Exponents and Scientific Notation

MODULE 2

ESSENTIAL QUESTION

How can you use scientific notation to solve real-world problems?

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Real-World Video

The distance from Earth to other planets, moons, and stars is a very great number of kilometers. To make it easier to write very large and very small numbers, we use scientific notation.

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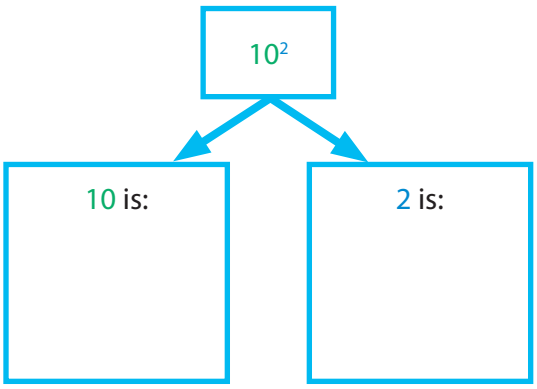
Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the Venn diagram. You can put more than one word in each section of the diagram.



Understand Vocabulary

Complete the sentences using the preview words.

1. A number produced by raising a base to an exponent is a _____.

2. _____ is a method of writing very large or very small numbers by using powers of 10.

3. A _____ is any number that can be expressed as a ratio of two integers.

Vocabulary

Review Words

- ✔ base *(base)*
- ✔ exponent *(exponente)*
- integers *(enteros)*
- ✔ positive number *(número positivo)*
- standard notation *(notación estándar)*

Preview Words

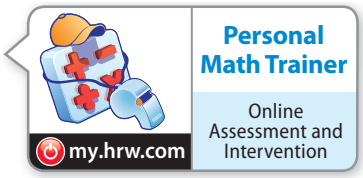
- power *(potencia)*
- rational number *(número racional)*
- real numbers *(número real)*
- scientific notation *(notación científica)*
- whole number *(número entero)*

Active Reading

Two-Panel Flip Chart Create a two-panel flip chart to help you understand the concepts in this module. Label one flap "Positive Powers of 10" and the other flap "Negative Powers of 10." As you study each lesson, write important ideas under the appropriate flap. Include sample problems that will help you remember the concepts later when you look back at your notes.

Are YOU Ready?

Complete these exercises to review skills you will need for this module.



Exponents

EXAMPLE $10^4 = 10 \times 10 \times 10 \times 10$ — Write the exponential expression as a product.

$= 10{,}000$ — Simplify.

Write each exponential expression as a decimal.

1. 10^2 _____ **2.** 10^3 _____ **3.** 10^5 _____ **4.** 10^7 _____

Multiply and Divide by Powers of 10

EXAMPLE $0.0478 \times 10^5 = 0.0478 \times 100{,}000$ — Identify the number of zeros in the power of 10. When multiplying, move the decimal point to the *right* the same number of places as the number of zeros.

$= 4{,}780$

$37.9 \div 10^4 = 37.9 \div 10{,}000$ — Identify the number of zeros in the power of 10. When dividing, move the decimal point to the *left* the same number of places as the number of zeros.

$= 0.00379$

Find each product or quotient.

5. 45.3×10^3 _____ **6.** $7.08 \div 10^2$ _____ **7.** 0.00235×10^6 _____ **8.** $3{,}600 \div 10^4$ _____

9. 0.5×10^2 _____ **10.** $67.7 \div 10^5$ _____ **11.** 0.0057×10^4 _____ **12.** $195 \div 10^6$ _____

Are YOU Ready? *(cont'd)*

Complete these exercises to review skills you will need for this module.

Exponents

13. Chris has an MP3 player that can store about 10^8 bytes of data. Explain how to determine the number of zeros in the decimal for 10^8. Then write the decimal and state the number of bytes in words, without using exponents.

14. Explain how to write 10^5 as a decimal.

15. To write 10^4 as a decimal, Tyrese multiplied 10×4 to get 40. Identify Tyrese's error, and write the correct value.

Multiply and Divide by Powers of 10

16. Explain how to find the product 0.8×10^3. Compare this product with 10^3.

17. Jake found that the quotient $9.236 \div 10^4$ is 0.0009236. If Jake's result is correct, justify his answer. If not, give the correct decimal.

18. To find the quotient $62.5 \div 10^5$, Gina first identified the number of zeros in the power of 10. When written as a decimal, 10^5 has five zeros. Describe how Gina can complete the problem.

LESSON

2.1 Integer Exponents

8.1.2.1

Students will develop and use the properties of integer exponents.

ESSENTIAL QUESTION

How can you develop and use the properties of integer exponents?

EXPLORE ACTIVITY 1

Using Patterns of Integer Exponents

The table below shows powers of 5, 4, and 3.

$5^4 = 625$	$5^3 = 125$	$5^2 = 25$	$5^1 = 5$	$5^0 =$ ☐	$5^{-1} =$ ☐	$5^{-2} =$ ☐
$4^4 = 256$	$4^3 = 64$	$4^2 = 16$	$4^1 = 4$	$4^0 =$ ☐	$4^{-1} =$ ☐	$4^{-2} =$ ☐
$3^4 = 81$	$3^3 = 27$	$3^2 = 9$	$3^1 = 3$	$3^0 =$ ☐	$3^{-1} =$ ☐	$3^{-2} =$ ☐

A What pattern do you see in the powers of 5?

_____

_____

B What pattern do you see in the powers of 4?

_____

_____

C What pattern do you see in the powers of 3?

_____

_____

D Complete the table for the values of 5^0, 5^{-1}, 5^{-2}.

E Complete the table for the values of 4^0, 4^{-1}, 4^{-2}.

F Complete the table for the values of 3^0, 3^{-1}, 3^{-2}.

Reflect

1. **Make a Conjecture** Write a general rule for the value of a^0. _____

2. **Make a Conjecture** Write a general rule for the value of a^{-n}. _____

EXPLORE ACTIVITY 2

Exploring Properties of Integer Exponents

A Complete the following equations.

$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^{\square}$

$(3 \cdot 3 \cdot 3 \cdot 3) \cdot 3 = 3^{\square} \cdot 3^{\square} = 3^{\square}$

$(3 \cdot 3 \cdot 3) \cdot (3 \cdot 3) = 3^{\square} \cdot 3^{\square} = 3^{\square}$

What pattern do you see when multiplying two powers with the same base?

_____

_____

Use your pattern to complete this equation: $5^2 \cdot 5^5 = 5^{\square}$.

B Complete the following equation:

$\frac{4^5}{4^3} = \frac{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}{4 \cdot 4 \cdot 4} = \frac{\cancel{4}^{1} \cdot \cancel{4}^{1} \cdot \cancel{4}^{1} \cdot 4 \cdot 4}{\cancel{4}_{1} \cdot \cancel{4}_{1} \cdot \cancel{4}_{1}} = 4 \cdot 4 = 4^{\square}$

What pattern do you see when dividing two powers with the same base?

_____

_____

_____

Use your pattern to complete this equation: $\frac{6^8}{6^3} = 6^{\square}$.

Math Talk

Mathematical Processes

Do the patterns you found in parts A–C apply if the exponents are negative? If so, give an example of each.

C Complete the following equations:

$(5^3)^2 = (5 \cdot 5 \cdot 5)^{\square} = (5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5) = 5^{\square}$

What pattern do you see when raising a power to a power?

_____

_____

Use your pattern to complete this equation: $(7^2)^4 = 7^{\square}$.

Reflect

Let *m* and *n* be integers.

3. **Make a Conjecture** Write a general rule for the value of $a^m \cdot a^n$. _____

4. **Make a Conjecture** Write a general rule for the value of $\frac{a^m}{a^n}$, $a \neq 0$. _____

5. **Make a Conjecture** Write a general rule for the value of $(a^m)^n$. _____

Applying Properties of Integer Exponents

You can use the general rules you found in the Explore Activities to simplify more complicated expressions.

EXAMPLE 1

Simplify each expression.

A $(5 - 2)^5 \cdot 3^{-8} + (5 + 2)^0$

$(3)^5 \cdot 3^{-8} + (7)^0$ Simplify within parentheses.

$3^{5+(-8)} + 1$ Use properties of exponents.

$3^{-3} + 1$ Simplify.

$\frac{1}{27} + 1 = 1\frac{1}{27}$ Apply the rule for negative exponents and add.

B $\frac{\left[(3 + 1)^2\right]^3}{(7 - 3)^2}$

$\frac{(4^2)^3}{4^2}$ Simplify within parentheses.

$\frac{4^6}{4^2}$ Use properties of exponents.

4^{6-2} Use properties of exponents.

$4^4 = 256$ Simplify.

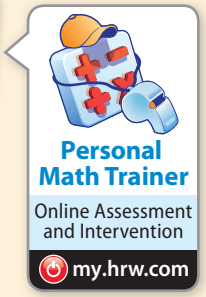
My Notes

YOUR TURN

Simplify each expression.

6. $\frac{\left[(6 - 1)^2\right]^2}{(3 + 2)^3}$ _____

7. $(2^2)^3 - (10 - 6)^3 \cdot 4^{-5}$ _____



Guided Practice

Find the value of each power. (Explore Activity 1)

1. $8^{-1} =$ _____ **2.** $6^{-2} =$ _____ **3.** $256^{0} =$ _____

4. $10^{2} =$ _____ **5.** $5^{4} =$ _____ **6.** $2^{-5} =$ _____

7. $4^{-5} =$ _____ **8.** $89^{0} =$ _____ **9.** $11^{-3} =$ _____

Use properties of exponents to write an equivalent expression. (Explore Activity 2)

10. $4 \cdot 4 \cdot 4 = 4^{\square}$

11. $(2 \cdot 2) \cdot (2 \cdot 2 \cdot 2) = 2^{\square} \cdot 2^{\square} = 2^{\square}$

12. $\frac{6^7}{6^5} = \frac{6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6}{6 \cdot 6 \cdot 6 \cdot 6 \cdot 6} = \square^{\square}$

13. $\frac{8^{12}}{8^9} = 8^{\square - \square} = \square^{\square}$

14. $5^{10} \cdot 5 \cdot 5 = 5^{\square}$

15. $7^8 \cdot 7^5 = \square^{\square}$

16. $(6^2)^4 = (6 \cdot 6)^{\square}$

$= (6 \cdot 6) \cdot (6 \cdot 6) \cdot \left(\square \cdot \square\right) \cdot$ _____

$= \square^{\square}$

17. $(3^3)^3 = (3 \cdot 3 \cdot 3)^3$

$= (3 \cdot 3 \cdot 3) \cdot \left(\square \cdot \square \cdot \square\right)$ _____

$= \square^{\square}$

Simplify each expression. (Example 1)

18. $(10 - 6)^3 \cdot 4^2 + (10 + 2)^2$ _____

19. $\frac{(12 - 5)^7}{\left[(3 + 4)^2\right]^2}$ _____

ESSENTIAL QUESTION CHECK-IN

20. Summarize the rules for multiplying powers with the same base, dividing powers with the same base, and raising a power to a power.

_____

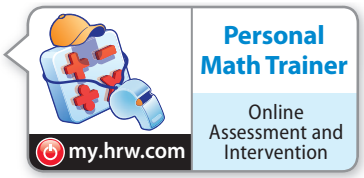
_____

_____

_____

Name _____ Class _____ Date _____

2.1 Independent Practice



21. Explain why the exponents cannot be added in the product $12^3 \cdot 11^3$.

_____

22. List three ways to express 3^5 as a product of powers.

_____

23. **Astronomy** The distance from Earth to the moon is about 22^4 miles. The distance from Earth to Neptune is about 22^7 miles. Which distance is the greater distance and about how many times greater is it?

_____

24. **Critique Reasoning** A student claims that $8^3 \cdot 8^{-5}$ is greater than 1. Explain whether the student is correct or not.

_____

_____

Find the missing exponent.

25. $(b^2)^{\square} = b^{-6}$

26. $x^{\square} \cdot x^6 = x^9$

27. $\dfrac{y^{25}}{y^{\square}} = y^6$

28. **Communicate Mathematical Ideas** Why do you subtract exponents when dividing powers with the same base?

_____

_____

_____

29. **Astronomy** The mass of the Sun is about 2×10^{27} metric tons, or 2×10^{30} kilograms. How many kilograms are in one metric ton?

_____

30. **Represent Real-World Problems** In computer technology, a kilobyte is 2^{10} bytes in size. A gigabyte is 2^{30} bytes in size. The size of a terabyte is the product of the size of a kilobyte and the size of a gigabyte. What is the size of a terabyte?

_____

31. Write equivalent expressions for $x^7 \cdot x^{-2}$ and $\frac{x^7}{x^2}$. What do you notice? Explain how your results relate to the properties of integer exponents.

A toy store is creating a large window display of different colored cubes stacked in a triangle shape. The table shows the number of cubes in each row of the triangle, starting with the top row.

Row	1	2	3	4
Number of cubes in each row	3	3^2	3^3	3^4

32. **Look for a Pattern** Describe any pattern you see in the table.

33. Using exponents, how many cubes will be in Row 6?
How many times as many cubes will be in Row 6 than in Row 3?

34. **Justify Reasoning** If there are 6 rows in the triangle, what is the total number of cubes in the triangle? Explain how you found your answer.

H.O.T. FOCUS ON HIGHER ORDER THINKING

35. **Critique Reasoning** A student simplified the expression $\frac{6^2}{36^2}$ as $\frac{1}{3}$. Do you agree with this student? Explain why or why not.

36. **Draw Conclusions** Evaluate $-a^n$ when $a = 3$ and $n = 2, 3, 4$, and 5. Now evaluate $(-a)^n$ when $a = 3$ and $n = 2, 3, 4$, and 5. Based on this sample, does it appear that $-a^n = (-a)^n$? If not, state the relationships, if any, between $-a^n$ and $(-a)^n$.

37. **Persevere in Problem Solving** A number to the 12th power divided by the same number to the 9th power equals 125. What is the number?

Work Area

LESSON 2.2

Scientific Notation with Positive Powers of 10

8.1.2.2

Students will use scientific notation to express very large quantities.

ESSENTIAL QUESTION

How can you use scientific notation to express very large quantities?

EXPLORE ACTIVITY Real World

Using Scientific Notation

Scientific notation is a method of expressing very large and very small numbers as a product of a number greater than or equal to 1 and less than 10, and a power of 10.

The weights of various sea creatures are shown in the table. Write the weight of the blue whale in scientific notation.

Sea Creature	Blue whale	Gray whale	Whale shark
Weight (lb)	250,000	68,000	41,200

A Move the decimal point in 250,000 to the left as many places as necessary to find a number that is greater than or equal to 1 and less than 10.

What number did you find? _____

B Divide 250,000 by your answer to **A**. Write your answer as a power of 10.

_____

C Combine your answers to **A** and **B** to represent 250,000.

$250{,}000 = \square \times 10^{\square}$

Repeat steps **A** through **C** to write the weight of the whale shark in scientific notation.

$41{,}200 = \square \times 10^{\square}$

Reflect

1. How many places to the left did you move the decimal point to write 41,200 in scientific notation? _____

2. What is the exponent on 10 when you write 41,200 in scientific notation?

_____

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Writing a Number in Scientific Notation

To translate between standard notation and scientific notation, you can count the number of places the decimal point moves.

Writing Large Quantities in Scientific Notation

When the number is greater than or equal to 10, use a positive exponent.	$84,000 = 8.4 \times 10^4$	*The decimal point moves 4 places to the left.*

EXAMPLE 1

Real World

The distance from Earth to the Sun is about 93,000,000 miles. Write this distance in scientific notation.

STEP 1 Move the decimal point in 93,000,000 to the left until you have a number that is greater than or equal to 1 and less than 10.

$9.30000000.$ *Move the decimal point 7 places to the left.*

9.3 *Remove extra zeros.*

STEP 2 Divide the original number by the result from Step 1.

$10{,}000{,}000$ *Divide 93,000,000 by 9.3.*

10^7 *Write your answer as a power of 10.*

STEP 3 Write the product of the results from Steps 1 and 2.

$93{,}000{,}000 = 9.3 \times 10^7$ miles *Write a product to represent 93,000,000 in scientific notation.*

Math Talk
Mathematical Processes
Is 12×10^7 written in scientific notation? Explain.

YOUR TURN

Write each number in scientific notation.

3. 6,400 _____

4. 570,000,000,000 _____

5. A light-year is the distance that light travels in a year and is equivalent to 9,461,000,000,000 km. Write this distance in scientific notation.

_____

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Writing a Number in Standard Notation

To translate between scientific notation and standard notation, move the decimal point the number of places indicated by the exponent in the power of 10. When the exponent is positive, move the decimal point to the right and add placeholder zeros as needed.

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My Notes

EXAMPLE 2

Write 3.5×10^6 in standard notation.

STEP 1 Use the exponent of the power of 10 to see how many places to move the decimal point. — 6 places

STEP 2 Place the decimal point. Since you are going to write a number greater than 3.5, move the decimal point to the *right*. Add placeholder zeros if necessary. — 3 5 0 0 0 0 0.

The number 3.5×10^6 written in standard notation is 3,500,000.

Reflect

6. Explain why the exponent in 3.5×10^6 is 6, while there are only 5 zeros in 3,500,000.

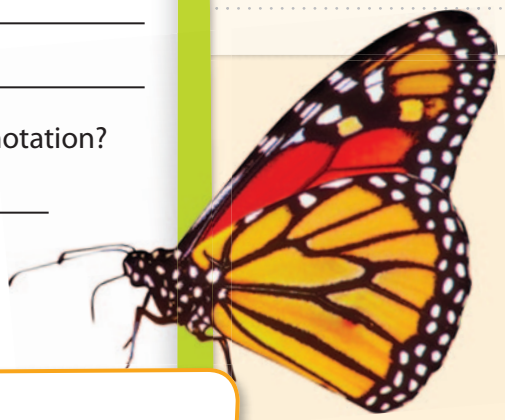
_____

_____

_____

7. What is the exponent on 10 when you write 5.3 in scientific notation?

_____



YOUR TURN

Write each number in standard notation.

8. 7.034×10^9

_____

9. 2.36×10^5

_____

10. The mass of one roosting colony of Monarch butterflies in Mexico was estimated at 5×10^6 grams. Write this mass in standard notation.

_____

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Guided Practice

Write each number in scientific notation. (Explore Activity and Example 1)

1. 58,927
Hint: Move the decimal left 4 places.

_____

2. 1,304,000,000
Hint: Move the decimal left 9 places.

_____

3. 6,730,000

_____

4. 13,300

_____

5. An ordinary quarter contains about 97,700,000,000,000,000,000,000 atoms.

_____

6. The distance from Earth to the Moon is about 384,000 kilometers.

_____

Write each number in standard notation. (Example 2)

7. 4×10^5
Hint: Move the decimal right 5 places.

_____

8. 1.8499×10^9
Hint: Move the decimal right 9 places.

_____

9. 6.41×10^3

_____

10. 8.456×10^7

_____

11. 8×10^5

_____

12. 9×10^{10}

_____

13. Diana calculated that she spent about 5.4×10^4 seconds doing her math homework during October. Write this time in standard notation. (Example 2)

_____

14. The town recycled 7.6×10^6 cans this year. Write the number of cans in standard notation. (Example 2)

_____

ESSENTIAL QUESTION CHECK-IN

15. Describe how to write 3,482,000,000 in scientific notation.

_____

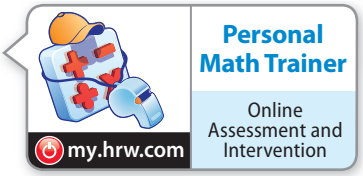
_____

_____

_____

Name _____ Class _____ Date _____

2.2 Independent Practice



Paleontology **Use the table for problems 16–21. Write the estimated weight of each dinosaur in scientific notation.**

Estimated Weight of Dinosaurs	
Name	**Pounds**
Argentinosaurus	220,000
Brachiosaurus	100,000
Apatosaurus	66,000
Diplodocus	50,000
Camarasaurus	40,000
Cetiosauriscus	19,850

16. *Apatosaurus* _____

17. *Argentinosaurus* _____

18. *Brachiosaurus* _____

19. *Camarasaurus* _____

20. *Cetiosauriscus* _____

21. *Diplodocus* _____

22. A single little brown bat can eat up to 1,000 mosquitoes in a single hour. Express in scientific notation how many mosquitoes a little brown bat might eat in 10.5 hours.

_____

23. **Multistep** Samuel can type nearly 40 words per minute. Use this information to find the number of hours it would take him to type 2.6×10^5 words.

_____

_____

24. **Entomology** A tropical species of mite named *Archegozetes longisetosus* is the record holder for the strongest insect in the world. It can lift up to 1.182×10^3 times its own weight.

a. If you were as strong as this insect, explain how you could find how many pounds you could lift.

_____

_____

_____

b. Complete the calculation to find how much you could lift, in pounds, if you were as strong as an *Archegozetes longisetosus* mite. Express your answer in both scientific notation and standard notation.

_____

_____

25. During a discussion in science class, Sharon learns that at birth an elephant weighs around 230 pounds. In four herds of elephants tracked by conservationists, about 20 calves were born during the summer. In scientific notation, express approximately how much the calves weighed all together.

_____

26. **Classifying Numbers** Which of the following numbers are written in scientific notation?

0.641×10^3 9.999×10^4

2×10^1 4.38×5^{10}

_____

27. **Explain the Error** Polly's parents' car weighs about 3500 pounds. Samantha, Esther, and Polly each wrote the weight of the car in scientific notation. Polly wrote 35.0×10^2, Samantha wrote 0.35×10^4, and Esther wrote 3.5×10^4.

a. Which of these girls, if any, is correct?

b. Explain the mistakes of those who got the question wrong.

28. **Justify Reasoning** If you were a biologist counting very large numbers of cells as part of your research, give several reasons why you might prefer to record your cell counts in scientific notation instead of standard notation.

Work Area

H.O.T. FOCUS ON HIGHER ORDER THINKING

29. **Draw Conclusions** Which measurement would be least likely to be written in scientific notation: number of stars in a galaxy, number of grains of sand on a beach, speed of a car, or population of a country? Explain your reasoning.

30. **Analyze Relationships** Compare the two numbers to find which is greater. Explain how you can compare them without writing them in standard notation first.

$$4.5 \times 10^6 \qquad 2.1 \times 10^8$$

31. **Communicate Mathematical Ideas** To determine whether a number is written in scientific notation, what test can you apply to the first factor, and what test can you apply to the second factor?

LESSON 2.3

Scientific Notation with Negative Powers of 10

8.1.2.3

Students will use scientific notation to express very small quantities.

ESSENTIAL QUESTION

How can you use scientific notation to express very small quantities?

EXPLORE ACTIVITY Real World

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Negative Powers of 10

You can use what you know about writing very large numbers in scientific notation to write very small numbers in scientific notation.

A typical human hair has a diameter of 0.000025 meter. Write this number in scientific notation.

A Notice how the decimal point moves in the list below. Complete the list.

$2.345 \times 10^0 = 2.345$ *It moves one place to the right with each increasing power of 10.*

$2.345 \times 10^1 = 23.45$

$2.345 \times 10^2 = 234.5$

$2.345 \times 10^{\square} = 2345.$

$2.345 \times 10^0 = 2.345$ *It moves one place to the left with each decreasing power of 10.*

$2.345 \times 10^{-1} = 0.2345$

$2.345 \times 10^{-2} = 0.02345$

$2.345 \times 10^{\square} = 0.002345$

B Move the decimal point in 0.000025 to the right as many places as necessary to find a number that is greater than or equal to 1 and less than 10. What number did you find? _____

C Divide 0.000025 by your answer to **B**. _____

Write your answer as a power of 10. _____

D Combine your answers to **B** and **C** to represent 0.000025 in scientific notation. _____

Reflect

1. When you move the decimal point, how can you know whether you are increasing or decreasing the number?

_____

2. Explain how the two steps of moving the decimal and multiplying by a power of 10 leave the value of the original number unchanged.

_____

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Writing a Number in Scientific Notation

To write a number less than 1 in scientific notation, move the decimal point right and use a negative exponent.

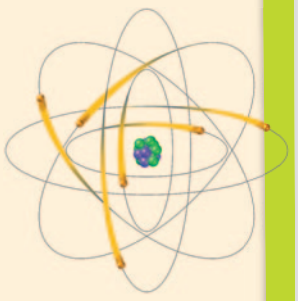
Writing Small Quantities in Scientific Notation

When the number is between 0 and 1, use a negative exponent.	$0.0\,7\,8\,3 = 7.83 \times 10^{-2}$	*The decimal point moves 2 places to the right.*

EXAMPLE 1

Real World

The average size of an atom is about 0.00000003 centimeter across. Write the average size of an atom in scientific notation.



Move the decimal point as many places as necessary to find a number that is greater than or equal to 1 and less than 10.

STEP 1 Place the decimal point. 3.0

STEP 2 Count the number of places you moved the decimal point. 8

STEP 3 Multiply 3.0 times a power of 10. 3.0×10^{-8}

Since 0.00000003 is less than 1, you moved the decimal point to the right and the exponent on 10 is negative.

The average size of an atom in scientific notation is 3.0×10^{-8}.

Reflect

3. **Critical Thinking** When you write a number that is less than 1 in scientific notation, how does the power of 10 differ from when you write a number greater than 1 in scientific notation?

_____

YOUR TURN

Write each number in scientific notation.

4. 0.0000829

_____

5. 0.000000302

_____

6. A typical red blood cell in human blood has a diameter of approximately 0.000007 meter. Write this diameter in scientific notation. _____

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Writing a Number in Standard Notation

To translate between scientific notation and standard notation with very small numbers, you can move the decimal point the number of places indicated by the exponent on the power of 10. When the exponent is negative, move the decimal point to the left.

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EXAMPLE 2

Real World

Platelets are one component of human blood. A typical platelet has a diameter of approximately 2.33×10^{-6} meter. Write 2.33×10^{-6} in standard notation.

STEP 1 Use the exponent of the power of 10 to see how many places to move the decimal point. — 6 places

STEP 2 Place the decimal point. Since you are going to write a number less than 2.33, move the decimal point to the *left*. Add placeholder zeros if necessary. — 0.000002 33

The number 2.33×10^{-6} in standard notation is 0.00000233.

Math Talk
Mathematical Processes
Describe the two factors that multiply together to form a number written in scientific notation.

Reflect

7. **Justify Reasoning** Explain whether 0.9×10^{-5} is written in scientific notation. If not, write the number correctly in scientific notation.

_____

_____

_____

8. Which number is larger, 2×10^{-3} or 3×10^{-2}? Explain.

_____

YOUR TURN

Write each number in standard notation.

9. 1.045×10^{-6} _____

10. 9.9×10^{-5} _____

11. Jeremy measured the length of an ant as 1×10^{-2} meter. Write this length in standard notation.

_____

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Guided Practice

Write each number in scientific notation. (Explore Activity and Example 1)

1. 0.000487
Hint: Move the decimal right 4 places.

2. 0.000028
Hint: Move the decimal right 5 places.

3. 0.000059

4. 0.0417

5. Picoplankton can be as small as 0.00002 centimeter.

6. The average mass of a grain of sand on a beach is about 0.000015 gram.

Write each number in standard notation. (Example 2)

7. 2×10^{-5}
Hint: Move the decimal left 5 places.

8. 3.582×10^{-6}
Hint: Move the decimal left 6 places.

9. 8.3×10^{-4}

10. 2.97×10^{-2}

11. 9.06×10^{-5}

12. 4×10^{-5}

13. The average length of a dust mite is approximately 0.0001 meter. Write this number in scientific notation. (Example 1)

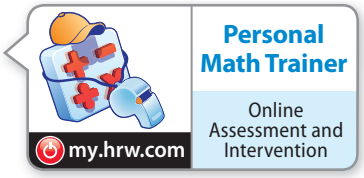
14. The mass of a proton is about 1.7×10^{-24} gram. Write this number in standard notation. (Example 2)

ESSENTIAL QUESTION CHECK-IN

15. Describe how to write 0.0000672 in scientific notation.

Name _____ Class _____ Date _____

2.3 Independent Practice



Use the table for problems 16–21. Write the diameter of the fibers in scientific notation.

Average Diameter of Natural Fibers	
Animal	**Fiber Diameter (cm)**
Vicuña	0.0008
Angora rabbit	0.0013
Alpaca	0.00277
Angora goat	0.0045
Llama	0.0035
Orb web spider	0.015

16. Alpaca

_____

17. Angora rabbit

_____

18. Llama

_____

19. Angora goat

_____

20. Orb web spider

_____

21. Vicuña

_____

22. **Make a Conjecture** Which measurement would be least likely to be written in scientific notation: the thickness of a dog hair, the radius of a period on this page, the ounces in a cup of milk? Explain your reasoning.

_____

_____

23. **Multiple Representations** Convert the length 7 centimeters to meters. Compare the numerical values when both numbers are written in scientific notation.

_____

_____

_____

_____

24. **Draw Conclusions** A graphing calculator displays 1.89×10^{12} as 1.89E12. How do you think it would display 1.89×10^{-12}? What does the E stand for?

_____

_____

25. **Communicate Mathematical Ideas** When a number is written in scientific notation, how can you tell right away whether or not it is greater than or equal to 1?

_____

_____

_____

26. The volume of a drop of a certain liquid is 0.000047 liter. Write the volume of the drop of liquid in scientific notation.

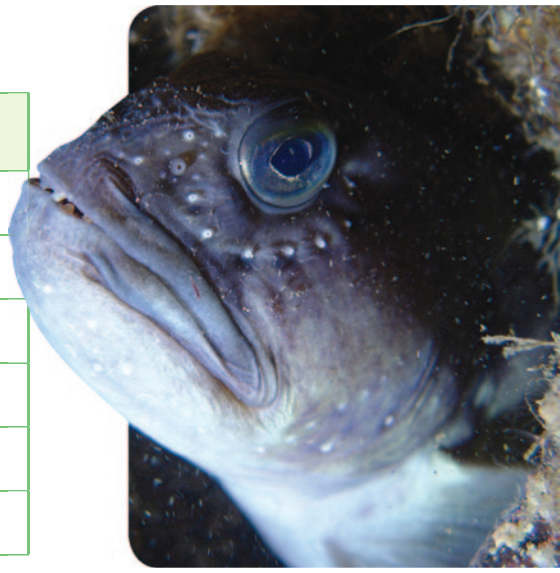
_____

27. **Justify Reasoning** If you were asked to express the weight in ounces of a ladybug in scientific notation, would the exponent of the 10 be positive or negative? Justify your response.

_____

_____

Physical Science **The table shows the length of the radii of several very small or very large items. Complete the table.**



	Item	Radius in Meters (Standard Notation)	Radius in Meters (Scientific Notation)
28.	The Moon	1,740,000	
29.	Atom of silver		1.25×10^{-10}
30.	Atlantic wolfish egg	0.0028	
31.	Jupiter		7.149×10^{7}
32.	Atom of aluminum	0.000000000182	
33.	Mars		3.397×10^{6}

34. List the items in the table in order from the smallest to the largest.

_____

_____

H.O.T. FOCUS ON HIGHER ORDER THINKING

35. **Analyze Relationships** Write the following diameters from least to greatest.
1.5×10^{-2} m 1.2×10^{2} m 5.85×10^{-3} m 2.3×10^{-2} m 9.6×10^{-1} m

_____

_____

36. **Critique Reasoning** Jerod's friend Al had the following homework problem:

Express 5.6×10^{-7} in standard form.

Al wrote 56,000,000. How can Jerod explain Al's error and how to correct it?

_____

_____

_____

37. **Make a Conjecture** Two numbers are written in scientific notation. The number with a positive exponent is divided by the number with a negative exponent. Describe the result. Explain your answer.

_____

_____

_____

Work Area

LESSON 2.4

Operations with Scientific Notation

8.1.2.4

Students will add, subtract, multiply, and divide using scientific notation.

ESSENTIAL QUESTION

How do you add, subtract, multiply, and divide using scientific notation?

Adding and Subtracting with Scientific Notation

Numbers in scientific notation can be added and subtracted, either directly or by rewriting them in standard form.

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EXAMPLE 1

Real World

The table below shows the population of the three largest countries in North America in 2011. Find the total population of these countries.

Country	United States	Canada	Mexico
Population	3.1×10^8	3.38×10^7	1.1×10^8

Method 1:

STEP 1 First, write each population with the same power of 10.

United States: 3.1×10^8

Canada: 0.338×10^8

Mexico: 1.1×10^8

STEP 2 Add the multipliers for each population.

$3.1 + 0.338 + 1.1 = 4.538$

STEP 3 Write the final answer in scientific notation: 4.538×10^8.

Method 2:

STEP 1 First, write each number in standard notation.

United States: 310,000,000

Canada: 33,800,000

Mexico: 110,000,000

STEP 2 Find the sum of the numbers in standard notation.

$310{,}000{,}000 + 33{,}800{,}000 + 110{,}000{,}000 = 453{,}800{,}000$

STEP 3 Write the answer in scientific notation: 4.538×10^8.

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YOUR TURN

1. Using the population table above, how many more people live in Mexico than in Canada? Write your answer in scientific notation.

_____

Multiplying and Dividing with Scientific Notation

Numbers in scientific notation can be multiplied and divided directly by using properties of exponents.

EXAMPLE 2 Problem Solving

When the Sun makes an orbit around the center of the Milky Way, it travels 2.025×10^{14} kilometers. The orbit takes 225 million years. At what rate does the Sun travel? Write your answer in scientific notation.

Analyze Information

The answer is the number of kilometers per year that the Sun travels around the Milky Way.

Formulate a Plan

Set up a division problem using $\text{Rate} = \frac{\text{Distance}}{\text{Time}}$ to represent the situation.

Solve

STEP 1 Substitute the values from the problem into the Rate formula.

$$\text{Rate} = \frac{2.025 \times 10^{14} \text{ kilometers}}{225{,}000{,}000 \text{ years}}$$

STEP 2 Write the expression for rate with years in scientific notation.

$$\text{Rate} = \frac{2.025 \times 10^{14} \text{ kilometers}}{2.25 \times 10^{8} \text{ years}}$$

$225 \text{ million} = 2.25 \times 10^8$

STEP 3 Find the quotient by dividing the decimals and using the laws of exponents.

$2.025 \div 2.25 = 0.9$ Divide the multipliers.

$\frac{10^{14}}{10^{8}} = 10^{14-8} = 10^{6}$ Divide the powers of 10.

STEP 4 Combine the answers to write the rate in scientific notation.

$\text{Rate} = 0.9 \times 10^6 = 9.0 \times 10^5$ km per year

Justify and Evaluate

Check your answer using multiplication.

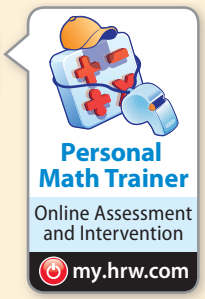
$900{,}000 \times 225{,}000{,}000 = 202{,}500{,}000{,}000{,}000$, or 2.025×10^{14}.
The answer is correct.

Math Talk
Mathematical Processes
Could you write 2.025×10^{14} in standard notation to do the division? Would this be a good way to solve the problem?

YOUR TURN

2. Light travels at a speed of 1.86×10^5 miles per second. It takes light from the Sun about 4.8×10^3 seconds to reach Saturn. Find the approximate distance from the Sun to Saturn. Write your answer in scientific notation. _____

3. Light travels at the speed of 1.17×10^7 miles per minute. Pluto's average distance from the Sun is 3,670,000,000 miles. On average, how long does it take sunlight to reach Pluto? Write your answer in scientific notation. _____



Scientific Notation on a Calculator

On many scientific calculators, you can enter numbers in scientific notation by using a function labeled "ee" or "EE". Usually, the letter "E" takes the place of "$\times 10$". So, the number 4.1×10^9 would appear as 4.1E9 on the calculator.

EXAMPLE 3

Real World

The table shows the approximate areas for three continents given in square meters. What is the total area of these three continents? Write the answer in scientific notation using more appropriate units.

Continent	Asia	Africa	Europe
Area (m^2)	4.4×10^{13}	3.02×10^{13}	1.04×10^{13}

Find $4.4 \times 10^{13} + 3.02 \times 10^{13} + 1.04 \times 10^{13}$.

Enter 4.4E13 + 3.02E13 + 1.04E13 on your calculator.

Write the results from your calculator: 8.46E13.

Write this number in scientific notation: 8.46×10^{13} m^2.

Square kilometers is more appropriate: 8.46×10^7 km^2.

Because 1 km = 1,000 m, 1 km^2 = $1{,}000^2$ m^2, or 10^6 m^2. Divide by 10^6.

YOUR TURN

Write each number using calculator notation.

4. 7.5×10^5 _____

5. 3×10^{-7} _____

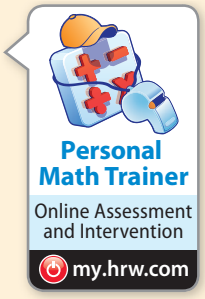
6. 2.7×10^{13} _____

Write each number using scientific notation.

7. 4.5E−1 _____

8. 5.6E12 _____

9. 6.98E−8 _____



Guided Practice

Add or subtract. Write your answer in scientific notation. (Example 1)

1. $4.2 \times 10^6 + 2.25 \times 10^5 + 2.8 \times 10^6$

$4.2 \times 10^6 + \square \times 10^{\square} + 2.8 \times 10^6$

$4.2 + \square + \square$

$\square \times 10^{\square}$

2. $8.5 \times 10^3 - 5.3 \times 10^3 - 1.0 \times 10^2$

$8.5 \times 10^3 - 5.3 \times 10^3 - \square \times 10^{\square}$

$\square - \square - \square$

$\square \times 10^{\square}$

3. $1.25 \times 10^2 + 0.50 \times 10^2 + 3.25 \times 10^2$

_____

4. $6.2 \times 10^5 - 2.6 \times 10^4 - 1.9 \times 10^2$

_____

Multiply or divide. Write your answer in scientific notation. (Example 2)

5. $(1.8 \times 10^9)(6.7 \times 10^{12})$ _____

6. $\frac{3.46 \times 10^{17}}{2 \times 10^9}$ _____

7. $(5 \times 10^{12})(3.38 \times 10^6)$ _____

8. $\frac{8.4 \times 10^{21}}{4.2 \times 10^{14}}$ _____

Write each number using calculator notation. (Example 3)

9. 3.6×10^{11} _____

10. 7.25×10^{-5} _____

11. 8×10^{-1} _____

Write each number using scientific notation. (Example 3)

12. 7.6E−4 _____

13. 1.2E16 _____

14. 9E1 _____

ESSENTIAL QUESTION CHECK-IN

15. How do you add, subtract, multiply, and divide numbers written in scientific notation?

_____

_____

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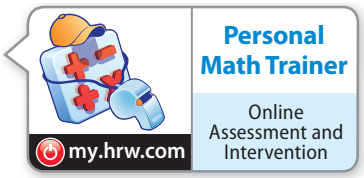
_____

_____

Name _____ Class _____ Date _____

2.4 Independent Practice



16. An adult blue whale can eat 4.0×10^7 krill in a day. At that rate, how many krill can an adult blue whale eat in 3.65×10^2 days?

17. A newborn baby has about 26,000,000,000 cells. An adult has about 4.94×10^{13} cells. How many times as many cells does an adult have than a newborn? Write your answer in scientific notation.

Represent Real-World Problems **The table shows the number of tons of waste generated and recovered (recycled) in 2010.**

	Paper	Glass	Plastics
Tons generated	7.131×10^7	1.153×10^7	3.104×10^7
Tons recovered	4.457×10^7	0.313×10^7	0.255×10^7

18. What is the total amount of paper, glass, and plastic waste generated?

19. What is the total amount of paper, glass, and plastic waste recovered?

20. What is the total amount of paper, glass, and plastic waste ***not*** recovered?

21. Which type of waste has the lowest recovery ratio?

Social Studies **The table shows the approximate populations of three countries.**

Country	China	France	Australia
Population	1.3×10^9	6.48×10^7	2.15×10^7

22. How many more people live in France than in Australia?

23. The area of Australia is 2.95×10^6 square miles. What is the approximate average number of people per square mile in Australia?

24. How many times greater is the population of China than the population of France? Write your answer in standard notation.

25. Mia is 7.01568×10^6 minutes old. Convert her age to more appropriate units using years, months, and days. Assume each month to have 30.5 days.

26. Courtney takes 2.4×10^4 steps during her a long-distance run. Each step covers an average of 810 mm. What total distance (in mm) did Courtney cover during her run? Write your answer in scientific notation. Then convert the distance to the more appropriate unit kilometers. Write that answer in standard form.

27. **Social Studies** The U.S. public debt as of October 2010 was $\$9.06 \times 10^{12}$. What was the average U.S. public debt per American if the population in 2010 was 3.08×10^8 people?

H.O.T. FOCUS ON HIGHER ORDER THINKING

28. **Communicate Mathematical Ideas** How is multiplying and dividing numbers in scientific notation different from adding and subtracting numbers in scientific notation?

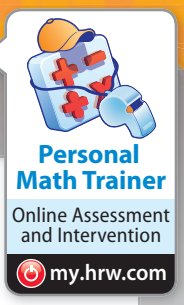
29. **Explain the Error** A student found the product of 8×10^6 and 5×10^9 to be 4×10^{15}. What is the error? What is the correct product?

30. **Communicate Mathematical Ideas** Describe a procedure that can be used to simplify $\frac{\left(4.87 \times 10^{12}\right) - \left(7 \times 10^{10}\right)}{\left(3 \times 10^7\right) + \left(6.1 \times 10^8\right)}$. Write the expression in scientific notation in simplified form.

Work Area

MODULE QUIZ

Ready to Go On?



2.1 Integer Exponents

Find the value of each power.

1. 3^{-4} _____ **2.** 35^{0} _____ **3.** 4^{4} _____

Use the properties of exponents to write an equivalent expression.

4. $8^{3} \cdot 8^{7}$ _____ **5.** $\frac{12^{6}}{12^{2}}$ _____ **6.** $(10^{3})^{5}$ _____

2.2 Scientific Notation with Positive Powers of 10

Convert each number to scientific notation or standard notation.

7. 2,000 _____ **8.** 91,007,500 _____

9. 1.0395×10^{9} _____ **10.** 4×10^{2} _____

2.3 Scientific Notation with Negative Powers of 10

Convert each number to scientific notation or standard notation.

11. 0.02 _____ **12.** 0.000701 _____

13. 8.9×10^{-5} _____ **14.** 4.41×10^{-2} _____

2.4 Operations with Scientific Notation

Perform the operation. Write your answer in scientific notation.

15. $7 \times 10^{6} - 5.3 \times 10^{6}$ _____ **16.** $3.4 \times 10^{4} + 7.1 \times 10^{5}$ _____

17. $(2 \times 10^{4})(5.4 \times 10^{6})$ _____ **18.** $\frac{7.86 \times 10^{9}}{3 \times 10^{4}}$ _____

19. Neptune's average distance from the Sun is 4.503×10^{9} km. Mercury's average distance from the Sun is 5.791×10^{7} km. About how many times farther from the Sun is Neptune than Mercury? Write your answer in scientific notation.

_____

ESSENTIAL QUESTION

20. How is scientific notation used in the real world?

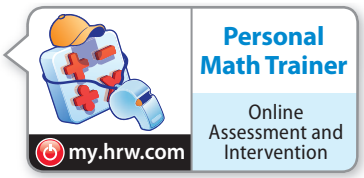
_____

_____

_____

MODULE 2 MIXED REVIEW

Assessment Readiness



Selected Response

1. Which of the following is equivalent to 6^{-3}?

Ⓐ 216 Ⓒ $-\frac{1}{216}$

Ⓑ $\frac{1}{216}$ Ⓓ -216

2. About 786,700,000 passengers traveled by plane in the United States in 2010. What is this number written in scientific notation?

Ⓐ $7{,}867 \times 10^5$ passengers

Ⓑ 7.867×10^2 passengers

Ⓒ 7.867×10^8 passengers

Ⓓ 7.867×10^9 passengers

3. In 2011, the population of Mali was about 1.584×10^7 people. What is this number written in standard notation?

Ⓐ 1.584 people

Ⓑ 1,584 people

Ⓒ 15,840,000 people

Ⓓ 158,400,000 people

4. The square root of a number is between 7 and 8. Which could be the number?

Ⓐ 72 Ⓒ 51

Ⓑ 83 Ⓓ 66

5. Each entry-level account executive in a large company makes an annual salary of $\$3.48 \times 10^4$. If there are 5.2×10^2 account executives in the company, how much do they make in all?

Ⓐ $\$6.69 \times 10^1$

Ⓑ $\$3.428 \times 10^4$

Ⓒ $\$3.532 \times 10^4$

Ⓓ $\$1.8096 \times 10^7$

6. Place the numbers in order from least to greatest.

$0.24, 4 \times 10^{-2}, 0.042, 2 \times 10^{-4}, 0.004$

Ⓐ $2 \times 10^{-4}, 4 \times 10^{-2}, 0.004, 0.042, 0.24$

Ⓑ $0.004, 2 \times 10^{-4}, 0.042, 4 \times 10^{-2}, 0.24$

Ⓒ $0.004, 2 \times 10^{-4}, 4 \times 10^{-2}, 0.042, 0.24$

Ⓓ $2 \times 10^{-4}, 0.004, 4 \times 10^{-2}, 0.042, 0.24$

7. Guillermo is $5\frac{5}{6}$ feet tall. What is this number of feet written as a decimal?

Ⓐ 5.7 feet Ⓒ 5.83 feet

Ⓑ $5.\overline{7}$ feet Ⓓ $5.8\overline{3}$ feet

8. A human hair has a width of about 6.5×10^{-5} meter. What is this width written in standard notation?

Ⓐ 0.00000065 meter

Ⓑ 0.0000065 meter

Ⓒ 0.000065 meter

Ⓓ 0.00065 meter

Mini-Task

9. Consider the following numbers: 7000, 700, 70, 0.7, 0.07, 0.007

a. Write the numbers in scientific notation.

_____

_____

b. Look for a pattern in the given list and the list in scientific notation. Which numbers are missing from the lists?

_____

c. Make a conjecture about the missing numbers.

_____

_____