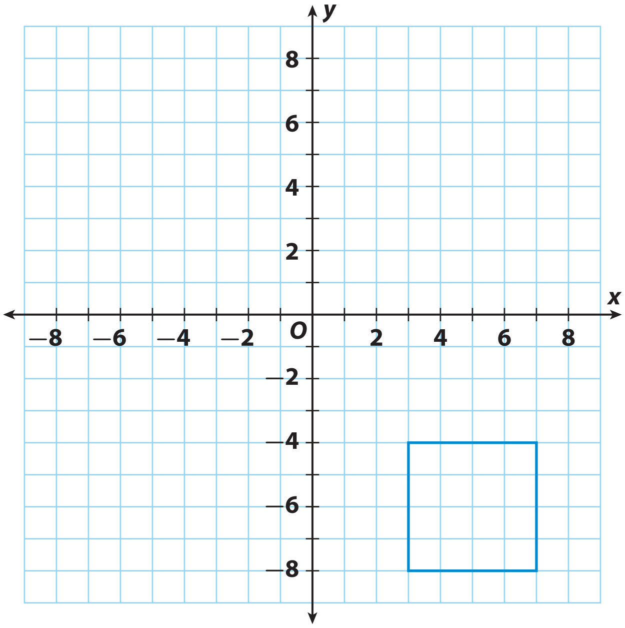
1. Apply the indicated sequence of transformations to the square. Apply each transformation to the image of the previous transformation. Label each image with the letter of the transformation applied. (Explore Activity)

A $(x, y) \rightarrow (-x, y)$

B Rotate the square 180° around the origin.

C $(x, y) \rightarrow (x - 5, y - 6)$

D $(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$



Identify a sequence of two transformations that will transform figure A into the given figure. (Example 1)

2. figure *B*

____

____

3. figure *C*

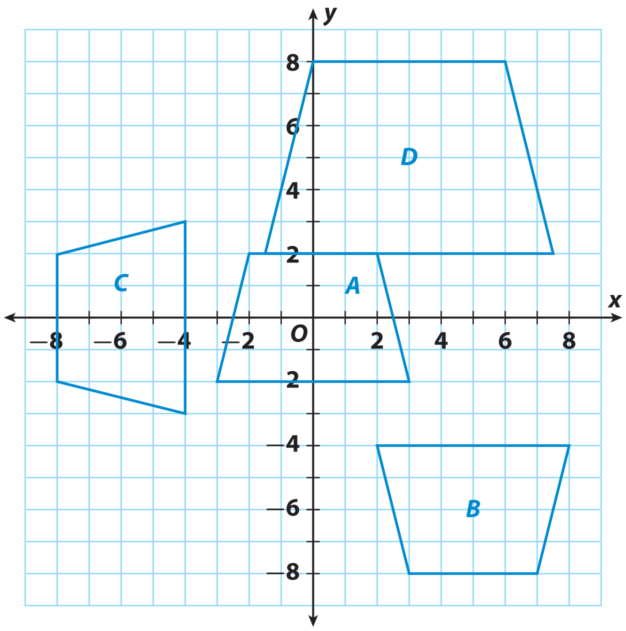
____

____

4. figure *D*

____

____



ESSENTIAL QUESTION CHECK-IN

5. If two figures are similar but not congruent, what do you know about the sequence of transformations used to create one from the other?

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Name ______________________ Class ______________ Date ____________

10.3 Independent Practice

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6. A designer creates a drawing of a triangular sign on centimeter grid paper for a new business. The drawing has sides measuring 6 cm, 8 cm, and 10 cm, and angles measuring 37°, 53°, and 90°. To create the actual sign shown, the drawing must be dilated using a scale factor of 40.

a. Find the lengths of the sides of the actual sign.

____

b. Find the angle measures of the actual sign.

____

c. The drawing has the hypotenuse on the bottom. The business owner would like it on the top. Describe two transformations that will do this.

____

____

d. The shorter leg of the drawing is currently on the left. The business owner wants it to remain on the left after the hypotenuse goes to the top. Which transformation in part c will accomplish this?

____

In Exercises 7–10, the transformation of a figure into its image is described. Describe the transformations that will transform the image back into the original figure. Then write them algebraically.

7. The figure is reflected across the *x*-axis and dilated by a scale factor of 3.

____

____

8. The figure is dilated by a scale factor of 0.5 and translated 6 units left and 3 units up.

____

____

9. The figure is dilated by a scale factor of 5 and rotated 90° clockwise.

____

____

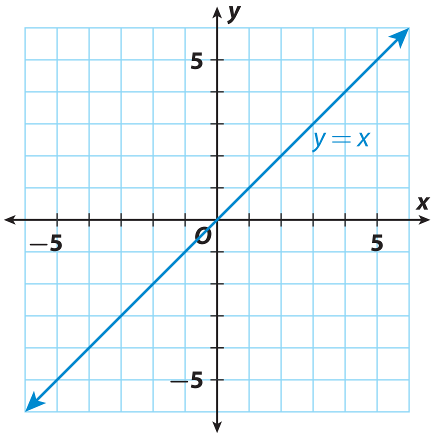
10. The figure is reflected across the y-axis and dilated by a scale factor of 4.

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

11. **Draw Conclusions** A figure undergoes a sequence of transformations that include dilations. The figure and its final image are congruent. Explain how this can happen.

12. **Multistep** As with geometric figures, graphs can be transformed through translations, reflections, rotations, and dilations. Describe how the graph of $y = x$ shown at the right is changed through each of the following transformations.



a. a dilation by a scale factor of 4

b. a translation down 3 units

c. a reflection across the y-axis

13. **Justify Reasoning** The graph of the line $y = x$ is dilated by a scale factor of 3 and then translated up 5 units. Is this the same as translating the graph up 5 units and then dilating by a scale factor of 3? Explain.

MODULE QUIZ

Ready to Go On?

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10.1 Properties of Dilations

Determine whether one figure is a dilation of the other. Justify your answer.

1. Triangle XYZ has angles measuring 54° and 29°. Triangle $X'Y'Z'$ has angles measuring 29° and 92°.

____

2. Quadrilateral $DEFG$ has sides measuring 16 m, 28 m, 24 m, and 20 m. Quadrilateral $D'E'F'G'$ has sides measuring 20 m, 35 m, 30 m, and 25 m.

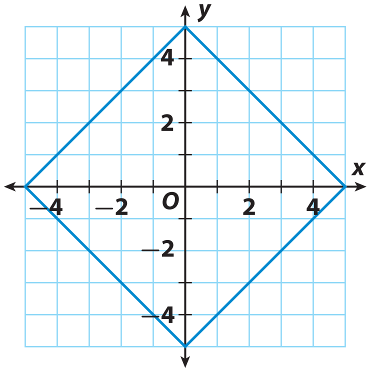
____

____

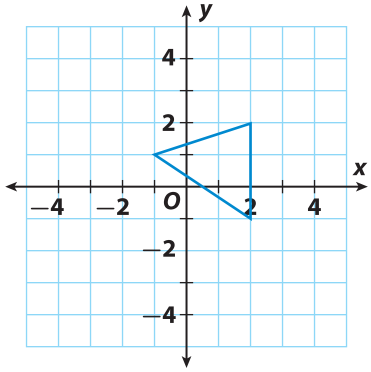
10.2 Algebraic Representations of Dilations

Dilate each figure with the origin as the center of dilation.

3. $(x, y) \rightarrow (0.8x, 0.8y)$



4. $(x, y) \rightarrow (2.5x, 2.5y)$



10.3 Similar Figures

5. Describe what happens to a figure when the given sequence of transformations is applied to it: $(x, y) \rightarrow (-x, y)$; $(x, y) \rightarrow (0.5x, 0.5y)$; $(x, y) \rightarrow (x - 2, y + 2)$

____

____

ESSENTIAL QUESTION

6. How can you use dilations to solve real-world problems?

____

MODULE 10 MIXED REVIEW

Assessment Readiness

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Selected Response

1. A rectangle has vertices (6, 4), (2, 4), (6, –2), and (2, –2). What are the coordinates of the vertices of the image after a dilation with the origin as its center and a scale factor of 1.5?

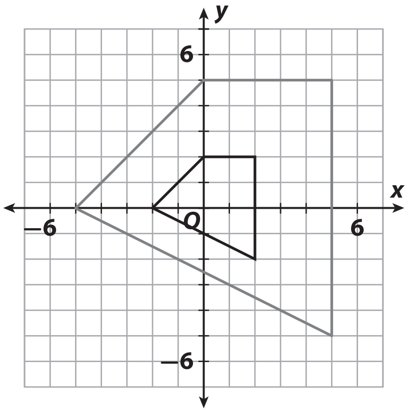
Ⓐ (9, 6), (3, 6), (9, –3), (3, –3)

Ⓑ (3, 2), (1, 2), (3, –1), (1, –1)

Ⓒ (12, 8), (4, 8), (12, –4), (4, –4)

Ⓓ (15, 10), (5, 10), (15, –5), (5, –5)

2. Which represents the dilation shown where the black figure is the preimage?



Ⓐ $(x, y) \rightarrow (1.5x, 1.5y)$

Ⓑ $(x, y) \rightarrow (2.5x, 2.5y)$

Ⓒ $(x, y) \rightarrow (3x, 3y)$

Ⓓ $(x, y) \rightarrow (6x, 6y)$

3. Identify the sequence of transformations that will reflect a figure over the *x*-axis and then dilate it by a scale factor of 3.

Ⓐ $(x, y) \rightarrow (-x, y)$; $(x, y) \rightarrow (3x, 3y)$

Ⓑ $(x, y) \rightarrow (-x, y)$; $(x, y) \rightarrow (x, 3y)$

Ⓒ $(x, y) \rightarrow (x, -y)$; $(x, y) \rightarrow (3x, y)$

Ⓓ $(x, y) \rightarrow (x, -y)$; $(x, y) \rightarrow (3x, 3y)$

4. Solve $-a + 7 = 2a - 8$.

Ⓐ $a = -3$

Ⓑ $a = -\frac{1}{3}$

Ⓒ $a = 5$

Ⓓ $a = 15$

5. Which equation does **not** represent a line with an *x*-intercept of 3?

Ⓐ $y = -2x + 6$

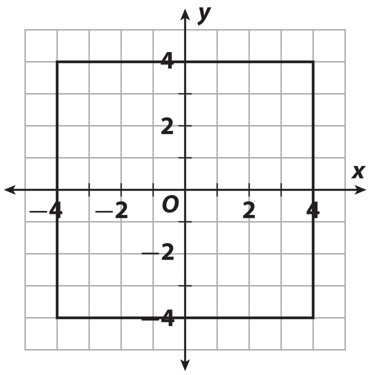
Ⓑ $y = -\frac{1}{3}x + 1$

Ⓒ $y = \frac{2}{3}x - 2$

Ⓓ $y = 3x - 1$

Mini-Task

6. The square is dilated under the dilation $(x, y) \rightarrow (0.25x, 0.25y)$.



a. Graph the image. What are the coordinates?

____

b. What is the length of a side of the image?

____

c. What are the perimeter and area of the preimage?

____

d. What are the perimeter and area of the image?

____