Why Is Teaching With Problem Solving Important to Student Learning?

Problem solving plays an important role in mathematics and should have a prominent role in the mathematics education of K−12 students. However, knowing how to incorporate problem solving meaningfully into the mathematics curriculum is not necessarily obvious to mathematics teachers. (The term "problem solving" refers to mathematical tasks that have the potential to provide intellectual challenges for enhancing students' mathematical understanding and development.) Fortunately, a considerable amount of research on teaching and learning mathematical problem solving has been conducted during the past 40 years or so and, taken collectively; this body of work provides useful suggestions for both teachers and curriculum writers. The following brief provides some directions on teaching with problem solving based on research.

What kinds of problem-solving activities should students be given?

Story or word problems often come to mind in a discussion about problem solving. However, this conception of problem solving is limited. Some "story problems" are not problematic enough for students and hence should only be considered as exercises for students to perform. For example, students may be asked to find the perimeter of a polygon, given the length of each side. They can mindlessly add these numbers and get the answer without understanding the concept of perimeter and the problem situation. However, some nonstory problems can be true problems, such as those found, for example, while playing mathematical games.

In general, when researchers use the term *problem solving* they are referring to mathematical tasks that have the potential to provide intellectual challenges that can enhance students' mathematical development. Such tasks—that is, problems— can promote students' conceptual understanding, foster their ability to reason and communicate mathematically, and capture their interests and curiosity (Hiebert & Wearne, 1993; Marcus & Fey, 2003; NCTM, 1991; van de Walle, 2003). Research recommends that students should be exposed to truly problematic tasks so that mathematical sense making is practiced (Marcus & Fey, 2003; NCTM, 1991; van de Walle, 2003). Mathematical problems that are truly problematic and involve significant mathematics have the potential to provide the intellectual contexts for students' mathematical development. However, only "worthwhile problems" give students the chance to solidify and extend what they know and stimulate mathematics learning. That said, what is a worthwhile problem? Regardless of the context, worthwhile tasks should be intriguing and contain a level of challenge that invites speculation and hard work. Most important, worthwhile mathematical tasks should direct students to investigate important mathematical ideas and ways of thinking toward the learning goals (NCTM, 1991). Lappan and Phillips (1998) developed a set of criteria for a good problem that they used to develop their middle school mathematics curriculum (Connected Mathematics), and there has been some research supporting the effectiveness of this curriculum for fostering students' conceptual understanding and problem solving (Cai, Moyer, Wang, & Nie, in press). Although there has been no research focusing specifically on the effectiveness of this set of criteria, the fact that the curriculum as a whole has been shown to be effective suggests that teachers might want to attend to this set in choosing, revising, and designing problems. See the following worthwhile-problem criteria:

1. The problem has important, useful mathematics embedded in it.
2. The problem requires higher-level thinking and problem solving.
3. The problem contributes to the conceptual development of students.
4. The problem creates an opportunity for the teacher to assess what his or her students are learning and where they are experiencing difficulty.
5. The problem can be approached by students in multiple ways using different solution strategies.
6. The problem has various solutions or allows different decisions or positions to be taken and defended.
7. The problem encourages student engagement and discourse.
8. The problem connects to other important mathematical ideas.
9. The problem promotes the skillful use of mathematics.

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10. The problem provides an opportunity to practice important skills.

Of course, it is not reasonable to expect that every problem that a teacher chooses should satisfy all 10 criteria; which criteria to consider should depend on a teacher's instructional goals. For example, some problems are used primarily because they provide students with an opportunity to practice a certain skill (criterion 10), say, solving a proportion, whereas others are used primarily to encourage students to collaborate with one another and justify their thinking (criteria 6 and 7). But researchers and curriculum developers alike tend to agree that the first four criteria (important mathematics, higher-level thinking, conceptual development, and opportunity to assess learning) should be considered essential in the selection of all problems. Indeed, these four can be regarded as the sine qua non of the criteria. The real value of these criteria is that they provide teachers with guidelines for making decisions about how to make problem solving a central aspect of their instruction.

The role of teachers is to revise, select, and develop tasks that are likely to foster the development of understandings and mastery of procedures in a way that also promotes the development of abilities to solve problems and reason and communicate mathematically (NCTM, 1991). The following example illustrates how a teacher can modify a standard textbook problem in a way that both engages students in learning important mathematics (criterion 1) and also enhances the development of their problem-solving abilities (criteria 2, 3, 4, and 5).

EXAMPLE. Original problem (Cai & Nie, 2007) (Grades 9−11): In the figure below, segment AB is parallel to segment CD. Show that the sum of the measures of $\angle A$, $\angle E$, and $\angle C$ is 360°.
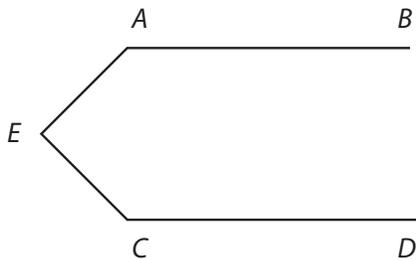
This problem might be found in any standard textbook. It clearly involves important mathematics, but in its present form, criteria 2, 3, 4, and 5 are not as clearly included. By making a quite modest revision, we can open up the problem and by doing so raise the cognitive demand (criterion 2) and also satisfy criteria 3 and 4: *Revised problem:* What is the

sum of the measures of $\angle A$, $\angle E$, and $\angle C$? In addition, we might ask students to find the sum of the three angles in different ways and make generalization of the problem by asking: What is the sum of the three angle measures if point E is at different locations (as shown in the figures below)?
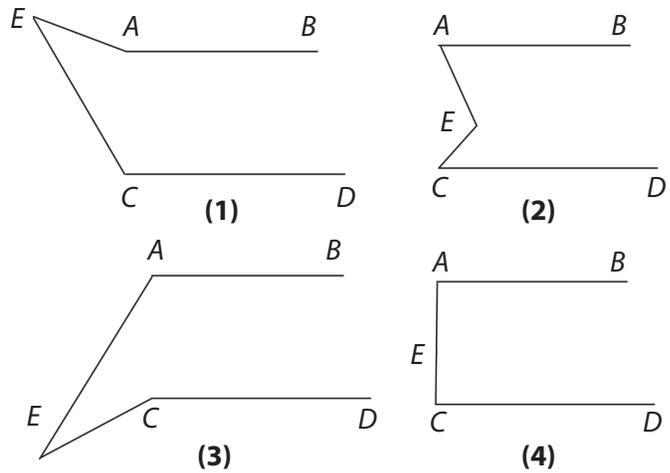
This example illustrates that modifying problems that already exist in textbooks is often a relatively easy thing to do but increases the learning opportunity for students. Indeed, the revised problems need not be complicated or have a fancy format. Readers may also see (Butts, 1980) how to revise a problem to be more problematic so that the learning opportunity for students is increased.

Should problem solving be taught as a separate topic in the mathematics curriculum or should it be integrated throughout the curriculum?

There is little or no evidence that students' problem-solving abilities are improved by isolating problem solving from learning mathematics concepts and procedures. That is, the common approach of first teaching the concepts and procedures, then assigning one-step "story" problems that are designed to provide practice on the content learned, then teaching problem solving as a collection of strategies such as "draw a picture" or "guess and check," and finally, if time, providing students with applied problems that will require the mathematics learned in the first step (Lesh & Zawojewski, 2007, p. 765), is not supported by research. In fact, the evidence has mounted over the past 30 years that such an approach does not improve students' problem solving to the point that today

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no research is being conducted with this approach as an instructional intervention (e.g., Begle, 1973; Charles & Silver, 1988; Lester, 1980; Schoenfeld, 1979). The implication of this change in perspective is that if we are to help students become successful problem solvers, we first need to change our views of problem solving as a topic that is added onto instruction after concepts and skills have been taught. One alternative is to make problem solving an integral part of mathematics learning. This alternative, often called teaching through problem solving, adopts the view that the connection between problem solving and concept learning is symbiotic (Lambdin, 2003): Students learn and understand mathematics through solving mathematically rich problems and problem-solving skills are developed through learning and understanding mathematics concepts and procedures (Schroeder & Lester, 1989).

In teaching through problem solving, learning takes place during the process of attempting to solve problems in which relevant mathematics concepts and skills are embedded (Lester & Charles, 2003; Schoen & Charles, 2003). As students solve problems, they can use any approach they can think of, draw on any piece of knowledge they have learned, and justify their ideas in ways that they feel are convincing. The learning environment of teaching through problem solving provides a natural setting for students to present various solutions to their group or class and learn mathematics through social interactions, meaning negotiation, and reaching shared understanding. Such activities help students clarify their ideas and acquire different perspectives of the concept or idea they are learning. Empirically, teaching mathematics through problem solving helps students go beyond acquiring isolated ideas toward developing increasingly connected and complex system of knowledge (e.g., Cai, 2003; Carpenter, Franke, Jacobs, Fennema, & Empson, 1998; Cobb et al. 1991; Hiebert & Wearne, 1993; Lambdin, 2003). The power of problem solving is that obtaining a successful solution requires students to refine, combine, and modify knowledge they have already learned.

It is important to point out that we are not saying that every task that students encounter must be problematic. If the goal of a lesson is to develop and master certain skills, some exercises are necessary. In addition, as we indicated before, teachers may modify existing less problematic problems to be "true" problems.

How can teachers orchestrate pedagogically sound, active problem solving in the classroom?

Picking the problem or task is only one part of teaching with problem solving. There is considerable evidence that even when teachers have good problems they may not be implemented as intended. Students' actual opportunities to learn depend not only on the type of mathematical tasks that teachers pose but also on the kinds of classroom discourse that takes place during problem solving, both between the teacher and students and among students. Discourse refers to the ways of representing, thinking, talking, and agreeing and disagreeing that teachers and students use to engage in instructional tasks. Considerable theoretical and empirical evidence exists supporting the connection between classroom discourse and student learning. The theoretical support comes from both constructivist and sociocultural perspectives of learning (e.g., Cobb, 1994; Hatano, 1988; Hiebert et al., 1997). As students explain and justify their thinking and challenge the explanations of their peers and teachers, they are also engaging in clarification of their own thinking and becoming owners of "knowing" (Lampert, 1990). The empirical evidence supporting the positive relationships between teachers' asking high-order questions and students' learning can be found in the work of Hiebert and Wearne (1993) and of Redfield and Rousseau 1981).

Then what is considered to be desirable discourse in mathematics teaching? To explore this question, let us compare the two teaching episodes shown below involving seventh-grade teachers and their students (Thompson, Philipp, Thompson, & Boyd, 1994). The teachers presented the following problem to their classes:

> At some time in the future John will be 38 years old. At that time he will be 3 times as old as Sally. Sally is now 7 years old. How old is John now?

Teaching Episode 1

T: Let's talk about this problem a bit. How is it that you thought about it?

S1: I divided 38 by 3 and I got 12 2/3. Then I subtracted 7 from 12 2/3 and got 5 2/3. [Pause] Then I subtracted that from 38 and got 32 1/3. [Pause] John is 32 1/3.

T: That's good! [Pause] Can you explain what you did in more detail? Why did you divide 38 by 3?

S1: [Appearing puzzled by the question, S1 looks back at her work. She looks again at the original problem.] Because I knew that John is older—3 times older.

T: Okay, and then what did you do?

S1: Then I subtracted 7 and got 5 2/3. [Pause] I took that away from 38, and that gave me 32 1/3.

T: Why did you take 5 2/3 away from 38?

S1: [Pause] To find out how old John is.

T: Okay, and you got 32 1/3 for John's age. That's good! [Pause]

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Teaching Episode 2

T: Let's talk about this problem a bit. How is it that you thought about the information in it?

S1: Well, you gotta start by dividing 38 by 3. Then you take away . . .

T: [Interrupting] Wait! Before going on, tell us about the calculations you did, explain to us why you did what you did. (Pause) What were you trying to find?

S1: Well, you know that John is 3 times as old as Sally, so you divide 38 by 3 to find out how old Sally is.

T: Do you all agree with S1's thinking?

[Several students say "Yes"; others nod their heads.]

S2: That's not gonna tell you how old Sally is now. It'll tell you how old Sally is when John is 38.

T: Is that what you had in mind, S1?

S1: Yes.

T: [To the rest of the class] What does the 38 stand for?

S2: John's age in the future.

T: So 38 is not how old John is now. It's how old John will be in the future. [Pause] The problem says that when John gets to be 38 he will be 3 times as old as Sally. Does that mean "3 times as old as Sally is now" or "3 times as old as Sally will be when John is 38"?

[Several students respond in unison, "When John is 38."]

T: Are we all clear on S2's reasoning? [Pause]

There are a number of similarities between the two teaching episodes that Thompson and colleagues analyzed. For example, both teachers opened their lessons with the same problem and with similar instructions. Both teachers pressed their students to give rationales for their calculation procedures. However, the two teaching episodes differed significantly in terms of how the teachers led the classroom discussion. For example, students in Teaching Episode 2 began to give explanations that were grounded in conceptions of the situation (i.e., in making sense of the situation presented in the problem). By contrast, the explanations given by students in Teaching Episode 1 remained strictly procedural. In addition, Teacher 1 was less persistent than Teacher 2 in probing the students' thinking. He accepted solutions consisting of calculation sequences. However, Teacher 2 persistently probed students' thinking whenever their responses were cast in terms of numbers and operations. The analysis clearly shows that mathematical tasks can be implemented differently, depending on the nature of classroom discourse (Knuth & Peressini, 2001; Sherin, 2000; Silver & Smith, 1996; Thompson et al., 1994).

There are a number of factors that can influence the implementation of worthwhile problems in classrooms (e.g., Henningsen & Stein, 1997). One of the predominant factors is the amount of time allocated to solving and discussing the prob-

lem. For example, Rowe (1974) found that the mean time that teachers waited between asking a question and, if no answer was forthcoming, intervening again was only 0.9 seconds. A wait time of less than one second prevented most students from taking part in the classroom discussion. Consequently, it is no wonder that many students believe that every problem should be solvable with little or no thinking (Lesh & Zawojewski, 2007). Another important barrier to meaningful problem solving experiences is that teachers often remove the challenges of a mathematical task by taking over the thinking and reasoning and telling students how to solve the problem. There is considerable evidence that many U.S. mathematics teachers think that they have the responsibility to remove the challenge (and the struggle) for their students when they are engaged in problem solving. In her study of eighth-grade students who were part of the Third International Mathematics and Science Study (TIMSS), Smith (2000) found that U.S. teachers almost always intervened to show students how to solve the problems they had been asked to solve, leaving the mathematics they were left to do rather straightforward. This stands in direct contrast to teachers in Germany and Japan, who allowed students much greater opportunities to struggle with the more challenging parts of the problems. Productive struggle with complex mathematical ideas is crucial to learning during problem solving. Finally, teachers are also responsible for listening carefully to students' ideas and asking them to clarify and justify their ideas orally and in writing, as well as monitoring their participation in discussions and deciding when and how to encourage each student to participate. The questions that teachers ask are also critical for orchestrating sound classroom discourse (Rasmussen, Yackel, & King, 2003; Stephan & Whitenack, 2003).

Conclusion

To help students become successful problem solvers, teachers must accept that students' problem-solving abilities often develop slowly, thereby requiring long-term, sustained attention to making problem solving an integral part of the mathematics program. Moreover, teachers must develop a problem-solving culture in classroom to make problem solving a regular and consistent part of one's classroom practice. Students must also buy into the importance of regularly engaging in challenging activities (Lester, 1994; Willoughby, 1990).

Developing students' abilities to solve problems is not only a fundamental part of mathematics learning across content areas but also an integral part of mathematics learning across grade levels. Beginning in preschool or kindergarten, students should be taught mathematics in a way that fosters understanding of mathematics concepts and procedures and

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solving problems. In fact, there is strong evidence that even very young students are quite capable of exploring problem situations and inventing strategies to solve the problems (e.g., Ben-Chaim et al., 1998; Cai, 2000; Carpenter et al., 1998; Kamii & Housman, 1989; Maher & Martino, 1996; Resnick, 1989). However, students cannot become successful problem solvers overnight. Helping students become successful problem solvers should be a long-term instructional goal, so effort should be made to reach this goal at every grade level, in every mathematical topic, and in every lesson.

Research clearly suggests that problem solving should not be taught as a separate topic in the mathematics curriculum. In fact, research tells us that teaching students to use general problem-solving strategies has little effect on their success as problem solvers. Thus, problem solving must be taught as an integral part of mathematics learning, and it requires a significant commitment in the curriculum at every grade level and in every mathematical topic. In addition to making a commitment to problem solving in the mathematics curriculum, teachers need to be strategic in selecting appropriate tasks and orchestrating classroom discourse to maximize learning opportunities. In particular, teachers should engage students in a variety of problem-solving activities: (a) finding multiple solution strategies for a given problem, (b) engaging in mathematical exploration, (c) giving reasons for their solutions, and (d) making generalizations. Focusing on problem solving in the classroom not only impacts the development of students' higher-order thinking skills but also reinforces positive attitudes. Finally, there is no evidence that we should worry that students sacrifice their basic skills if teachers focus on developing their mathematical problem-solving skills.

By Jinfa Cai and Frank Lester
Judith Reed Quander, Series Editor

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