Transformations and Congruence

? ESSENTIAL QUESTION

How can you use transformations and congruence to solve real-world problems?

Real-World Video

When a marching band lines up and marches across the field, they are modeling a translation. As they march, they maintain size and orientation. A translation is one type of transformation.

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Animated Math
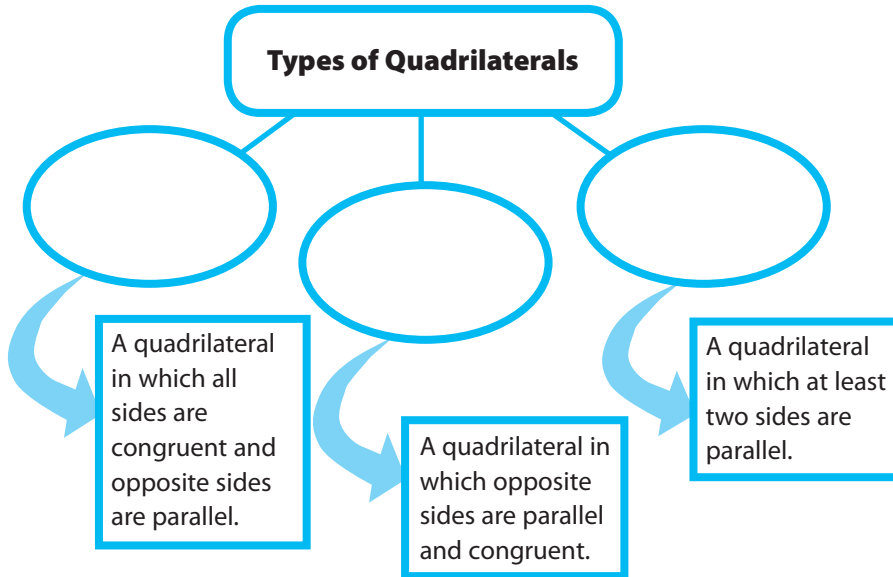Interactively explore key concepts to see how math works.

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Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the graphic organizer. You will put one word in each oval.

```
                    Types of Quadrilaterals
                   /          |            \
              (       )    (       )    (       )
                 ↓              ↓              ↓
```

A quadrilateral in which all sides are congruent and opposite sides are parallel.

A quadrilateral in which opposite sides are parallel and congruent.

A quadrilateral in which at least two sides are parallel.

Understand Vocabulary

Match the term on the left to the correct expression on the right.

1. transformation   **A.** A function that describes a change in the position, size, or shape of a figure.

2. reflection   **B.** A function that slides a figure along a straight line.

3. translation   **C.** A transformation that flips a figure across a line.

Active Reading

Booklet Before beginning the module, create a booklet to help you learn the concepts in this module. Write the main idea of each lesson on each page of the booklet. As you study each lesson, write important details that support the main idea, such as vocabulary and formulas. Refer to your finished booklet as you work on assignments and study for tests.

Are YOU Ready?

Complete these exercises to review skills you will need for this module.

Integer Operations

EXAMPLE

$-3 - (-6) = -3 + 6$

$= |-3| - |6|$

$= 3$

To subtract an integer, add its opposite. The signs are different, so find the difference of the absolute values: $6 - 3 = 3$. Use the sign of the number with the greater absolute value.

Find each difference.

1. $5 - (-9)$

_____

2. $-6 - 8$

_____

3. $2 - 9$

_____

4. $-10 - (-6)$

_____

5. $3 - (-11)$

_____

6. $12 - 7$

_____

7. $-4 - 11$

_____

8. $0 - (-12)$

_____

Measure Angles

EXAMPLE
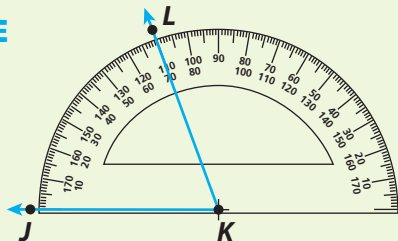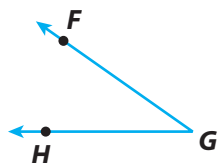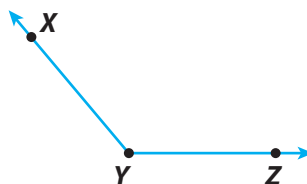

$m\angle JKL = 70°$

Place the center point of the protractor on the angle's vertex.

Align one ray with the base of the protractor.

Read the angle measure where the other ray intersects the semicircle.

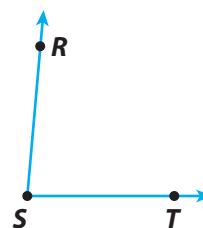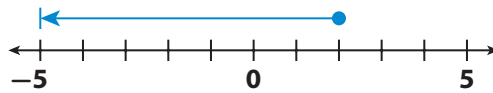**Use a protractor to measure each angle.**

9.

_____

10.

_____

11.

_____

Complete these exercises to review skills you will need for this module.
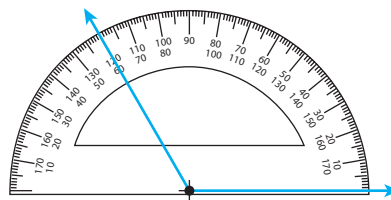
Integer Operations

12. Explain how to find the difference $6 - 8$.

13. The number line graph represents the difference of two integers. Interpret the graph to find the difference expression, and then give the solution.
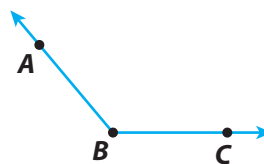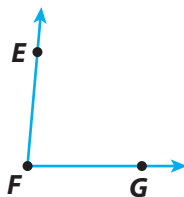

$$-5 \qquad 0 \qquad 5$$

Measure Angles

14. Explain how to measure an angle using a protractor. What is the measure of the angle shown?

15. Marco used a protractor to measure the angles shown. He concluded that the angle on the left measures 95° and the angle on the right measures 50°. What was Marco's error? Give the correct angle measures.

ESSENTIAL QUESTION

How do you describe the properties of translation and their effect on the congruence and orientation of figures?
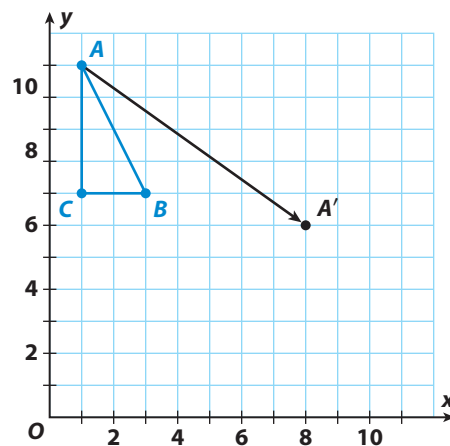
EXPLORE ACTIVITY 1

Exploring Translations

You learned that a function is a rule that assigns exactly one output to each input. A **transformation** is a function that describes a change in the position, size, or shape of a figure. The input of a transformation is the **preimage**, and the output of a transformation is the **image**.

A **translation** is a transformation that slides a figure along a straight line.

The triangle shown on the grid is the preimage (input). The arrow shows the motion of a translation and how point *A* is translated to point *A′*.

A Trace triangle *ABC* onto a piece of paper. Cut out your traced triangle.

B Slide your triangle along the arrow to model the translation that maps point *A* to point *A′*.

C The image of the translation is the triangle produced by the translation. Sketch the image of the translation.

D The vertices of the image are labeled using prime notation. For example, the image of *A* is *A′*. Label the images of points *B* and *C*.

E Describe the motion modeled by the translation.

Move _____ units right and _____ units down.

F Check that the motion you described in part **E** is the same motion that maps point *A* onto *A′*, point *B* onto *B′*, and point *C* onto *C′*.
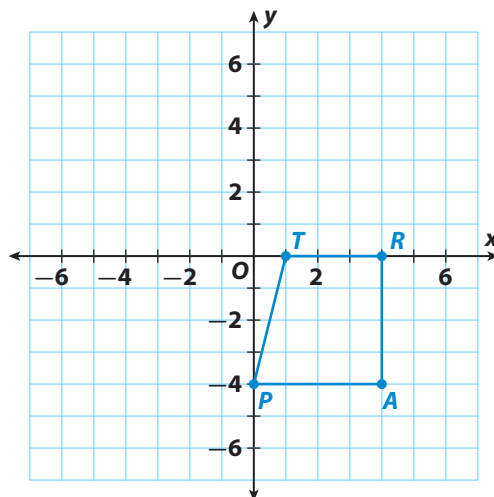
Reflect

1. How is the orientation of the triangle affected by the translation?

_____

Properties of Translations

Use trapezoid *TRAP* to investigate the properties of translations.

A Trace the trapezoid onto a piece of paper. Cut out your traced trapezoid.

B Place your trapezoid on top of the trapezoid in the figure. Then translate your trapezoid 5 units to the left and 3 units up. Sketch the image of the translation by tracing your trapezoid in this new location. Label the vertices of the image *T′*, *R′*, *A′*, and *P′*.

C Use a ruler to measure the sides of trapezoid *TRAP* in centimeters.

$TR =$ _____ $RA =$ _____ $AP =$ _____ $TP =$ _____

D Use a ruler to measure the sides of trapezoid *T′R′A′P′* in centimeters.

$T′R′ =$ _____ $R′A′ =$ _____ $A′P′ =$ _____ $T′P′ =$ _____

E What do you notice about the lengths of corresponding sides of the two figures?

_____

F Use a protractor to measure the angles of trapezoid *TRAP*.

$m\angle T =$ _____ $m\angle R =$ _____ $m\angle A =$ _____ $m\angle P =$ _____

G Use a protractor to measure the angles of trapezoid *T′R′A′P′*.

$m\angle T′ =$ _____ $m\angle R′ =$ _____ $m\angle A′ =$ _____ $m\angle P′ =$ _____

H What do you notice about the measures of corresponding angles of the two figures?

_____

I Which sides of trapezoid *TRAP* are parallel? How do you know?

_____

Which sides of trapezoid *T′R′A′P′* are parallel? _____

What do you notice? _____

_____

Reflect

2. **Make a Conjecture** Use your results from parts **E**, **H**, and **I** to make a conjecture about translations.

_____

_____

3. Two figures that have the same size and shape are called *congruent*. What can you say about translations and congruence?

_____

Graphing Translations

To translate a figure in the coordinate plane, translate each of its vertices. Then connect the vertices to form the image.

EXAMPLE 1

The figure shows triangle *XYZ*. Graph the image of the triangle after a translation of 4 units to the right and 1 unit up.

STEP 1 Translate point *X*.

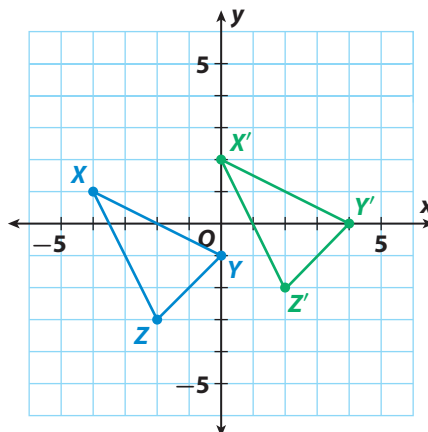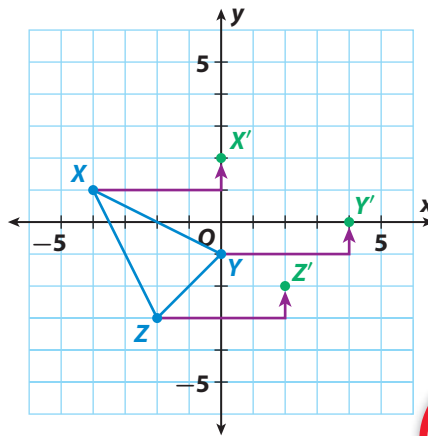Count right 4 units and up 1 unit and plot point *X'*.

STEP 2 Translate point *Y*.

Count right 4 units and up 1 unit and plot point *Y'*.

STEP 3 Translate point *Z*.

Count right 4 units and up 1 unit and plot point *Z'*.

STEP 4 Connect *X'*, *Y'*, and *Z'* to form triangle *X'Y'Z'*.

Each vertex is moved 4 units right and 1 unit up.

Math Talk
Mathematical Processes
Is the image congruent to the preimage? How do you know?

Lesson 9.1   **281**

YOUR TURN

4. The figure shows parallelogram *ABCD*. Graph the image of the parallelogram after a translation of 5 units to the left and 2 units down.
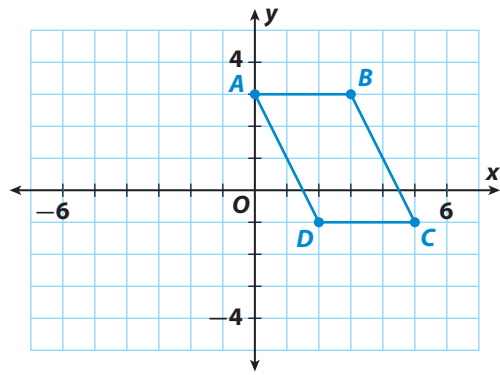
Guided Practice

1. Vocabulary A _____ is a change in the position, size, or shape of a figure.

2. Vocabulary When you perform a transformation of a figure on the coordinate plane, the input of the transformation is called

the _____, and the output of the transformation is

called the _____.

3. Joni translates a right triangle 2 units down and 4 units to the right. How does the orientation of the image of the triangle compare with the orientation of the preimage? **(Explore Activity 1)**

_____

4. Rashid drew rectangle *PQRS* on a coordinate plane. He then translated the rectangle 3 units up and 3 units to the left and labeled the image *P'Q'R'S'*. How do rectangle *PQRS* and rectangle *P'Q'R'S'* compare? **(Explore Activity 2)**
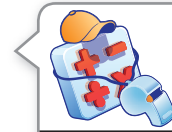
_____

5. The figure shows trapezoid *WXYZ*. Graph the image of the trapezoid after a translation of 4 units up and 2 units to the left. **(Example 1)**
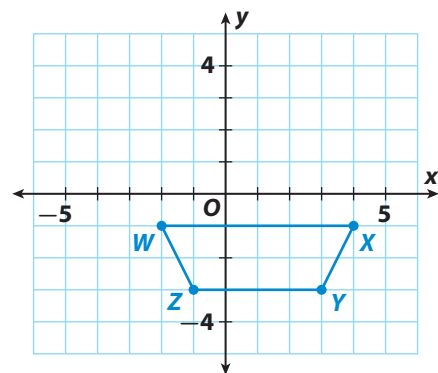
? **ESSENTIAL QUESTION CHECK-IN**

6. What are the properties of translations?
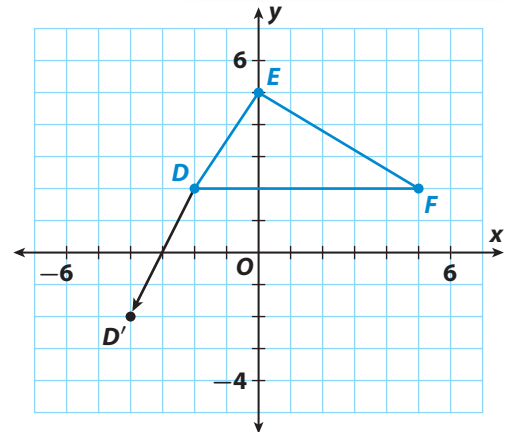
_____

_____

9.1 Independent Practice

7. The figure shows triangle *DEF*.

a. Graph the image of the triangle after the translation that maps point *D* to point *D′*.

b. How would you describe the translation?

_____

_____

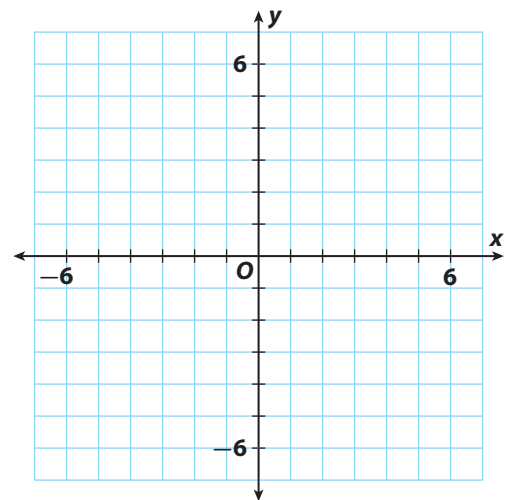**c.** How does the image of triangle *DEF* compare with the preimage?

_____

8. a. Graph quadrilateral *KLMN* with vertices *K*(−3, 2), *L*(2, 2), *M*(0, −3), and *N*(−4, 0) on the coordinate grid.

b. On the same coordinate grid, graph the image of quadrilateral *KLMN* after a translation of 3 units to the right and 4 units up.

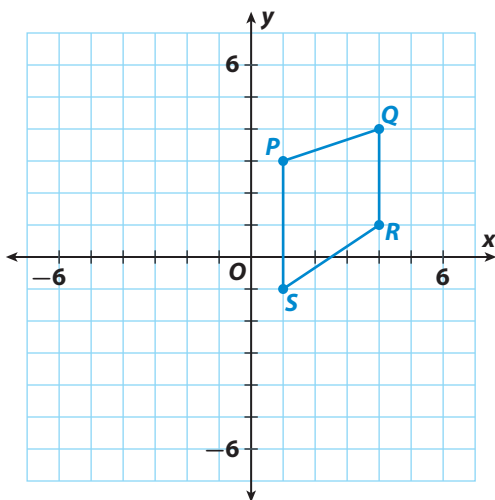**c.** Which side of the image is congruent to side $\overline{LM}$?

_____
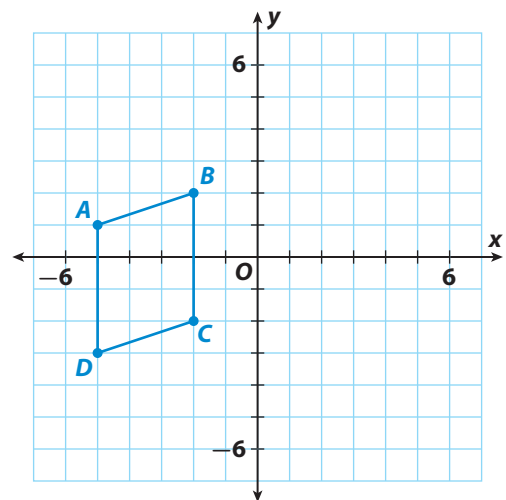
Name three other pairs of congruent sides.

_____

Draw the image of the figure after each translation.

9. 4 units left and 2 units down

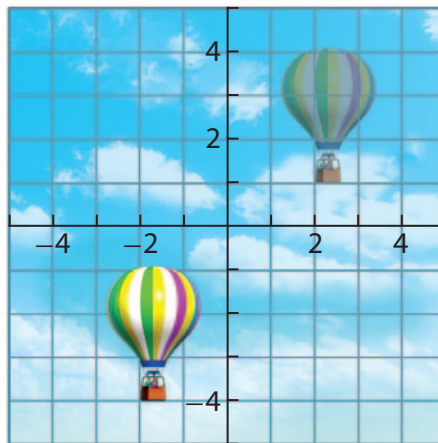**10.** 5 units right and 3 units up

11. The figure shows the ascent of a hot air balloon. How would you describe the translation?

_____

_____

**12.** **Critical Thinking** Is it possible that the orientation of a figure could change after it is translated? Explain.

_____

_____

_____

FOCUS ON HIGHER ORDER THINKING

13. a. **Multistep** Graph triangle *XYZ* with vertices *X*(−2, −5), *Y*(2, −2), and *Z*(4, −4) on the coordinate grid.

b. On the same coordinate grid, graph and label triangle *X′Y′Z′*, the image of triangle *XYZ* after a translation of 3 units to the left and 6 units up.

c. Now graph and label triangle *X″Y″Z″*, the image of triangle *X′Y′Z′* after a translation of 1 unit to the left and 2 units down.

d. **Analyze Relationships** How would you describe the translation that maps triangle *XYZ* onto triangle *X″Y″Z″*?

_____

_____
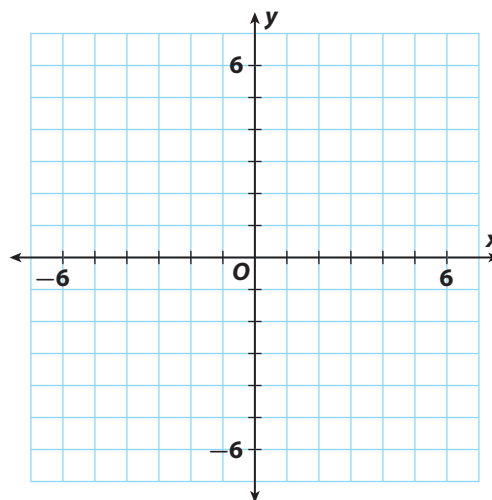
14. **Critical Thinking** The figure shows rectangle *P′Q′R′S′*, the image of rectangle *PQRS* after a translation of 5 units to the right and 7 units up. Graph and label the preimage *PQRS*.

15. **Communicate Mathematical Ideas** Explain why the image of a figure after a translation is congruent to its preimage.
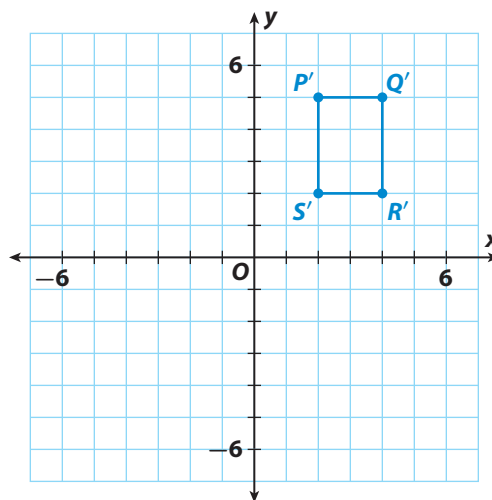
_____

_____

_____

_____

LESSON
9.2

8.4.9.2

Students will describe the properties of reflection and their effect on the congruence and orientation of figures.

# Properties of Reflections

? **ESSENTIAL QUESTION**

How do you describe the properties of reflection and their effect on the congruence and orientation of figures?

EXPLORE ACTIVITY 1

Exploring Reflections

A **reflection** is a transformation that flips a figure across a line. The line is called the **line of reflection**. Each point and its image are the same distance from the line of reflection.

The triangle shown on the grid is the preimage. You will explore reflections across the *x*- and *y*-axes.
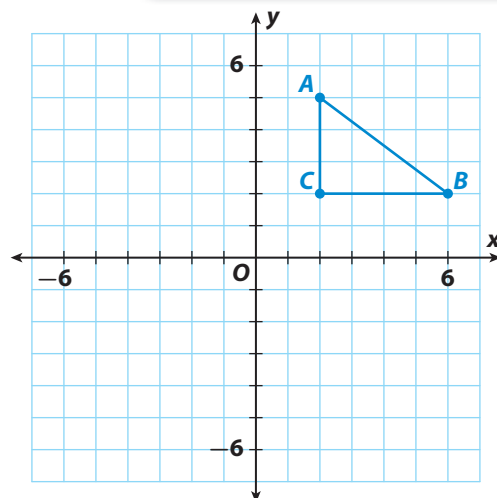
A Trace triangle *ABC* and the *x*- and *y*-axes onto a piece of paper.

B Fold your paper along the *x*-axis and trace the image of the triangle on the opposite side of the *x*-axis. Unfold your paper and label the vertices of the image *A′*, *B′*, and *C′*.

C What is the line of reflection for this transformation?

_____

D Find the perpendicular distance from each point to the line of reflection.

Point *A* _____   Point *B* _____   Point *C* _____

E Find the perpendicular distance from each point to the line of reflection.

Point *A′* _____   Point *B′* _____   Point *C′* _____

**F** What do you notice about the distances you found in **D** and **E** ?

_____

Reflect

1. Fold your paper from **A** along the *y*-axis and trace the image of triangle *ABC* on the opposite side. Label the vertices of the image *A″*, *B″*, and *C″*. What is the line of reflection for this transformation? _____

2. How does each image in your drawings compare with its preimage?

_____

Properties of Reflections

Use trapezoid *TRAP* to investigate the properties of reflections.
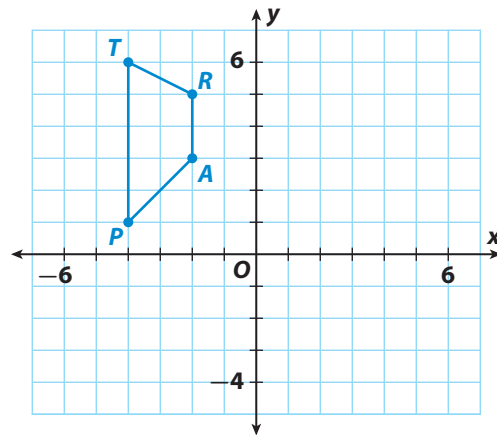
A Trace the trapezoid onto a piece of paper. Cut out your traced trapezoid.

B Place your trapezoid on top of the trapezoid in the figure. Then reflect your trapezoid across the *y*-axis. Sketch the image of the reflection by tracing your trapezoid in this new location. Label the vertices of the image *T′*, *R′*, *A′*, and *P′*.

C Use a ruler to measure the sides of trapezoid *TRAP* in centimeters.

$TR = $ _____ $RA = $ _____ $AP = $ _____ $TP = $ _____

D Use a ruler to measure the sides of trapezoid *T′R′A′P′* in centimeters.

$T′R′ = $ _____ $R′A′ = $ _____ $A′P′ = $ _____ $T′P′ = $ _____

E What do you notice about the lengths of corresponding sides of the two figures?

_____

F Use a protractor to measure the angles of trapezoid *TRAP*.

$m\angle T = $ _____ $m\angle R = $ _____ $m\angle A = $ _____ $m\angle P = $ _____

G Use a protractor to measure the angles of trapezoid *T′R′A′P′*.

$m\angle T′ = $ _____ $m\angle R′ = $ _____ $m\angle A′ = $ _____ $m\angle P′ = $ _____

H What do you notice about the measures of corresponding angles of the two figures?

_____

I Which sides of trapezoid *TRAP* are parallel? _____

Which sides of trapezoid *T′R′A′P′* are parallel? _____
What do you notice?

_____

_____

Reflect

3. **Make a Conjecture** Use your results from **E**, **H**, and **I** to make a conjecture about reflections.

_____

_____

_____

Graphing Reflections

To reflect a figure across a line of reflection, reflect each of its vertices. Then connect the vertices to form the image. Remember that each point and its image are the same distance from the line of reflection.

Math On the Spot
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EXAMPLE 1

The figure shows triangle *XYZ*. Graph the image of the triangle after a reflection across the *x*-axis.

My Notes

STEP 1   Reflect point *X*.

Point *X* is 3 units below the *x*-axis. Count 3 units above the *x*-axis and plot point *X′*.

STEP 2   Reflect point *Y*.

Point *Y* is 1 unit below the *x*-axis. Count 1 unit above the *x*-axis and plot point *Y′*.
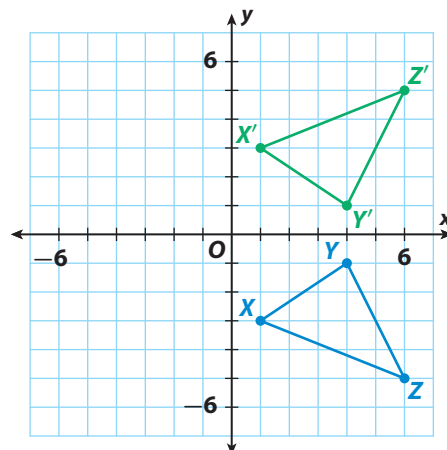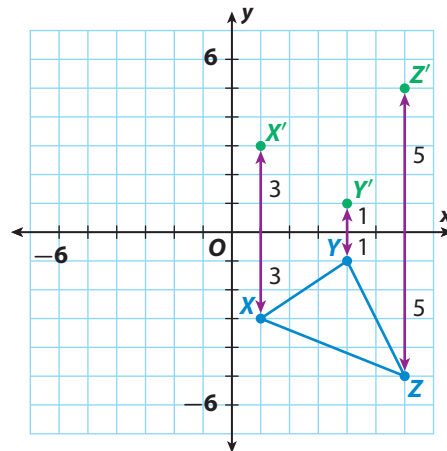
STEP 3   Reflect point *Z*.

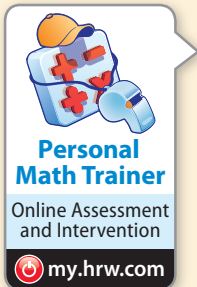Point *Z* is 5 units below the *x*-axis. Count 5 units above the *x*-axis and plot point *Z′*.

STEP 4   Connect *X′*, *Y′*, and *Z′* to form triangle *X′Y′Z′*.

Each vertex of the image is the same distance from the x-axis as the corresponding vertex in the original figure.
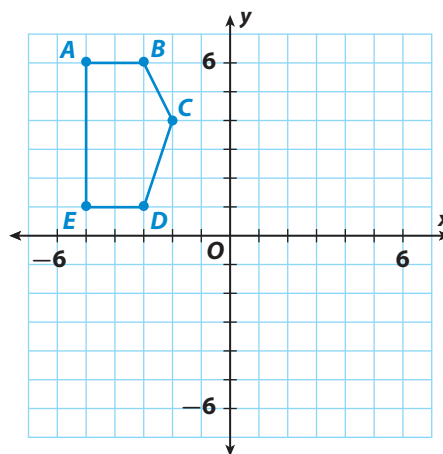
© Houghton Mifflin Harcourt Publishing Company

YOUR TURN

4. The figure shows pentagon *ABCDE*.
Graph the image of the pentagon
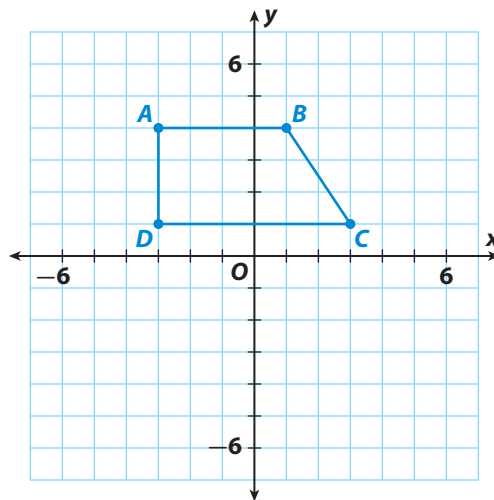after a reflection across the *y*-axis.

Guided Practice

1. Vocabulary A reflection is a transformation that flips a figure across

a line called the _____.

2. The figure shows trapezoid *ABCD*. **(Explore Activities 1 and 2 and Example 1)**

a. Graph the image of the trapezoid after a reflection
across the *x*-axis. Label the vertices of the image.

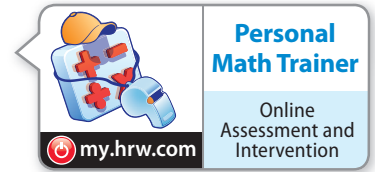**b.** How do trapezoid *ABCD* and trapezoid *A′B′C′D′*
compare?

_____

c. What If? Suppose you reflected trapezoid *ABCD*
across the *y*-axis. How would the orientation of
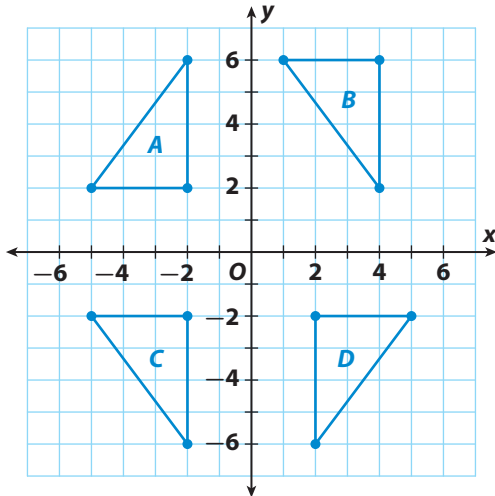the image of the trapezoid compare with the
orientation of the preimage?

_____

_____

_____

_____

? ESSENTIAL QUESTION CHECK-IN

3. What are the properties of reflections?

_____

_____

9.2 Independent Practice

The graph shows four right triangles. Use the graph for Exercises 4–7.
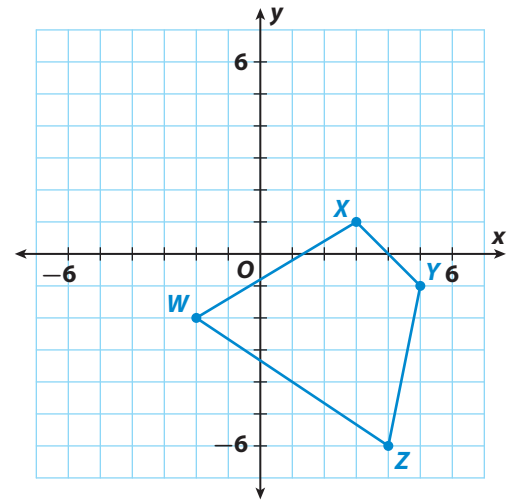

4. Which two triangles are reflections of each other across the *x*-axis?

_____

5. For which two triangles is the line of reflection the *y*-axis?

_____

6. Which triangle is a translation of triangle *C*? How would you describe the translation?

_____

_____

7. Which triangles are congruent? How do you know?

_____

_____

_____

_____

8. a. Graph quadrilateral *WXYZ* with vertices *W*(−2, −2), *X*(3, 1), *Y*(5, −1), and *Z*(4, −6) on the coordinate grid.

b. On the same coordinate grid, graph quadrilateral *W′X′Y′Z′*, the image of quadrilateral *WXYZ* after a reflection across the *x*-axis.

c. Which side of the image is congruent to side $\overline{YZ}$?

_____

Name three other pairs of congruent sides.

_____

_____

**d.** Which angle of the image is congruent to ∠*X*?

_____

Name three other pairs of congruent angles.

_____

_____

9. Critical Thinking Is it possible that the image of a point after a reflection could be the same point as the preimage? Explain.

_____

_____

10. a. Graph the image of the figure shown after a reflection across the *y*-axis.

b. On the same coordinate grid, graph the image of the figure you drew in part **a** after a reflection across the *x*-axis.

c. Make a Conjecture What other sequence of transformations would produce the same final image from the original preimage? Check your answer by performing the transformations. Then make a conjecture that generalizes your findings.

_____

_____

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_____
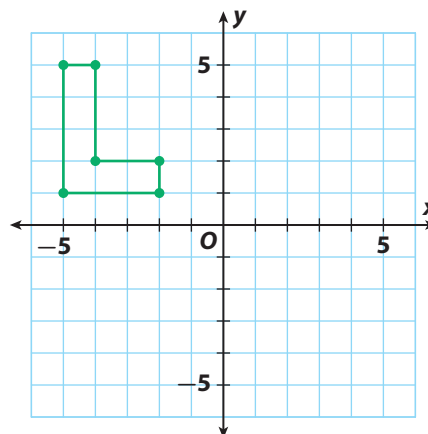
11. a. Graph triangle *DEF* with vertices *D*(2, 6), *E*(5, 6), and *F*(5, 1) on the coordinate grid.

b. Next graph triangle *D'E'F'*, the image of triangle *DEF* after a reflection across the *y*-axis.

c. On the same coordinate grid, graph triangle *D''E''F''*, the image of triangle *D'E'F'* after a translation of 7 units down and 2 units to the right.

d. Analyze Relationships Find a different sequence of transformations that will transform triangle *DEF* to triangle *D''E''F''*.

_____

_____

_____

_____
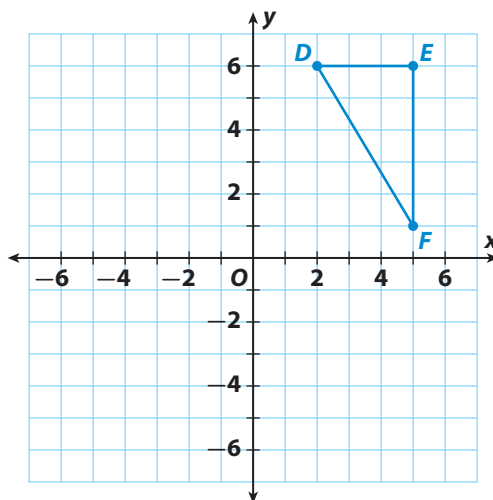
LESSON
9.3
Properties of Rotations

8.4.9.3
Students will describe the properties of rotation and their effect on the congruence and orientation of figures.

ESSENTIAL QUESTION

How do you describe the properties of rotation and their effect on the congruence and orientation of figures?

EXPLORE ACTIVITY 1

Exploring Rotations

A **rotation** is a transformation that turns a figure around a given point called the **center of rotation**. The image has the same size and shape as the preimage.

The triangle shown on the grid is the preimage. You will use the origin as the center of rotation.

A Trace triangle *ABC* onto a piece of paper. Cut out your traced triangle.

B Rotate your triangle 90° counterclockwise about the origin. The side of the triangle that lies along the *x*-axis should now lie along the *y*-axis.

C Sketch the image of the rotation. Label the images of points *A*, *B*, and *C* as *A′*, *B′*, and *C′*.

D Describe the motion modeled by the rotation.

Rotate _____ degrees _____ about the origin.

E Check that the motion you described in **D** is the same motion that maps point *A* onto *A′*, point *B* onto *B′*, and point *C* onto *C′*.
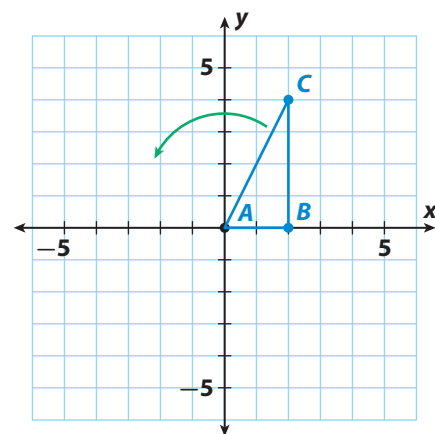
Reflect

1. **Communicate Mathematical Ideas** How are the size and the orientation of the triangle affected by the rotation?
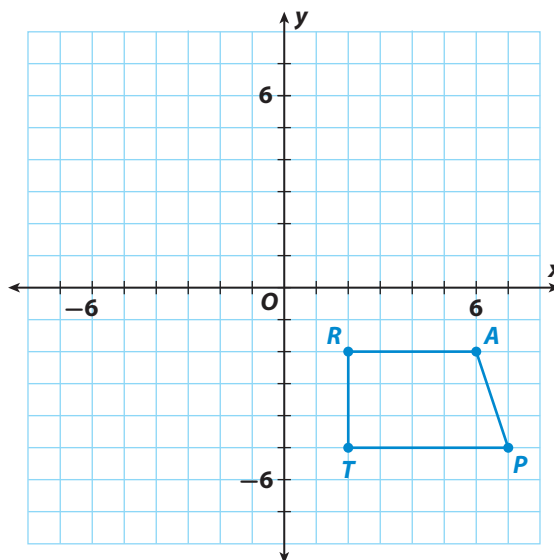
_____

_____

_____

2. Rotate triangle *ABC* 90° clockwise about the origin. Sketch the result on the coordinate grid above. Label the image vertices *A″*, *B″*, and *C″*.

Properties of Rotations

Use trapezoid *TRAP* to investigate the properties of rotations.

A Trace the trapezoid onto a piece of paper. Include the portion of the *x*- and *y*-axes bordering the third quadrant. Cut out your tracing.

B Place your trapezoid and axes on top of those in the figure. Then use the axes to help rotate your trapezoid 180° counterclockwise about the origin. Sketch the image of the rotation of your trapezoid in this new location. Label the vertices of the image *T′*, *R′*, *A′*, and *P′*.

C Use a ruler to measure the sides of trapezoid *TRAP* in centimeters.

$TR =$ _____ $RA =$ _____

$AP =$ _____ $TP =$ _____

D Use a ruler to measure the sides of trapezoid *T′R′A′P′* in centimeters.

$T′R′ =$ _____ $R′A′ =$ _____

$A′P′ =$ _____ $T′P′ =$ _____

E What do you notice about the lengths of corresponding sides of the two figures?

_____

F Use a protractor to measure the angles of trapezoid *TRAP*.

$m\angle T =$ _____ $m\angle R =$ _____ $m\angle A =$ _____ $m\angle P =$ _____

G Use a protractor to measure the angles of trapezoid *T′R′A′P′*.

$m\angle T′ =$ _____ $m\angle R′ =$ _____ $m\angle A′ =$ _____ $m\angle P′ =$ _____

H What do you notice about the measures of corresponding angles of the two figures?

_____

I Which sides of trapezoid *TRAP* are parallel? _____

Which sides of trapezoid *T′R′A′P′* are parallel? _____

What do you notice? _____

_____

Reflect

3. **Make a Conjecture** Use your results from **E**, **H**, and **I** to make a conjecture about rotations.

_____

_____

4. Place your tracing back in its original position. Then perform a 180° *clockwise* rotation about the origin. Compare the result with the result of the transformation in **B**.

_____

_____

Graphing Rotations

To rotate a figure in the coordinate plane, rotate each of its vertices. Then connect the vertices to form the image.

EXAMPLE 1

The figure shows triangle *ABC*. Graph the image of triangle *ABC* after a rotation of 90° clockwise.

STEP 1 Rotate the figure clockwise from the *y*-axis to the *x*-axis. Point *A* will still be at (0, 0).

Point *B* is 2 units to the left of the *y*-axis, so point *B*′ is 2 units above the *x*-axis.

Point *C* is 2 units to the right of the *y*-axis, so point *C*′ is 2 units below the *x*-axis.

STEP 2 Connect *A*′, *B*′, and *C*′ to form the image triangle *A*′*B*′*C*′.

Reflect

5. Is the image congruent to the preimage? How do you know?

_____

_____

_____

_____
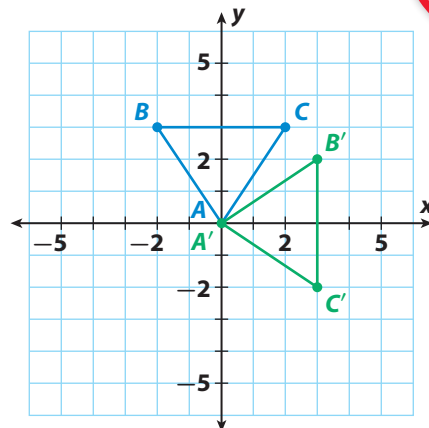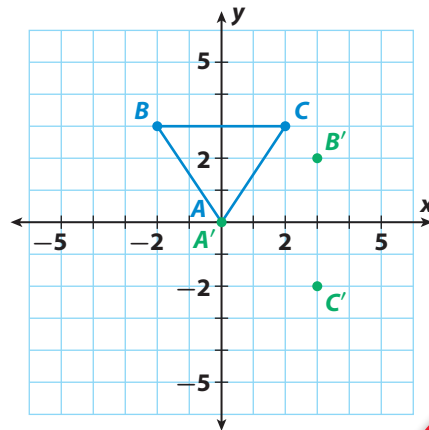
Math Talk
Mathematical Processes
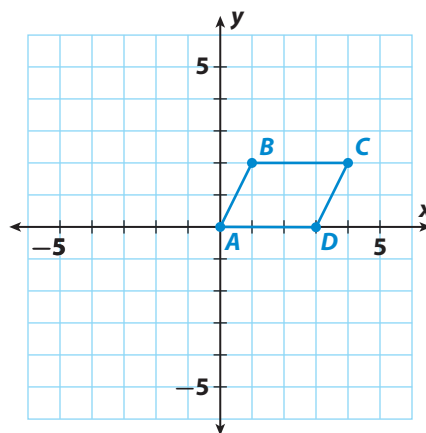How is the orientation of the triangle affected by the rotation?

YOUR TURN

Graph the image of quadrilateral *ABCD* after each rotation.

6. 180°

7. 270° clockwise

8. Find the coordinates of Point *C* after a 90° counterclockwise rotation followed by a 180° rotation.

_____

Guided Practice

1. **Vocabulary** A rotation is a transformation that turns a figure around a

   given _____ called the center of rotation.

Siobhan rotates a right triangle 90° counterclockwise about the origin.

2. How does the orientation of the image of the triangle compare with the orientation of the preimage? **(Explore Activity 1)**
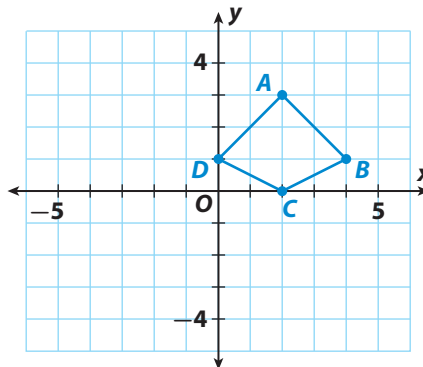
   _____

3. Is the image of the triangle congruent to the preimage? **(Explore Activity 2)**

   _____

Draw the image of the figure after the given rotation about the origin. (Example 1)

4. 90° counterclockwise
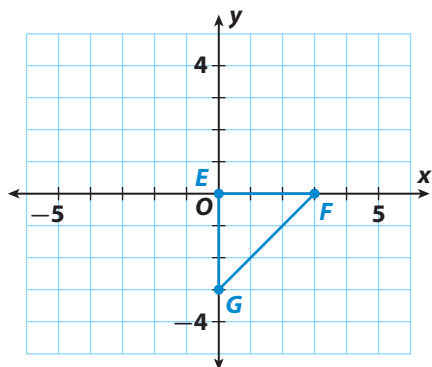
5. 180°

❓ ESSENTIAL QUESTION CHECK-IN

6. What are the properties of rotations?

   _____

9.3 Independent Practice

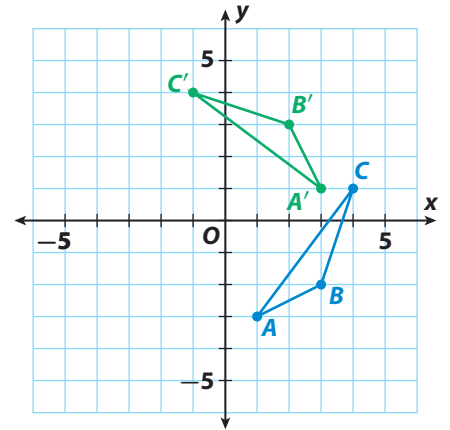**7.** The figure shows triangle *ABC* and a rotation of the triangle about the origin.

a. How would you describe the rotation?
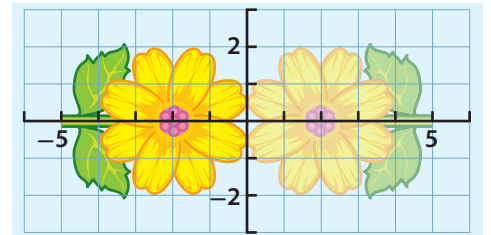
_____

_____

b. What are the coordinates of the image?

_____ , _____ , _____

**8.** The graph shows a figure and its image after a transformation.

a. How would you describe this as a rotation?

_____

_____

b. Can you describe this as a transformation other than a rotation? Explain.

_____

_____

9. What type of rotation will preserve the orientation of the H-shaped figure in the grid?

_____

10. A point with coordinates $(-2, -3)$ is rotated 90° clockwise about the origin. What are the coordinates of its image?
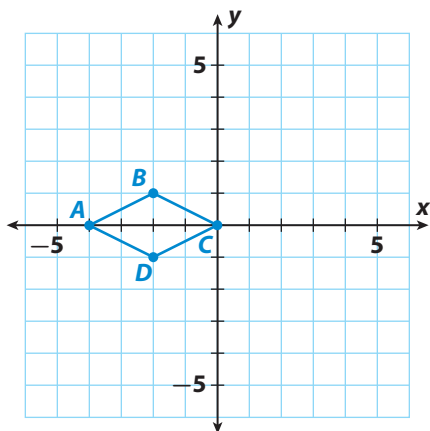
_____
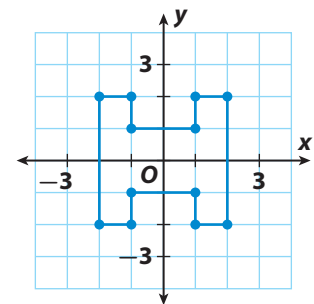
Complete the table with rotations of 180° or 90°. Include the direction of rotation for rotations of 90°.
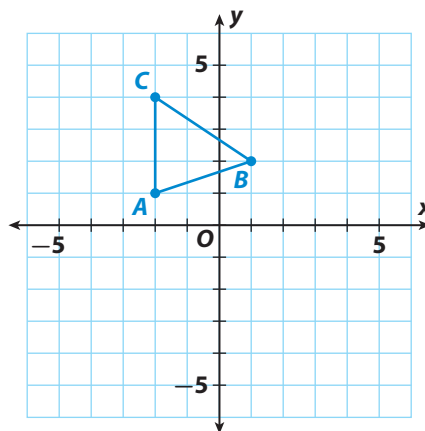
	Shape in quadrant	Image in quadrant	Rotation
11.	I	IV	
12.	III	I	
13.	IV	III	

Draw the image of the figure after the given rotation about the origin.

14. 180°

15. 270° counterclockwise

16. Is there a rotation for which the orientation of the image is always the same as that of the preimage? If so, what?
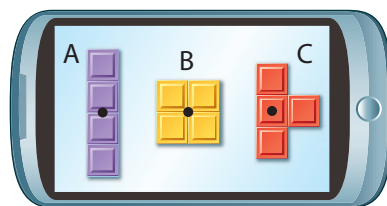
_____

H.O.T. **FOCUS ON HIGHER ORDER THINKING**

Work Area

17. **Problem Solving** Lucas is playing a game where he has to rotate a figure for it to fit in an open space. Every time he clicks a button, the figure rotates 90 degrees clockwise. How many times does he need to click the button so that each figure returns to its original orientation?

Figure A _____

Figure B _____

Figure C _____

18. **Make a Conjecture** Triangle *ABC* is reflected across the *y*-axis to form the image *A′B′C′*. Triangle *A′B′C′* is then reflected across the *x*-axis to form the image *A″B″C″*. What type of rotation can be used to describe the relationship between triangle *A″B″C″* and triangle *ABC*?

_____

19. **Communicate Mathematical Ideas** Point *A* is on the *y*-axis. Describe all possible locations of image *A′* for rotations of 90°, 180°, and 270°. Include the origin as a possible location for *A*.

_____

_____

_____

_____

Algebraic Representations of Transformations

? ESSENTIAL QUESTION

How can you describe the effect of a translation, rotation, or reflection on coordinates using an algebraic representation?

Math On the Spot
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Algebraic Representations of Translations

The rules shown in the table describe how coordinates change when a figure is translated up, down, right, and left on the coordinate plane.

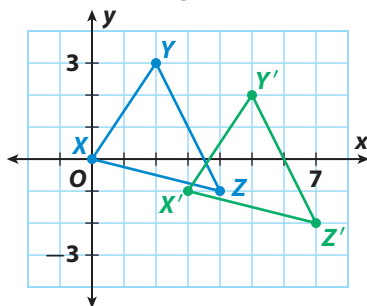| Translations	
Right a units	Add a to the x-coordinate: $(x, y) \rightarrow (x + a, y)$
Left a units	Subtract a from the x-coordinate: $(x, y) \rightarrow (x - a, y)$
Up b units	Add b to the y-coordinate: $(x, y) \rightarrow (x, y + b)$
Down b units	Subtract b from the y-coordinate: $(x, y) \rightarrow (x, y - b)$

EXAMPLE 1

Triangle XYZ has vertices $X(0, 0)$, $Y(2, 3)$, and $Z(4, -1)$. Find the vertices of triangle $X'Y'Z'$ after a translation of 3 units to the right and 1 unit down. Then graph the triangle and its image.

Add 3 to the x-coordinate of each vertex and subtract 1 from the y-coordinate of each vertex.

STEP 1 Apply the rule to find the vertices of the image.

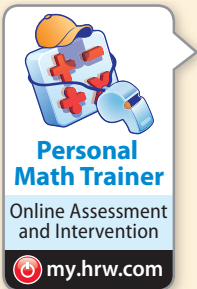| Vertices of △XYZ	Rule: $(x + 3, y - 1)$	Vertices of △$X'Y'Z'$
$X(0, 0)$	$(0 + 3, 0 - 1)$	$X'(3, -1)$
$Y(2, 3)$	$(2 + 3, 3 - 1)$	$Y'(5, 2)$
$Z(4, -1)$	$(4 + 3, -1 - 1)$	$Z'(7, -2)$

STEP 2 Graph triangle XYZ and its image.

Math Talk
Mathematical Processes

When you translate a figure to the left or right, which coordinate do you change?

1. A rectangle has vertices at $(0, -2)$, $(0, 3)$, $(3, -2)$, and $(3, 3)$. What are the coordinates of the vertices of the image after the translation $(x, y) \rightarrow (x - 6, y - 3)$? Describe the translation.

_____

_____

Algebraic Representations of Reflections

The signs of the coordinates of a figure change when the figure is reflected across the *x*-axis and *y*-axis. The table shows the rules for changing the signs of the coordinates after a reflection.

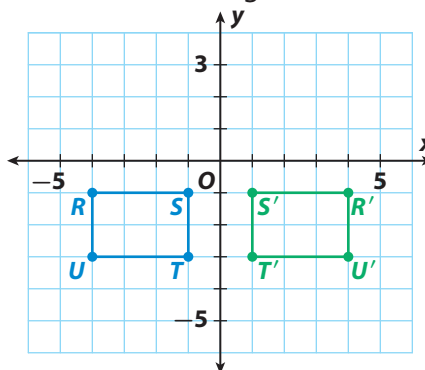| Reflections	
Across the *x*-axis	Multiply each *y*-coordinate by -1: $(x, y) \rightarrow (x, -y)$
Across the *y*-axis	Multiply each *x*-coordinate by -1: $(x, y) \rightarrow (-x, y)$

EXAMPLE 2

My Notes

Rectangle *RSTU* has vertices $R(-4, -1)$, $S(-1, -1)$, $T(-1, -3)$, and $U(-4, -3)$. Find the vertices of rectangle $R'S'T'U'$ after a reflection across the *y*-axis. Then graph the rectangle and its image.

STEP 1 Apply the rule to find the vertices of the image.

> Multiply the *x*-coordinate of each vertex by -1.

Vertices of *RSTU*	Rule: $(-1 \cdot x, y)$	Vertices of *R'S'T'U'*
$R(-4, -1)$	$(-1 \cdot (-4), -1)$	$R'(4, -1)$
$S(-1, -1)$	$(-1 \cdot (-1), -1)$	$S'(1, -1)$
$T(-1, -3)$	$(-1 \cdot (-1), -3)$	$T'(1, -3)$
$U(-4, -3)$	$(-1 \cdot (-4), -3)$	$U'(4, -3)$

STEP 2 Graph rectangle *RSTU* and its image.

2. Triangle *ABC* has vertices *A*(−2, 6), *B*(0, 5), and *C*(3, −1). Find the vertices of triangle *A′B′C′* after a reflection across the *x*-axis.

Algebraic Representations of Rotations

When points are rotated about the origin, the coordinates of the image can be found using the rules shown in the table.

| Rotations	
90° clockwise	Multiply each *x*-coordinate by −1; then switch the *x*- and *y*-coordinates: $(x, y) \rightarrow (y, -x)$
90° counterclockwise	Multiply each *y*-coordinate by −1; then switch the *x*- and *y*-coordinates: $(x, y) \rightarrow (-y, x)$
180°	Multiply both coordinates by −1: $(x, y) \rightarrow (-x, -y)$

EXAMPLE 3

Quadrilateral *ABCD* has vertices at *A*(−4, 2), *B*(−3, 4), *C*(2, 3), and *D*(0, 0). Find the vertices of quadrilateral *A′B′C′D′* after a 90° clockwise rotation. Then graph the quadrilateral and its image.

Multiply the x-coordinate of each vertex by −1, and then switch the x- and y-coordinates.

STEP 1 Apply the rule to find the vertices of the image.

Vertices of *ABCD*	Rule: $(y, -x)$	Vertices of *A′B′C′D′*
A(−4, 2)	$(2, -1 \cdot (-4))$	*A′*(2, 4)
B(−3, 4)	$(4, -1 \cdot (-3))$	*B′*(4, 3)
C(2, 3)	$(3, -1 \cdot 2)$	*C′*(3, −2)
D(0, 0)	$(0, -1 \cdot 0)$	*D′*(0, 0)

STEP 2 Graph the quadrilateral and its image.
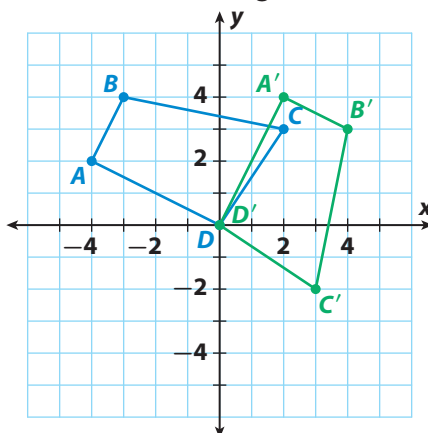
Reflect

3. **Communicate Mathematical Ideas** How would you find the vertices of an image if a figure were rotated 270° clockwise? Explain.

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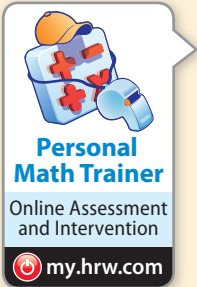
YOUR TURN

4. A triangle has vertices at $J(-2, -4)$, $K(1, 5)$, and $L(2, 2)$. What are the coordinates of the vertices of the image after the triangle is rotated 90° counterclockwise?

_____

Guided Practice

1. Triangle XYZ has vertices $X(-3, -2)$, $Y(-1, 0)$, and $Z(1, -6)$. Find the vertices of triangle $X'Y'Z'$ after a translation of 6 units to the right. Then graph the triangle and its image. **(Example 1)**

_____

2. Describe what happens to the x- and y-coordinates after a point is reflected across the x-axis. **(Example 2)**

_____

_____

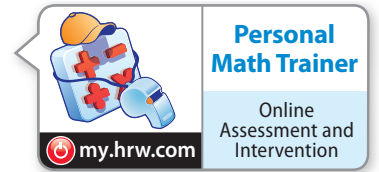3. Use the rule $(x, y) \rightarrow (y, -x)$ to graph the image of the triangle at right. Then describe the transformation. **(Example 3)**

_____
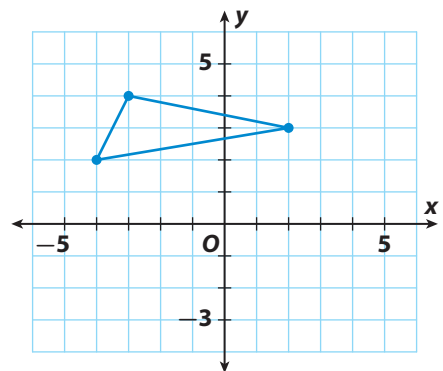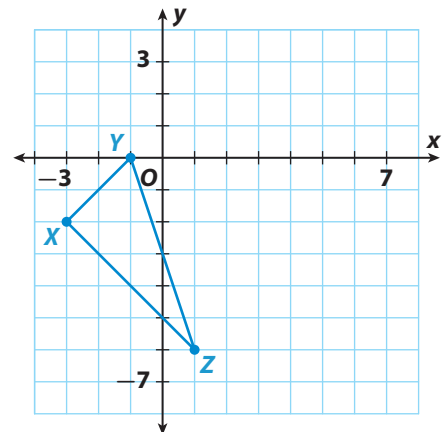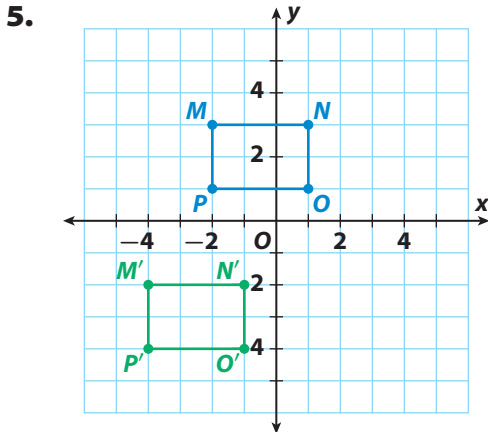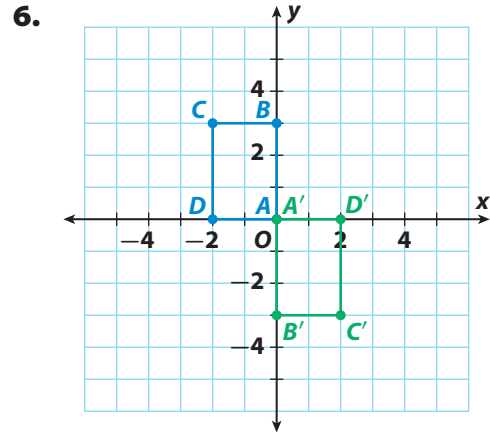
❓ ESSENTIAL QUESTION CHECK-IN

4. How do the x- and y-coordinates change when a figure is translated right a units and down b units?

_____

_____

9.4 Independent Practice

Write an algebraic rule to describe each transformation.
Then describe the transformation.

5.

_____

_____

6.

_____

_____

7. Triangle *XYZ* has vertices *X*(6, −2.3), *Y*(7.5, 5), and *Z*(8, 4). When translated, *X′* has coordinates (2.8, −1.3). Write a rule to describe this transformation. Then find the coordinates of *Y′* and *Z′*.

_____

8. Point *L* has coordinates (3, −5). The coordinates of point *L′* after a reflection are (−3, −5). Without graphing, tell which axis point *L* was reflected across. Explain your answer.

_____

_____

_____

9. Use the rule $(x, y) \rightarrow (x - 2, y - 4)$ to graph the image of the rectangle. Then describe the transformation.

_____

_____

10. Parallelogram *ABCD* has vertices $A(-2, -5\frac{1}{2})$, $B(-4, -5\frac{1}{2})$, *C*(−3, −2), and *D*(−1, −2). Find the vertices of parallelogram *A′B′C′D′* after a translation of $2\frac{1}{2}$ units down.

_____

_____
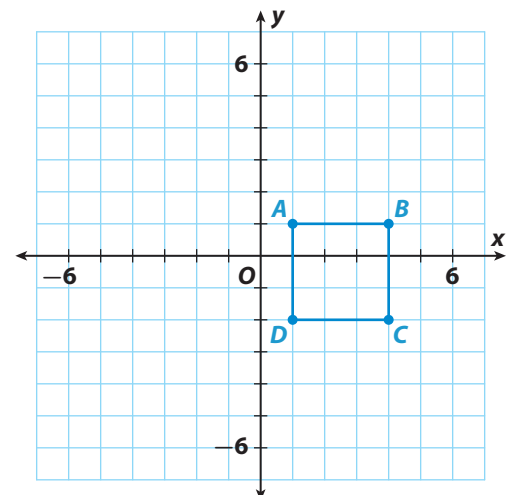
11. Alexandra drew the logo shown on half-inch graph paper. Write a rule that describes the translation Alexandra used to create the shadow on the letter A.

_____
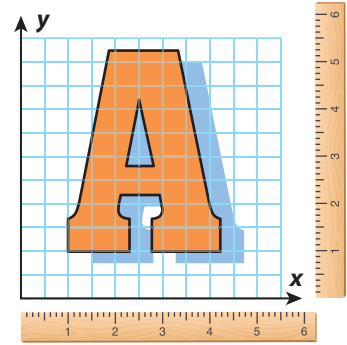
12. Kite *KLMN* has vertices at *K*(1, 3), *L*(2, 4), *M*(3, 3), and *N*(2, 0). After the kite is rotated, *K′* has coordinates (−3, 1). Describe the rotation, and include a rule in your description. Then find the coordinates of *L′*, *M′*, and *N′*.

_____

_____

H.O.T. FOCUS ON HIGHER ORDER THINKING

13. Make a Conjecture Graph the triangle with vertices (−3, 4), (3, 4), and (−5, −5). Use the transformation (*y*, *x*) to graph its image.
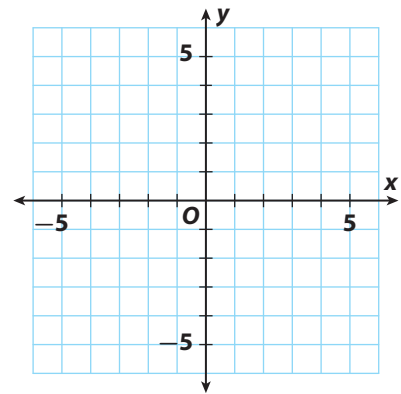
a. Which vertex of the image has the same coordinates as a vertex of the original figure? Explain why this is true.

_____

_____

b. What is the equation of a line through the origin and this point?

_____

c. Describe the transformation of the triangle.

_____

14. Critical Thinking Mitchell says the point (0, 0) does not change when reflected across the *x*- or *y*-axis or when rotated about the origin. Do you agree with Mitchell? Explain why or why not.

_____

_____

_____

15. Analyze Relationships Triangle *ABC* with vertices *A*(−2, −2), *B*(−3, 1), and *C*(1, 1) is translated by (*x*, *y*) → (*x* − 1, *y* + 3). Then the image, triangle *A′B′C′*, is translated by (*x*, *y*) → (*x* + 4, *y* − 1), resulting in *A″B″C″*.

a. Find the coordinates for the vertices of triangle *A″B″C″*.

_____

b. Write a rule for one translation that maps triangle *ABC* to triangle *A″B″C″*.

_____

**Work Area**

9.5 Congruent Figures

? ESSENTIAL QUESTION

What is the connection between transformations and figures that have the same shape and size?

EXPLORE ACTIVITY

Combining Transformations

Apply the indicated series of transformations to the triangle. Each transformation is applied to the image of the previous transformation, not the original figure. Label each image with the letter of the transformation applied.
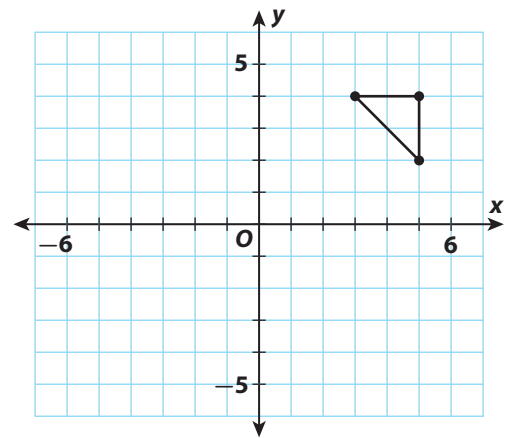
A Reflection across the *x*-axis

B $(x, y) \rightarrow (x - 3, y)$

C Reflection across the *y*-axis

D $(x, y) \rightarrow (x, y + 4)$

E Rotation 90° clockwise around the origin

F Compare the size and shape of the final image to that of the original figure.

_____

_____

Reflect

1. Which transformation(s) change the orientation of figures? Which do not?

_____

2. **Make a Conjecture** Two figures have the same size and shape. What does this indicate about the figures?

_____

_____

_____

Congruent Figures

Recall that segments and their images have the same length and angles and their images have the same measure under a translation, reflection, or rotation. Two figures are said to be **congruent** if one can be obtained from the other by a sequence of translations, reflections, and rotations. Congruent figures have the same size and shape.

When you are told that two figures are congruent, there must be a sequence of translations, reflections, and/or rotations that transforms one into the other.
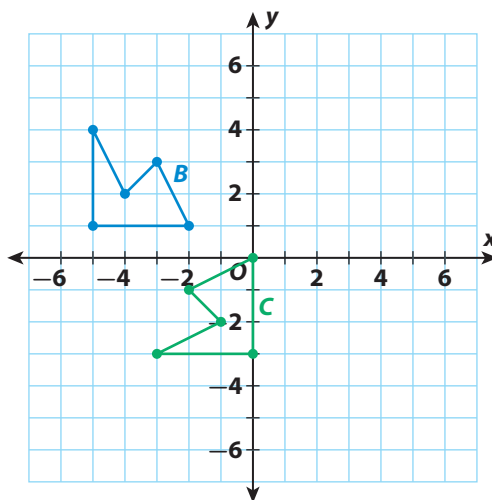
EXAMPLE 1

A Identify a sequence of transformations that will transform figure *A* into figure *B*.
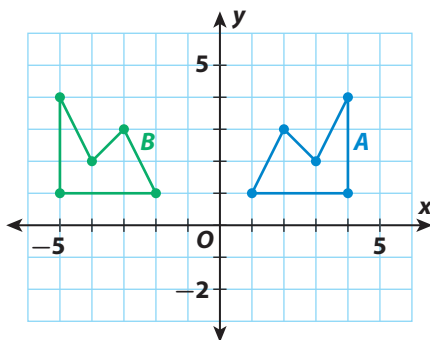
To transform figure *A* into figure *B*, you need to reflect it over the *y*-axis and translate one unit to the left. A sequence of transformations that will accomplish this is $(x, y) \rightarrow (-x, y)$ and $(x, y) \rightarrow (x - 1, y)$.

B Identify a sequence of transformations that will transform figure *B* into figure *C*.

Math Talk

Mathematical Processes

How do you know that the sequence of transformations in Parts B and C must include a rotation?

Any sequence of transformations that changes figure *B* into figure *C* will need to include a rotation. A 90° counterclockwise rotation around the origin would result in the figure being oriented as figure *C*.

However, the rotated figure would be 2 units below and 1 unit to the left of where figure *C* is. You would need to translate the rotated figure up 2 units and right 1 unit.
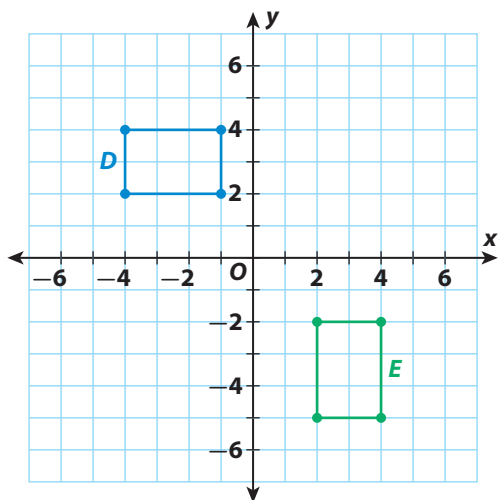
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The sequence of transformations is a 90° counterclockwise rotation about the origin, $(x, y) \rightarrow (-y, x)$, followed by $(x, y) \rightarrow (x + 1, y + 2)$.

C Identify a sequence of transformations that will transform figure *D* into figure *E*.


A sequence of transformations that changes figure *D* to figure *E* will need to include a rotation. A 90° clockwise rotation around the origin would result in the figure being oriented as figure *E*.

However, the rotated figure would be 6 units above where figure *E* is. You would need to translate the rotated figure down 6 units.

The sequence of transformations is a 90° clockwise rotation about the origin, $(x, y) \rightarrow (y, -x)$, followed by $(x, y) \rightarrow (x, y - 6)$.

YOUR TURN

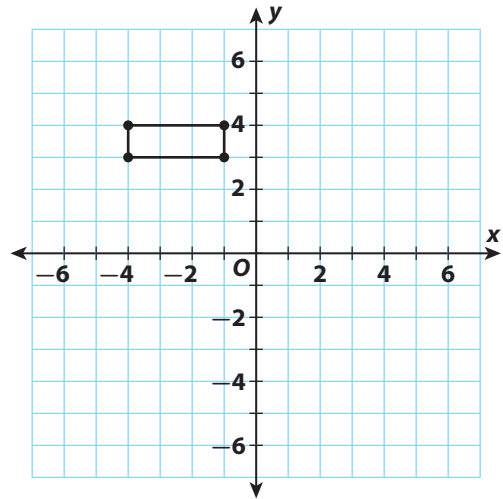**3.** Identify a sequence of transformations that will transform figure *A* into figure *B*.
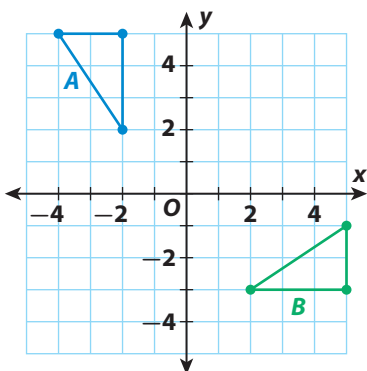
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and Intervention

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1. Apply the indicated series of transformations to the rectangle. Each transformation is applied to the image of the previous transformation, not the original figure. Label each image with the letter of the transformation applied. **(Explore Activity)**

  a. Reflection across the *y*-axis

  b. Rotation 90° clockwise around the origin

  c. $(x, y) \rightarrow (x - 2, y)$

  d. Rotation 90° counterclockwise around the origin

 **e.** $(x, y) \rightarrow (x - 7, y - 2)$

Identify a sequence of transformations that will transform figure *A* into figure *C*. (Example 1)

2. What transformation is used to transform figure *A* into figure *B*?

_____

_____

3. What transformation is used to transform figure *B* into figure *C*?

_____

_____

4. What sequence of transformations is used to transform figure *A* into figure *C*? Express the transformations algebraically.
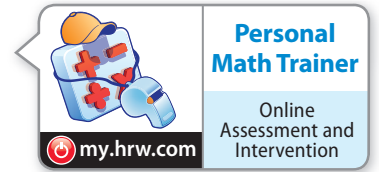
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_____

5. Vocabulary What does it mean for two figures to be congruent?

_____

_____
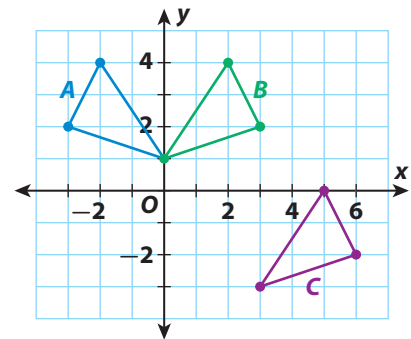
? ESSENTIAL QUESTION CHECK-IN

6. After a sequence of translations, reflections, and rotations, what is true about the first figure and the final figure?

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9.5 Independent Practice

For each given figure A, graph figures B and C using the given sequence of transformations. State whether figures A and C have the same or different orientation.
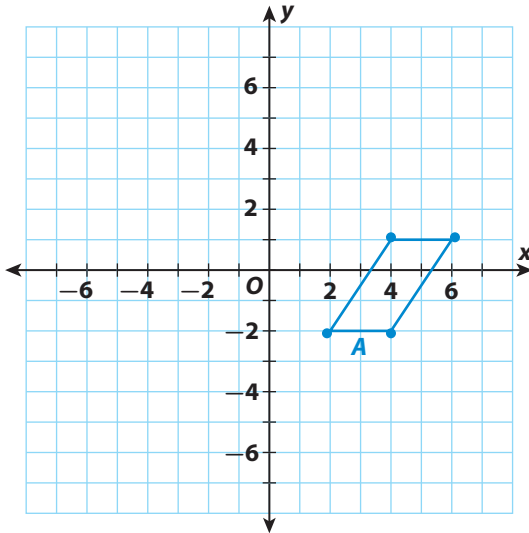
7.

Figure B: a translation of 1 unit to the right and 3 units up

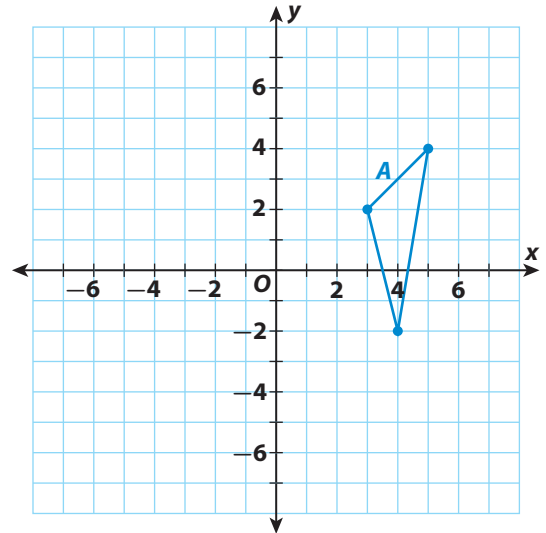Figure C: a 90° clockwise rotation around the origin

8.

Figure B: a reflection across the y-axis
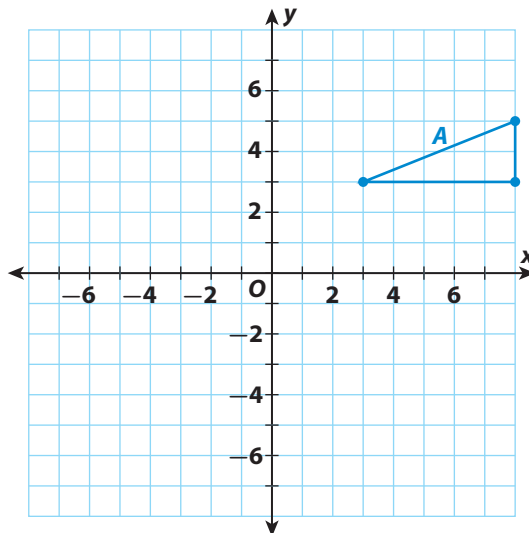
Figure C: a 180° rotation around the origin

9.

Figure B: a reflection across the y-axis
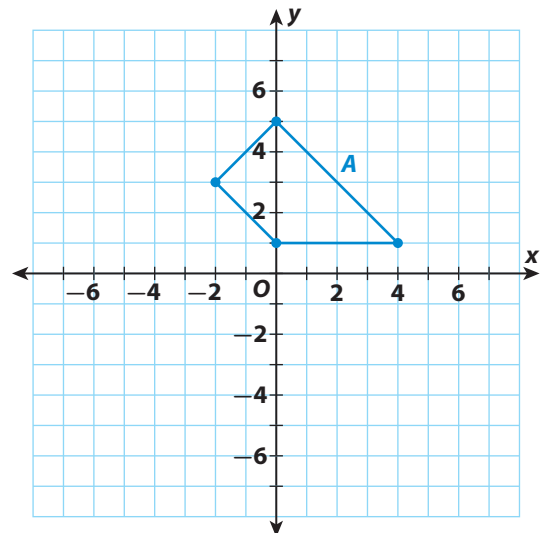
Figure C: a translation 2 units down

10.

Figure B: a translation 2 units up

Figure C: a rotation of 180° around the origin

11. Represent Real-World Problems A city planner wanted to place the new town library at site *A*. The mayor thought that it would be better at site *B*. What transformations were applied to the building at site *A* to relocate the building to site *B*? Did the mayor change the size or orientation of the library?

_____

_____

_____
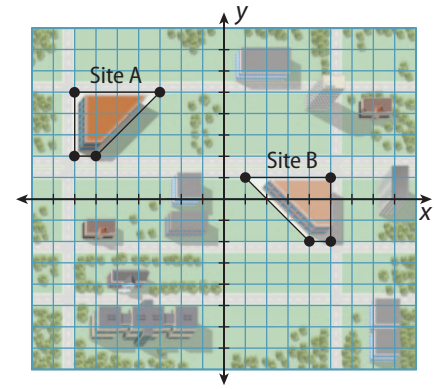
12. Persevere in Problem Solving Find a sequence of three transformations that can be used to obtain figure *D* from figure *A*. Graph the figures *B* and *C* that are created by the transformations.

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H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area
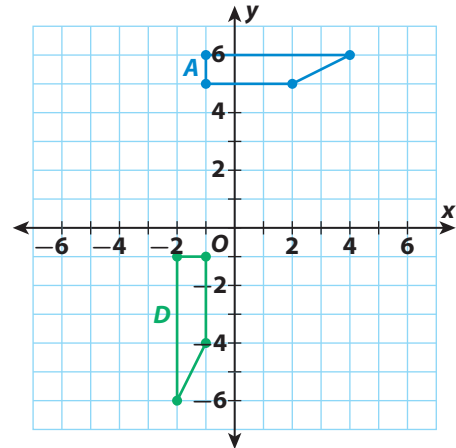
13. Counterexamples The Commutative Properties for Addition and Multiplication state that the order of two numbers being added or multiplied does not change the sum or product. Are translations and rotations commutative? If not, give a counterexample.
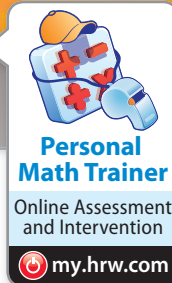
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14. Multiple Representations For each representation, describe a possible sequence of transformations.

a. $(x, y) \rightarrow (-x - 2, y + 1)$

_____

_____

**b.** $(x, y) \rightarrow (y, -x - 3)$

_____

_____

Ready to Go On?

9.1–9.3 Properties of Translations, Reflections, and Rotations

1. Graph the image of triangle *ABC* after a translation of 6 units to the right and 4 units down. Label the vertices of the image *A′*, *B′*, and *C′*.

2. On the same coordinate grid, graph the image of triangle *ABC* after a reflection across the *x*-axis. Label the vertices of the image *A″*, *B″*, and *C″*.

3. Graph the image of *HIJK* after it is rotated 180° about the origin. Label the vertices of the image *H′I′J′K′*.
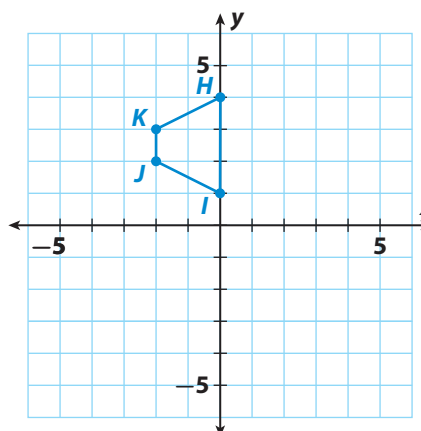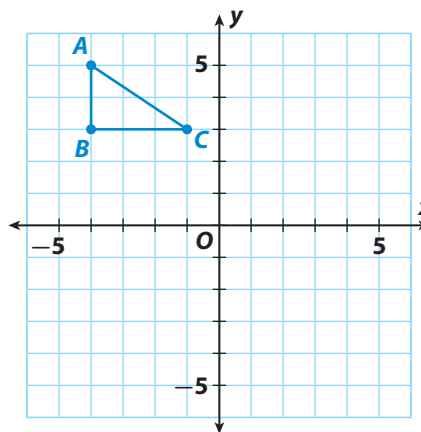
9.4 Algebraic Representations of Transformations

4. A triangle has vertices at (2, 3), (−2, 2), and (−3, 5). What are the coordinates of the vertices of the image after the translation $(x, y) \rightarrow (x + 4, y - 3)$?
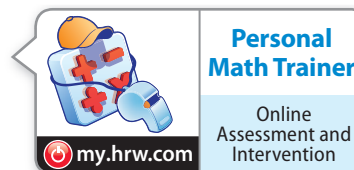
_____

9.5 Congruent Figures

5. Vocabulary Translations, reflections, and rotations produce a figure

that is _____ to the original figure.

6. Use the coordinate grid for Exercise 3. Reflect *H′I′J′K′* over the *y*-axis, then rotate it 180° about the origin. Label the new figure *H″I″J″K″*.
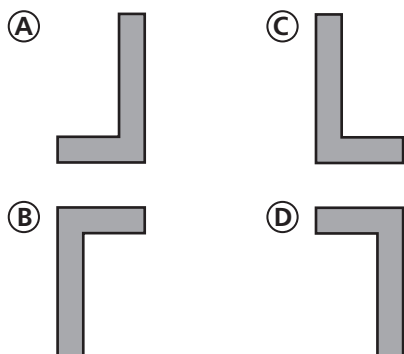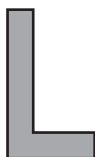
? ESSENTIAL QUESTION

7. How can you use transformations to solve real-world problems?

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Assessment Readiness

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Selected Response

1. What would be the orientation of the figure L after a translation of 8 units to the right and 3 units up?

Ⓐ

Ⓒ

Ⓑ

Ⓓ

2. Figure A is reflected over the *y*-axis and then lowered 6 units. Which sequence describes these transformations?

Ⓐ $(x, y) \rightarrow (x, -y)$ and $(x, y) \rightarrow (x, y - 6)$

Ⓑ $(x, y) \rightarrow (-x, y)$ and $(x, y) \rightarrow (x, y - 6)$

Ⓒ $(x, y) \rightarrow (x, -y)$ and $(x, y) \rightarrow (x - 6, y)$

Ⓓ $(x, y) \rightarrow (-x, y)$ and $(x, y) \rightarrow (x - 6, y)$

3. What quadrant would the triangle be in after a rotation of 90° counterclockwise about the origin?
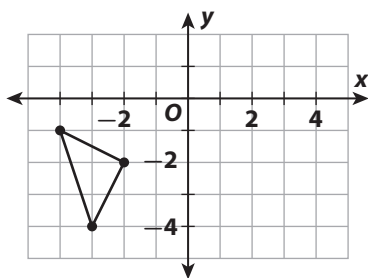
Ⓐ I   Ⓑ II   Ⓒ III   Ⓓ IV

4. Which rational number is greater than $-3\frac{1}{3}$ but less than $-\frac{4}{5}$?

Ⓐ -0.4 Ⓒ -0.19

Ⓑ $-\frac{9}{7}$ Ⓓ $-\frac{22}{5}$

5. Which of the following is **not** true of a trapezoid that has been reflected across the *x*-axis?

Ⓐ The new trapezoid is the same size as the original trapezoid.

Ⓑ The new trapezoid is the same shape as the original trapezoid.

Ⓒ The new trapezoid is in the same orientation as the original trapezoid.

Ⓓ The *x*-coordinates of the new trapezoid are the same as the *x*-coordinates of the original trapezoid.

6. A triangle with coordinates $(6, 4)$, $(2, -1)$, and $(-3, 5)$ is translated 4 units left and rotated 180° about the origin. What are the coordinates of its image?

Ⓐ $(2, 4), (-2, -1), (-7, 5)$

Ⓑ $(4, 6), (-1, 2), (5, -3)$

Ⓒ $(4, -2), (-1, 2), (5, 7)$

Ⓓ $(-2, -4), (2, 1), (7, -5)$

Mini-Task

7. A rectangle with vertices $(3, -2)$, $(3, -4)$, $(7, -2)$, $(7, -4)$ is reflected across the *x*-axis and then rotated 90° counterclockwise.

a. In what quadrant does the image lie?

_____

b. What are the vertices of the image?

_____

c. What other transformations produce the same image?

_____

_____