Phase 1: Constructing Meaning and Counting Strategies for Multiplication and Division

Phase I of Multiplication and Division is designed for children who have constructed meaning and have developed understanding of addition and related subtraction. Children not demonstrating the assumptions listed should continue to work in the phases of addition and subtraction.

Assumptions: Children ready for phase 1 of multiplication and division should demonstrate an understanding of:

- Number recognition to 81
- One to one correspondence
- Decomposing two digit numbers
- Constructing sets
- Meaning of number
- Understanding of zero
- How to navigate a number line moving forwards and backwards

Constructing meaning and counting strategies is about helping children connect different meanings, interpretations, and relationships to multiplication and division so that they can effectively use these operations in real-world situations. This phase involves students developing a deep understanding of operation and the many different, but related, meanings these operations take on in contexts.

As in addition and subtraction, ways of counting objects remains important in multiplication and division. For most situations in the early grades, each object is counted. The objects are counted together or separated to solve problems. Children have learned to count ten objects, one by one and are beginning to understand this is a group. This idea of ten objects becoming one group of ten is called unitizing. "Unitizing requires that children use number to count not only objects but also groups - and to count them simultaneously." (Fosnot, 2001, p. 11) Children need to shift from their thought process of cardinality (the last object counted and the last number spoken was the quantity) to unitizing (how many groups and how many in each group). "When children are attempting to understand "how many", the initial strategy they use is most often counting" (Fosnot, 2001, p. 34). Counting by ones becomes an inefficient strategy for solving multiplication and division problems. "Experience with making and counting groups, especially in contextual situations, are extremely helpful" (Van De Walle, 2007, p. 154). Forcing students to utilize a more efficient strategy that they have not discovered for themselves may actually interfere with their construction of meaning.

Constructing meaning of Multiplication/Division

Multiplication and division involve making/counting groups of equal size. Understanding how to count objects and groups is necessary for children in order to grasp multiplication and division. The whole is now seen, "as a number of groups of a number of objects - for example, four groups of six, or 4 X 6. The parts together become the new whole, and the parts (the groups) and the whole can be considered simultaneously. The relationship of these parts to the whole explains the reciprocal relationship between division and multiplication" (Fosnot, 2001, p. 36). When you know two parts, you can find the whole and when you know the whole and one part, you can find the other part.

Constructing Meaning of Multiplication/Division Problem Structures

All multiplication and division problems fall into two categories: equal groups and comparison:

> ***Equal groups***
> These are the most common problems that we use - putting things into equal groups or separating amounts into equal groups.
> ***Comparison***
> The other type of problems involve two different sets, one set consists of multiple copies of the other, one set being a particular multiple of the other or how many times larger is one than the other.

Teaching Multiplication and Division

"It remains important to use contextual problems whenever reasonable instead of more sterile story problems" (Van De Walle, 2007, p. 154). Effective instruction will begin by using contextual problems. Contextual problems should arise from children's everyday lives (field trips, ballgames, classroom experiences, community events) (Van de Walle, 2007). Teachers should use both multiplication and division simultaneously in order to show the relationship between the two. "…understanding the connection between multiplication and division is critical to understanding the part/whole relationship in the multiplicative structure" (Fosnot, 2007, p. 51). Teachers should only work on a few problems each time and discuss student thinking. During initial instruction, students usually apply direct modeling or counting strategies. In direct modeling, students "Act out" each part of the situation from the problem by using manipulatives or pictures. This is a natural strategy most students use to solve mathematical problems. Some students will use a more efficient strategy such as counting on, skip counting or repeated addition. "When children begin to form groups, they often use skip counting or repeated addition strategies to calculate the whole" (Fosnot, 2001, p. 49).

> Story problems are often designed with little context and written to get children to do the procedures that the teacher wants them to do. "Context problems, on the other hand, are connected as closely as possible to children's lives, rather than 'school mathematics.' They are designed to anticipate and develop children's mathematical modeling of the real world."
> Fosnot and Dolk (2001) p. 24

During the constructing meaning phase, it is important for students to be able to visualize the problem. Students need to be able to model what they know about the problem. Fosnot explains models as, …representations of relationships that mathematicians have constructed over time as they have reflected on how one thing can be changed into another and as they have generalized ideas, strategies, and representations across contexts. Visual models for multiplication and division are hundreds charts, number lines, sets and arrays.

Symbolic representation for multiplication can occur naturally for some children as repeated addition. If this does not occur the teacher needs to assist students in making this connection. Symbolic representation should continue to be linked to pictorial representation and models as long as students need this support.
This is the time to introduce the multiplication/division signs and to explain what they mean.

Note: Problems involving remainders should not be avoided. Dealing with them is a natural part of division. In real life, most division problems have remainders and need to be dealt with in context. Children need to encounter lots of problems in which the remainder can be interpreted differently.

Multiplication is represented by

factor x factor equals product.

Note: The usual convention is that 4 X 8 refers solely to four sets of eight. This convention is rigid and doesn't allow for other interpretations. 4 X 8 can be read "four sets of 8 objects", "four eights", "four objects in 8 sets" and many other ways as the context changes. (Van de Walle, p. 154)

Division can be represented in three different ways.

$24 \div 6$

$\frac{24}{6}$

$6\overline{)24}$

Dividend divided by divisor equals quotient

Caution: Avoid the Key Word Strategy

Often children are taught to look for "key words" in story problems. Some classrooms even post lists of key words with their corresponding meanings. When problems are written in a overly simple and formulaic way, this approach may appear effective. However, this can be a misleading strategy for children for several reasons:

- The key word could indicate an incorrect operation. An example might be that a child may see the word "left" in a problem and automatically assume that subtraction is the correct operation.
- Many problems have no key words.
- A child who relies on key words is left with no strategy to solve the problem. The child simply pulls out the numbers from the problem and becomes very procedural losing the context of the problem.
- Using problems in context allows children to make sense of number and operation.
- Using key words does not allow children to make sense of number and operation in a problem.

The most important approach to solving contextual problems is "to analyze the structure and make sense of it" (Van de Walle, 2006, p160).

References