# Phase 2: Reasoning Strategies for Addition & Subtraction

Phase 2 is critical for focusing on the acquisition of basic math facts. This phase involves the students in seeking efficient strategies and acquiring the ability to look at numbers and operations in many equivalent ways. Students continually look for ways to use known facts to solve facts that they have not yet retained. Students identify various relationships both between strategies and between operations. Students also develop the properties for addition and subtraction.

## The Role of Relationships in Learning Basic Facts

All facts are conceptually related. Students can derive new or unknown facts from those that they already know. "All children can construct efficient mental tools that will help them. Number relationships provide the foundation for strategies that help students remember basic facts" (Van de Walle, 2007, p. 165). For example, knowing how numbers are related to 5 and 10 allows the student to solve 7 + 5 (since 7 is 3 away from 10, take 3 from 5 to make 10 + 2 = 12).

"Fact mastery relies significantly on how well students have constructed relationships on numbers and how well they understand the operations (Phase 1)" (Van de Walle, 2007, p. 165). Students should learn or be taught a collection of strategies or thought patterns for various groups of basic facts. Students are guided to develop strategies that are useful and meaningful to them, although not all students will develop the same approaches. Class discussions following the use of manipulatives are vital for students to construct relationships and clarify their understanding. In these discussions, students may represent, display, explain, clarify, and argue for their solutions. Through the discussions, "the explicit construction of links between understood actions on the objects and related symbol procedures" are established (Ma, 1999, p. 20).

> "Children who commit the facts to memory easily are able to do this because they have constructed relationships among them and between addition and subtraction in general, and they use these relationships as shortcuts. Memorizing facts with flashcards or through drills and practice worksheets will not develop these relationships" (Fosnot & Dolk, 2001, p. 98).

When relationships are the focus, students have far fewer facts to remember and have a quick way to come up with the answers (Fosnot & Dolk, 2001).

## The Role of Invented Strategies

> "Invented strategies are flexible methods of computing that vary with the numbers and the situation. Successful use of the strategies requires that they be understood by the one that is using them—hence, the term *invented*. Strategies may be invented by a peer or the class as a whole; they may even be suggested by the teacher. However, they must be constructed by the student" (Van de Walle, 2007, p. 216).

“Not all students invent their own strategies. Strategies invented by classmates are shared, explored, and tried out by others” (Van de Walle, 2007, p 218). There is no sequence for teachers to follow that will dictate the order problems should be posed to students. Teachers must learn to listen to the kind of reasoning that students are using and the strategies that are being suggested during classroom discussions. Teachers will discover many variations of thought processes among the students (Van de Walle, 2007).

“When students attempt to apply a strategy that they have not invented, they are essentially following rules rather than thinking about number relationships…Furthermore, when students are working on a specific strategy drill, they often forget to use a strategy and revert to inefficient counting methods. When strategies are imposed on students, they often develop these ideas in a rote manner rather then integrating a strategy with their own ideas” (Van de Walle, 2007, p. 166).

In order for teachers to guide students to the invention of effective strategies, teachers need to have a command of as many good strategies as possible, even if the teachers have never used some of the strategies themselves. With this knowledge, teachers will be able to recognize effective strategies as their students develop them and help other students capitalize on these new ideas as they are shared within the classroom (Van de Walle, 2007).

Teachers need to plan lessons in which specific strategies are likely to be developed. Using contextual problems (story problems) designed in such a manner that students develop a strategy as they solve it, is one important way. “Don’t expect to have a strategy introduced and understood with just one story problem or one exposure. Children need many opportunities to make a strategy their own. No student should be forced to adopt someone else’s strategy, but every student should be required to understand strategies that are brought to the discussion” (Van de Walle, 2007, p. 167).

Another approach involves teachers creating a lesson that examines a specific set of facts for which a particular strategy is appropriate. Using this approach, teachers can discuss how all of the facts are alike or suggest an approach to see if students are able to use it with similar facts. “There is a huge temptation to simply tell students about a strategy and then have them practice it. Though this can be effective for some students, many others will not personally relate to (the teacher’s) ideas or may not be ready for them. Continue to discuss the strategies that are invented in class and plan lessons that encourage strategies” (Van de Walle, 2007, p.167).

**Assumptions:**
Students will be able to:

- Count forward and backward to 20
- Count 20 objects and know that the last number word tells how many
- Read and write the numerals 0 to 20
- State the number before and after up to 20

# Reasoning Strategies for Addition Facts

> "Children do not learn facts all at once, and they use selected number facts and derived facts (relationships between numbers and known facts) at the direct modeling and counting levels. Although not all children use derived facts consistently, derived facts play an important role in solving problems and in learning number facts at a recall level. It is much easier for a child to learn to recall number facts if the child understands the relationships among number facts" (Carpenter et al., 1999, p. 30). "Class discussion on such derived-fact strategies helps students learn from their peers and also legitimizes the use of strategies, thus encouraging the invention of further strategies. Class discussion should examine the relative advantages of different strategies for various problems" (Isaacs & Carroll, 1999, p.510)

**One-More Than and Two-More Than Facts**
Join or Part-Part-Whole problem structure problems, in which one of the addends is a 1 or 2, can be used in the classroom as a direct application of this relationship. Students should explain how they got their answer. These responses provide the teacher with much information about students' number sense. Teacher should be prepared for the opportunity to talk about this idea of one-more or two-more than as students share. (Van de Walle, 2007).

**Facts with Zero (Identity or Zero Property)**
When you add zero to any number, the sum is that same number, for example, 2 + 0 = 2. Facts with zero problems are typically easy for students. Some students will over-generalize the idea that answers to addition are bigger. Using story problems involving zero are helpful to explain these problems. Teachers should use drawings that show two parts with one part empty (Van de Walle, 2007).

**Doubles**
There are only 10 double facts for students to learn from 0 + 0 to 9 + 9. These facts are relatively easy for students to learn and become a powerful anchor for students to learn other facts, such as near double facts. When doubling a number, a student can also count by twos. For example 8 + 8 is equivalent to eight twos or 16. This underlies the relationship between even and odd numbers (Van de Walle, 2007).

**Near Doubles**
Near doubles are also called the "doubles-plus-one" or "doubles-minus-one" facts. All combinations where one addend is one more than the other are included in this group. The strategy involves adding one after doubling the smaller number or subtracting one after doubling the larger number. For example 4 + 5, start by doubling 4, then add one to 8 to get 9. A student can also start by doubling 5, then subtract one from 10 to get 9. Students must have a command of the doubles strategy before focusing on these facts (Van de Walle, 2007).

**Make-Ten Facts**
Students should be able to think of the numbers 11 through 18 as ten and some more. All of these facts have at least one addend of 8 or 9. This strategy has students build onto the 8 or 9 up to 10 and then add the rest. For example 9 + 3, start with the 9, then take 1 from the 3 to make

10, and that leaves 2 more for 12. Knowing the combinations that make ten is critical for children to be able to solve problems like 7 + 4 by making a ten (7 + 3) and then adding one. Otherwise they will just count on from seven using their fingers. Students may have other ways of using 10 to add with addends of 8 or 9. For example, with the fact 9 + 7, some students may solve this fact by adding 10 + 7 and subtract 1. This is an acceptable strategy for making ten. Tens frames and chips activities help students see this connection to ten (Van de Walle, 2007).

### The Last Six Facts

The previous strategies (One or Two More Than, Zeros, Doubles, Near Doubles, and Make-Ten) cover 88 of the 100 addition facts. These strategies are applications of important number relationships. There are 12 remaining facts that really are 6 facts and their respective "turnarounds" (Van de Walle, 2007, p.172).

### Order Property (Commutative)

The order property (or commutative property) for addition says that it "makes no difference in which order two numbers are added. Although the order property may seem obvious to us (simply reverse the two piles of counters on the part-part-whole mat), it may not be obvious to children. Since it is quite useful in problem solving, mastering basic facts, and mental mathematics, there is value in spending some time helping children construct the relationship. The name of the property is not important, but the idea is" (Van de Walle, 2007, p. 149).

## Strategies for Subtraction Facts

Subtraction facts have been proven to be more difficult for students to learn over addition facts. Students are often taught a "count-count-count" approach for subtraction. For example 12 – 4, count 12, count off and remove 4, count what's left (8). There is little evidence that students find this approach helpful in mastering their subtraction facts. They need other more efficient strategies (Van de Walle, 2007).

"'Think-Addition' is the most powerful way to think of subtraction facts. Rather than 13 'take away 6', which requires counting backward while also keeping track of how many counts, students can think 6 and what makes 13. They might add up to 10 or they may think double 6 is 12 so it must be 13" (Van de Walle, 2007, p. 165).

One big idea that children need to develop is that subtraction and addition are related. Children need to know that either operation can be used. Further, children need to understand that when in context the comparison and removal problem structures can both involve subtraction.

### Think Addition

There is an important relationship between parts and wholes, as well as, between addition and subtraction that makes learning subtraction facts easier for students. When students see 8 – 2, you want them to think spontaneously, "Two and what makes 8?" Story problems that promote think-addition are those that sound like addition story problems but have a missing addend (Van de Walle, 2007).

If Think-Addition is to be used efficiently by students, it is essential that students learn their addition facts. There is evidence suggesting that students learn very few subtraction facts, without first mastering the corresponding addition facts. Mastery of 4 + 5 can be thought of as a prerequisite knowledge for learning the subtraction facts of 9 – 5 and 9 – 4 (Van de Walle, 2007).

If students struggle with facts involving 0, 1, and 2 (examples 7 – 2, 5 – 1, 6 – 0), it would be important for the teacher to investigate their number concepts. These subtraction facts are closely related to basic number relationships and too often a child that says 8 – 0 is 7 may have over generalized that subtraction means making smaller (Van de Walle, 2007). Comparison situations involve two distinct sets or quantities and the difference between them. The same kind of model can be used whether the difference or one of the two quantities is unknown. Some children may on their own construct the idea of comparison as a part-part-whole problem structure.

**Strategies for Sums Greater than 10**

Of the 36 hard subtraction facts with sums of 11 or more, there are different strategies that children may use to solve these facts. Van de Walle (2007) mentions three strategies that build upon the relationship between addition and subtraction and 10 as an anchor (Build Up Through 10, Think-Addition, and Back Down Through 10) to help students with these facts. These strategies are not "required" strategies, as children may use think-addition or different strategies to solve these facts (p.174-175).