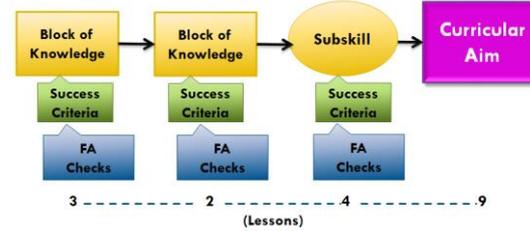




# Learning Progressions



## Learning Progression Model



**Learning Progression:** A carefully sequenced set of building blocks consisting of subskills and bodies of enabling knowledge that, it is believed, students must master en route to mastering a more remote curricular aim.

Popham, 2008

### YOUNG MATHEMATICIANS AT WORK

Constructing Multiplication and Division



Catherine Twomey Fosnot  
Maarten Dolk  
Copyrighted Material

### YOUNG MATHEMATICIANS AT WORK

Constructing Number Sense, Addition, and Subtraction



Catherine Twomey Fosnot  
Maarten Dolk  
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Progressions for the Common Core  
State Standards in Mathematics (draft)

©The Common Core Standards Writing Team  
29 May 2011

## K, Counting and Cardinality; K-5, Operations and Algebraic Thinking

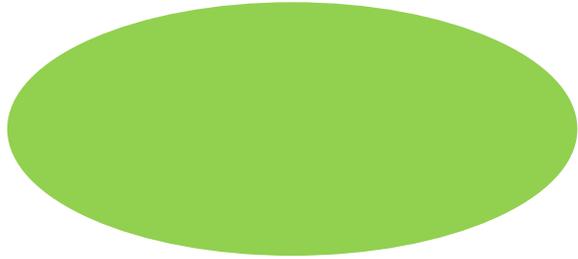
Counting and Cardinality and Operations and Algebraic Thinking are about understanding and using numbers. Counting and Cardinality underlies Operations and Algebraic Thinking as well as Number and Operations in Base Ten. It begins with early counting and telling how many in one group of objects. Addition, subtraction, multiplication, and division grow from these early roots. From its very beginnings, this Progression involves important ideas that are neither trivial nor obvious; these ideas need to be taught, in ways that are interesting and engaging to young students.

The Progression in Operations and Algebraic Thinking deals with the basic operations—the kinds of quantitative relationships they model and consequently the kinds of problems they can be used to solve as well as their mathematical properties and relationships. Although most of the standards organized under the OA heading involve whole numbers, the importance of the Progression is much more general because it describes concepts, properties, and representations that extend to other number systems, to measures, and to algebra. For example, if the mass of the sun is  $x$  kilograms, and the mass of the rest of the solar system is  $y$  kilograms, then the mass of the solar system as a whole is the sum  $x + y$  kilograms. In this example of additive reasoning, it doesn't matter whether  $x$  and  $y$  are whole numbers, fractions, decimals, or even variables. Likewise, a property such as distributivity holds for all the number systems that students will study in K-12, including complex numbers. The generality of the concepts involved in Operations and Algebraic Thinking means that students' work in this area should be designed to help them extend arithmetic beyond whole numbers (see the NF and NBT Progressions) and understand and apply expressions and equations in later grades (see the EE Progression). Addition and subtraction are the first operators studied. In-

Draft, 5/29/11, comment at [commoncoretools.wordpress.com](http://commoncoretools.wordpress.com)



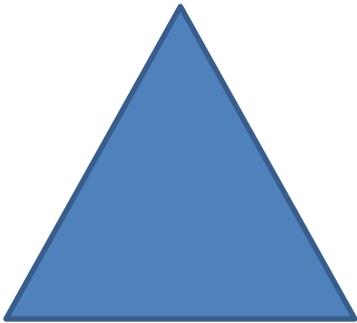
# Constructing a Landscape



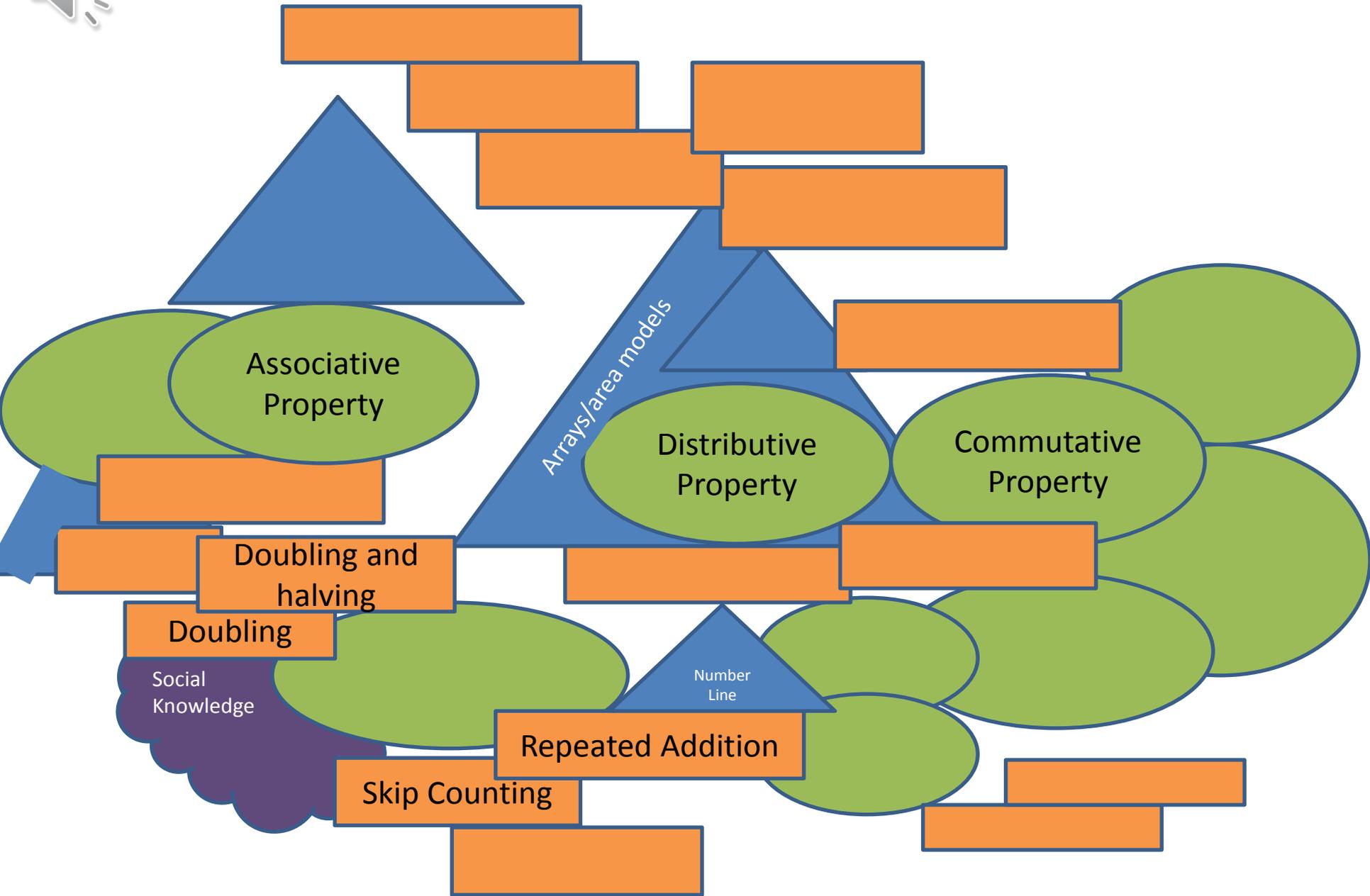
Big Ideas



Strategies



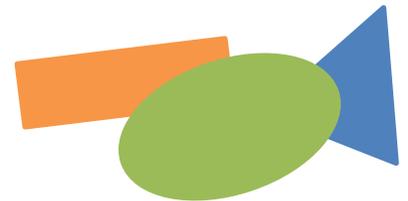
Mathematical Models



# Big Ideas, Strategies, Models

*Working in your group:*

- Which are you familiar with? Which does your group need to know more about?
- How might you group these or place these in a “landscape” to show the mathematical connections?
- Do any build on each other?

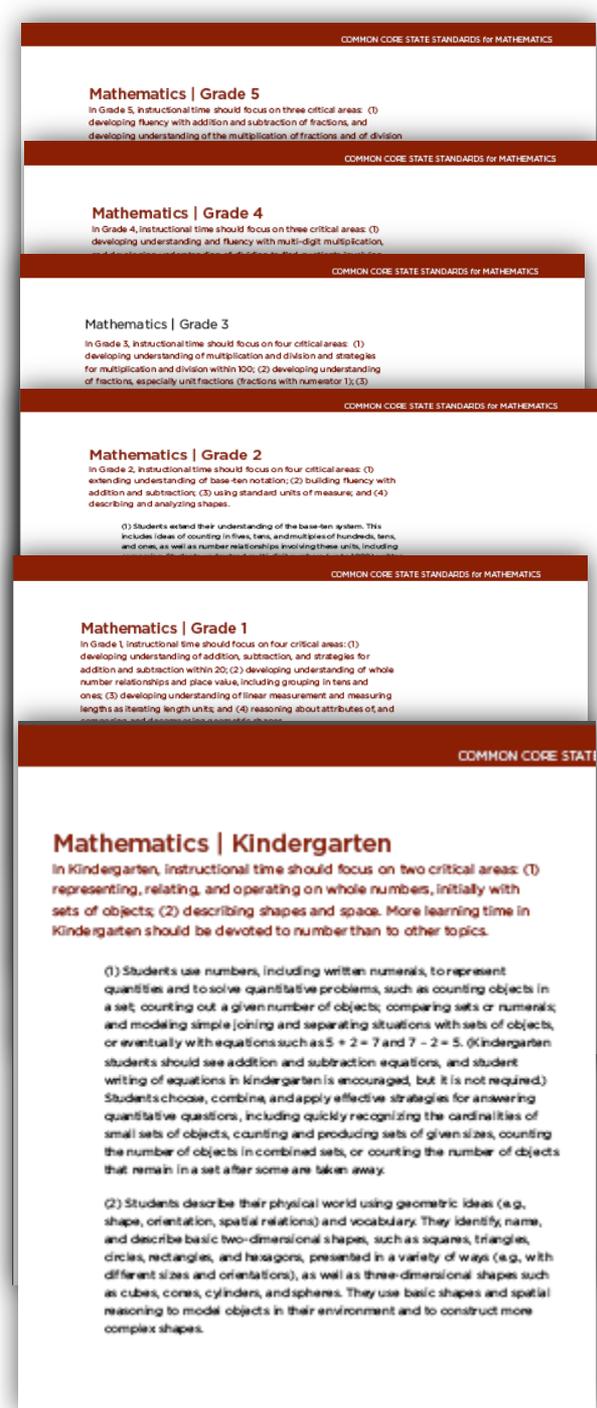




# Activity: “Big Idea”

## Step 1: Critical Content

- Identify big ideas, strategies, and models related to  $(x \text{ and } \div)$  /  $(+ \text{ and } -)$
- Discuss with your face partner which critical content area(s) focus(es) on  $(x \text{ and } \div)$  /  $(+ \text{ and } -)$





# Activity: “Big Idea”

## Step 2:

- Quickly skim and scan cluster headings and standards related to multiplication and division or addition and subtraction.



Packet Page

13

Operations and Algebraic Thinking 5.OA

Write and interpret numerical expressions.

Operations and Algebraic Thinking 4.OA

Use the four operations with whole numbers to solve problems.

1. Interpret a multiplication equation as a comparison, e.g., interpret  $35 = 5 \times 7$  as a statement that 35 is 5 times as many as 7 and 7 times as

Operations and Algebraic Thinking 3.OA

Represent and solve problems involving multiplication and division.

1. Interpret products of whole numbers, e.g., interpret  $5 \times 7$  as the total

Operations and Algebraic Thinking 2.OA

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting

Operations and Algebraic Thinking 1.OA

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart,

Counting and Cardinality K.CC

Know number names and the count sequence.

1. Count to 100 by ones and by tens.
2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Count to tell the number of objects.

4. Understand the relationship between numbers and quantities; connect counting to cardinality.
  - a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
  - b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
  - c. Understand that each successive number name refers to a quantity that is one larger.
5. Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.

Compare numbers.

6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.<sup>1</sup>
7. Compare two numbers between 1 and 10 presented as written numerals.

Operations and Algebraic Thinking K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

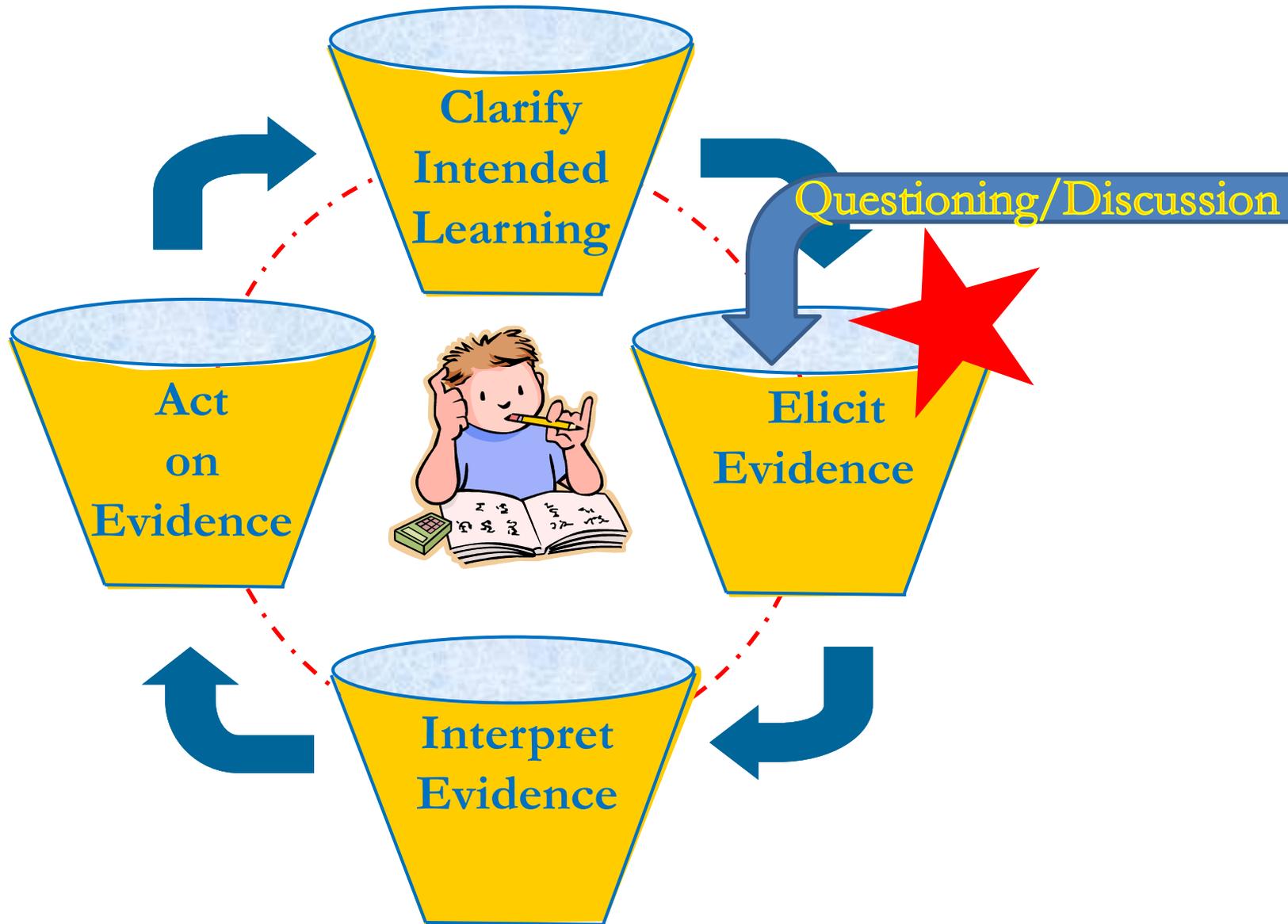
1. Represent addition and subtraction with objects, fingers, mental images, drawings<sup>2</sup>, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g.,  $5 = 2 + 3$  and  $5 = 4 + 1$ ).
4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
5. Fluently add and subtract within 5.

<sup>1</sup>Include groups with up to ten objects.

<sup>2</sup>Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

A man in a white dress shirt and a dark tie with light-colored polka dots is holding a yellow sign with the words "BREAK TIME" written in large, dark, block letters. He is sitting at a desk with a white mug and a black object, possibly a stapler or paperweight, visible. The background is a plain, light-colored wall with some greenery on the left.

**BREAK  
TIME**



# Formative Assessment Process



# Eliciting Evidence of Learning

We have to know where we are before deciding where we need to go.



**Learning Target:** Students will develop a conceptual understanding of counting and cardinality. (GRADE K)



In preparation for the beginning of the school year, we were organizing our supplies and we found a large container of paperclips that need to be inventoried.

Considering strategies used by **kindergarten students**, with your table group, how might this task be solved? Record your strategy on chart paper.

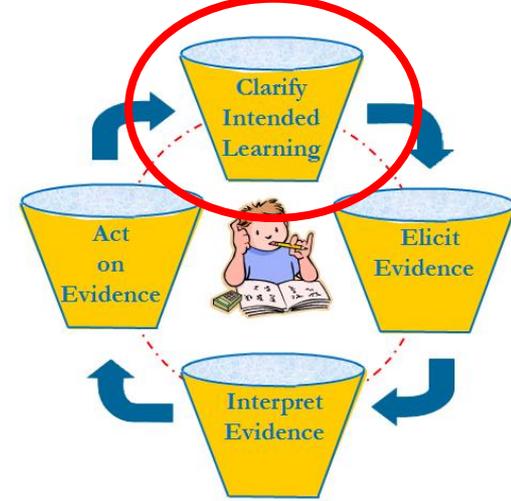
# Task Activity – Kindergarten



## Gallery Walk: Making Connections to your Classroom

- As you observe solutions offered by other groups, consider what you might expect to observe with your students?
- Based on experience, what typical steps and missteps do you anticipate?

# Populate your Landscape



Based on observations that you made during the gallery walk and your work with this task; relook at your sort and make connections between the big ideas, strategies, and models.

# Populate the Landscape

Strategies

Big Ideas

Model

Skip counting

Model with Groups

Uses 1-9 sequence when counting

Models quantities with tallies

One-to-one correspondence

Model with symbols to represent amounts

One-to-one tagging

Need for Organization & Keeping Track

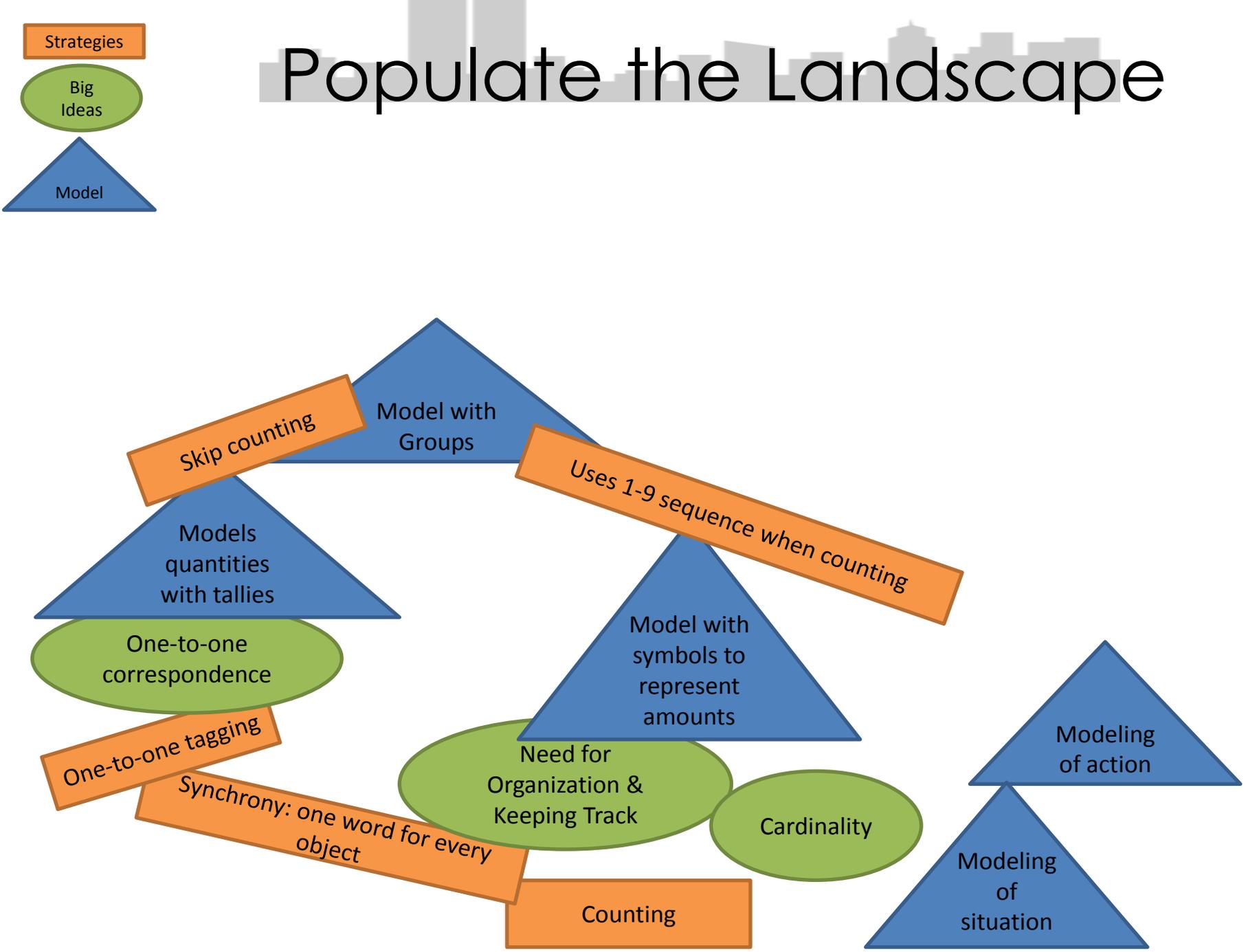
Cardinality

Synchrony: one word for every object

Counting

Modeling of action

Modeling of situation



# Norms for Watching Teaching Videos

- Teaching is multi-faceted.
  - *The video doesn't show everything.*
- Teaching is incredibly hard work!
  - *Assume positive intent.*
- No lesson is ever perfect.
  - *Focus on what you can use to improve your classroom instruction.*
  - *Focus on the student's and teacher's interaction with the Mathematics.*

# Observing Students: Discussion

- What do you think Jodi's mathematical goal is in choosing this particular task?
- Given the investigation that Jodi developed, what mathematical ideas and strategies do you expect to see as the children set to work?

# Observing Students: Preview Video 2 and 3

While watching the videos...

- Think about some of the strategies from the landscape you see Jodi's students using.
- Find one example of how Jodi is using formative assessment to move her students along the landscape.

# Observing Students: Discussion Video 2 and 3

- Think about some of the strategies from the landscape you see Jodi's students using.
- Find one example of how Jodi is using formative assessment to move her students along the landscape.

# Observing Students: Discussion Video 2 and 3

- Think about some of the strategies from the landscape you see Jodi's students using.
- Find one example of how Jodi is using formative assessment to move her students along the landscape.



# Populating Your Landscape

- Consider what big ideas, strategies or models we can now add to our landscape as a result of our learning.

# Populate the Landscape

Strategies

Big Ideas

Model

Place determines value

Unitizing

Using 10 structure

Systematic production of arrangements

Counting on

Model with Groups

Skip counting

Uses 1-9 sequence when counting

Models quantities with tallies

One-to-one correspondence

Model with symbols

Making 10s

One-to-one tagging

Need for Organization & Keeping Track

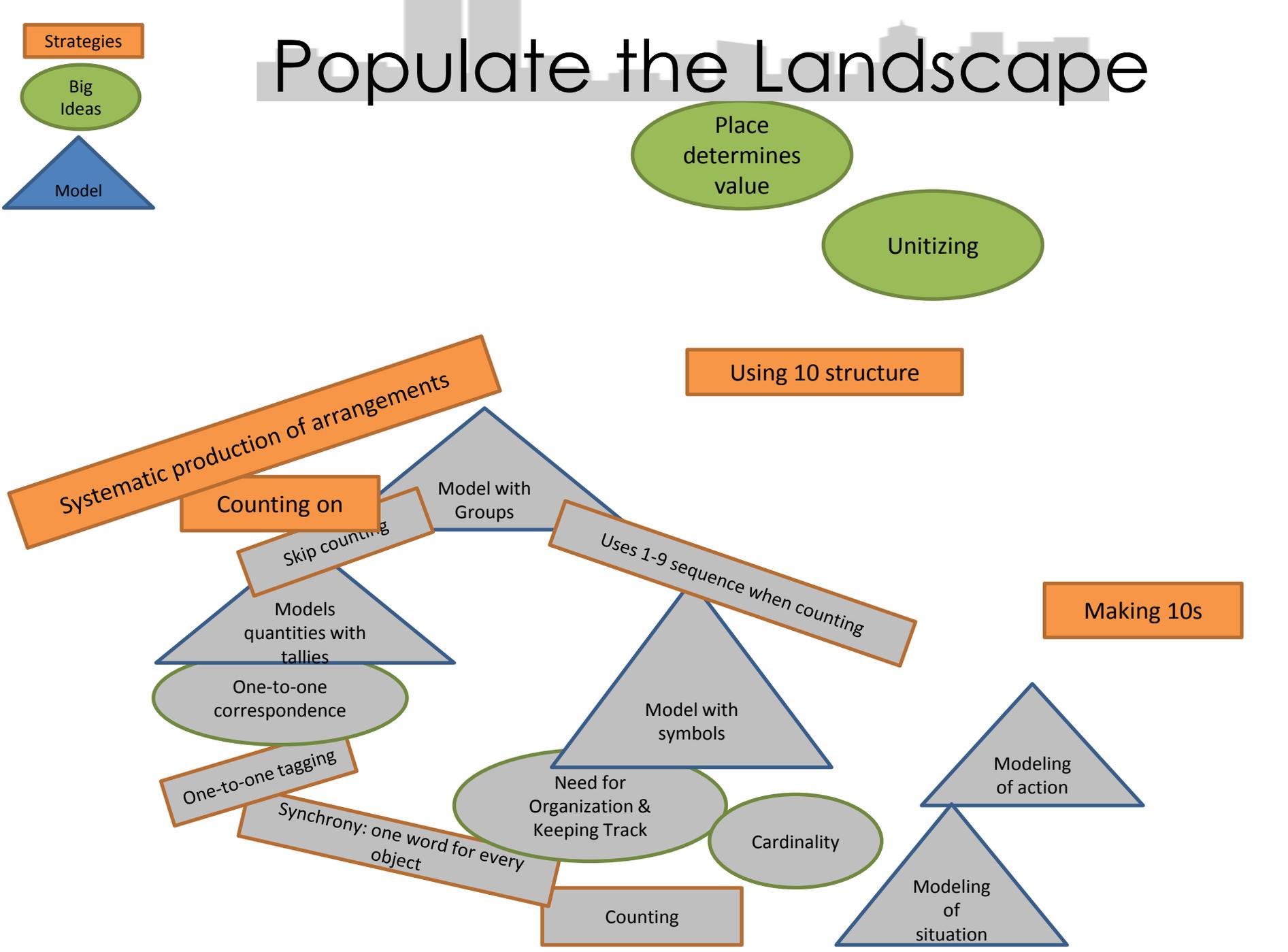
Cardinality

Modeling of action

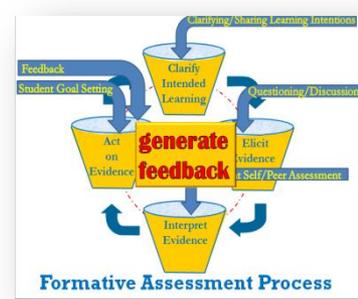
Synchrony: one word for every object

Counting

Modeling of situation



# Balcony View



1

Quiet Reflection: (3 min.)  
Looking at your note taker, what formative assessment practices did you notice in these video clips?

2

Discuss in your table groups.





# Reviewing Task Considerations

- ✓ Does the task expose students' current levels of understanding in **relation to** the mathematics **learning target**?
- ✓ Is it **problematic** for students?
- ✓ Does the **cognitive demand** enable students opportunities to expose the depth of their knowledge?
- ✓ Does the task have **multiple entry and exit points**?
- ✓ Is the context and the mathematics of the task **relevant** to students?

# **LUNCH**

**11:30 to 12:30**



- See you promptly at 12:30.
- Remember to sign in when you return.



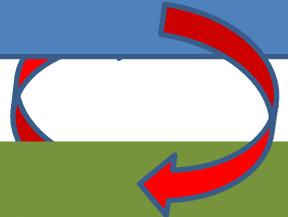
Explore ways in which FA attributes and practices move learning forward.



-Participants will construct a progression of learning.  
-Participants will discuss FA practices observed.



How to embed formative assessment practices into daily math instruction.



Context of Learning:  
x and  $\div$   
+ and -

# Bringing it Back to Your Classroom

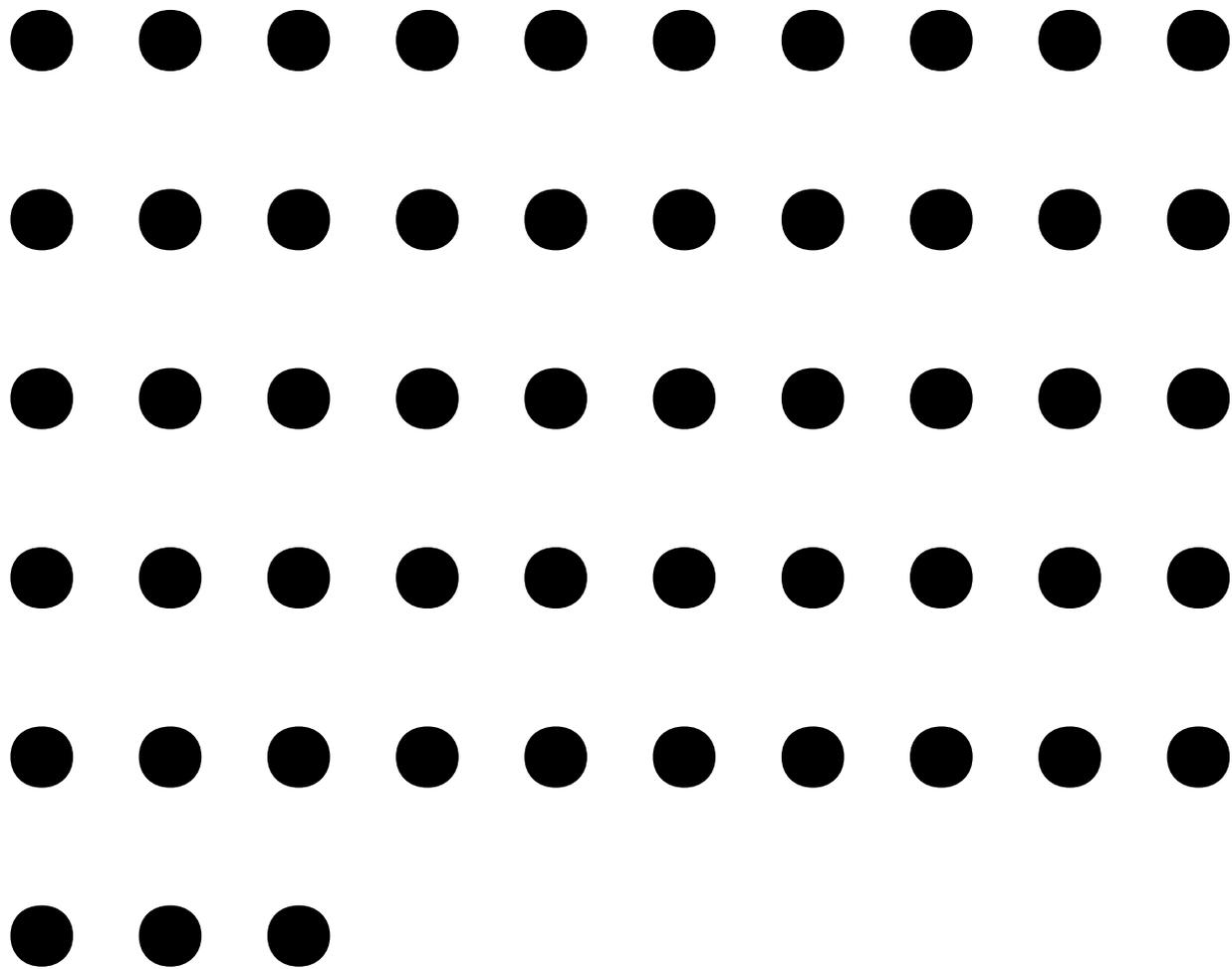
- Discuss with your table partners how this would look in your classroom at the beginning of the year.
  - How would you differentiate the task for students who are unable to count to 10? To 5?
  - What would these students count and how would you facilitate their learning?
  - How would you use your data from Kindergarten Portfolio Math Task 1 to structure this in your classroom?
  - How could the children organize their data?
- How would you adjust this task for mid year? End of year?
  - When would students be responsible for adding labels to the items being inventoried?

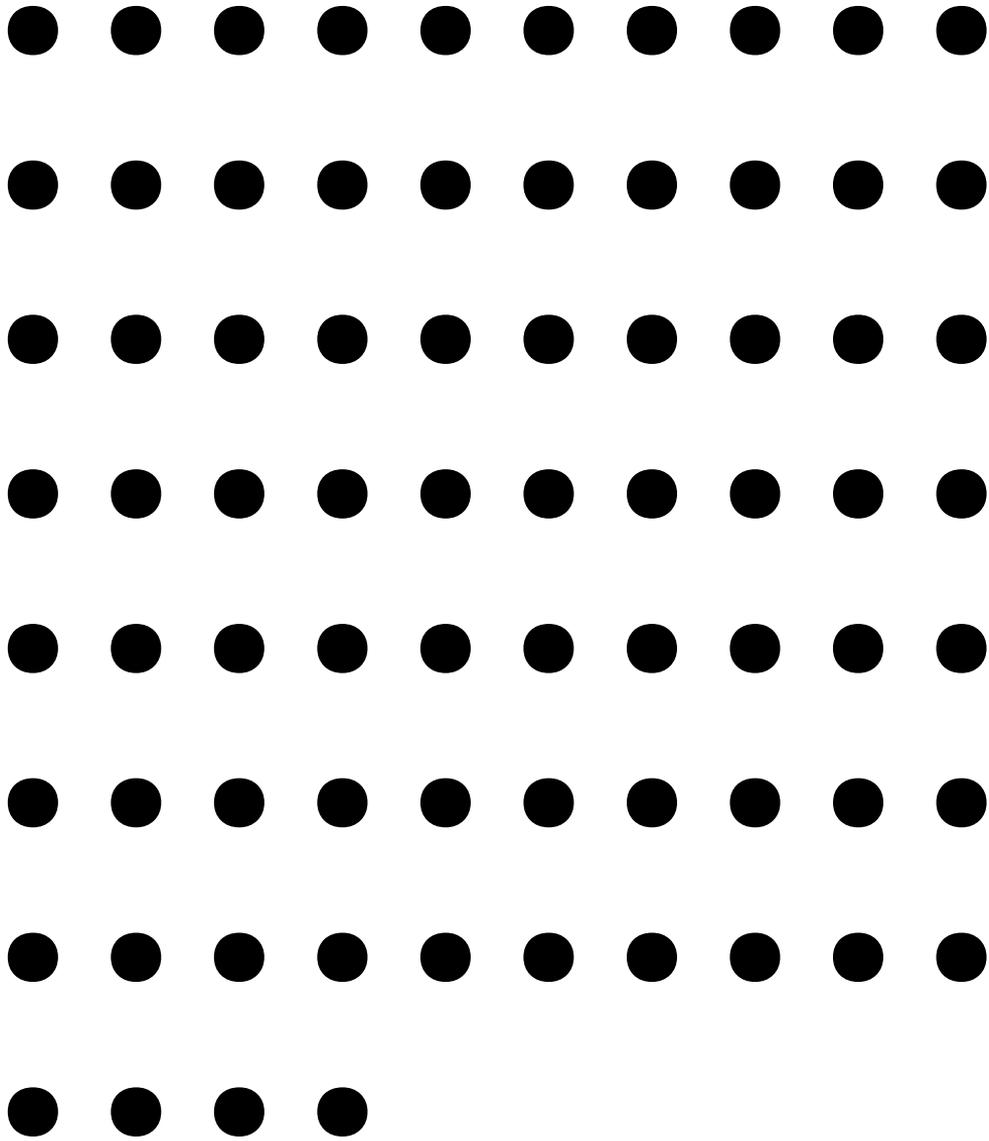
# Observing with Purpose



How Many Do You See?







# TIPS for Class Discussions

Engaging all learners and keeping cognitive demand high

- Helping Individual Students Clarify and Share Their Own Thoughts.
- Helping Students Orient to the Thinking of Others.
- Helping Students Deepen Their Own Reasoning.
- Helping Students Engage with the Reasoning of Others.



# Populating Your Landscape

“The landmarks in this journey are not necessarily sequential. Many paths can be taken toward this horizon. Some landmarks are, of course, precursors to others.” ~Cathy Fosnot

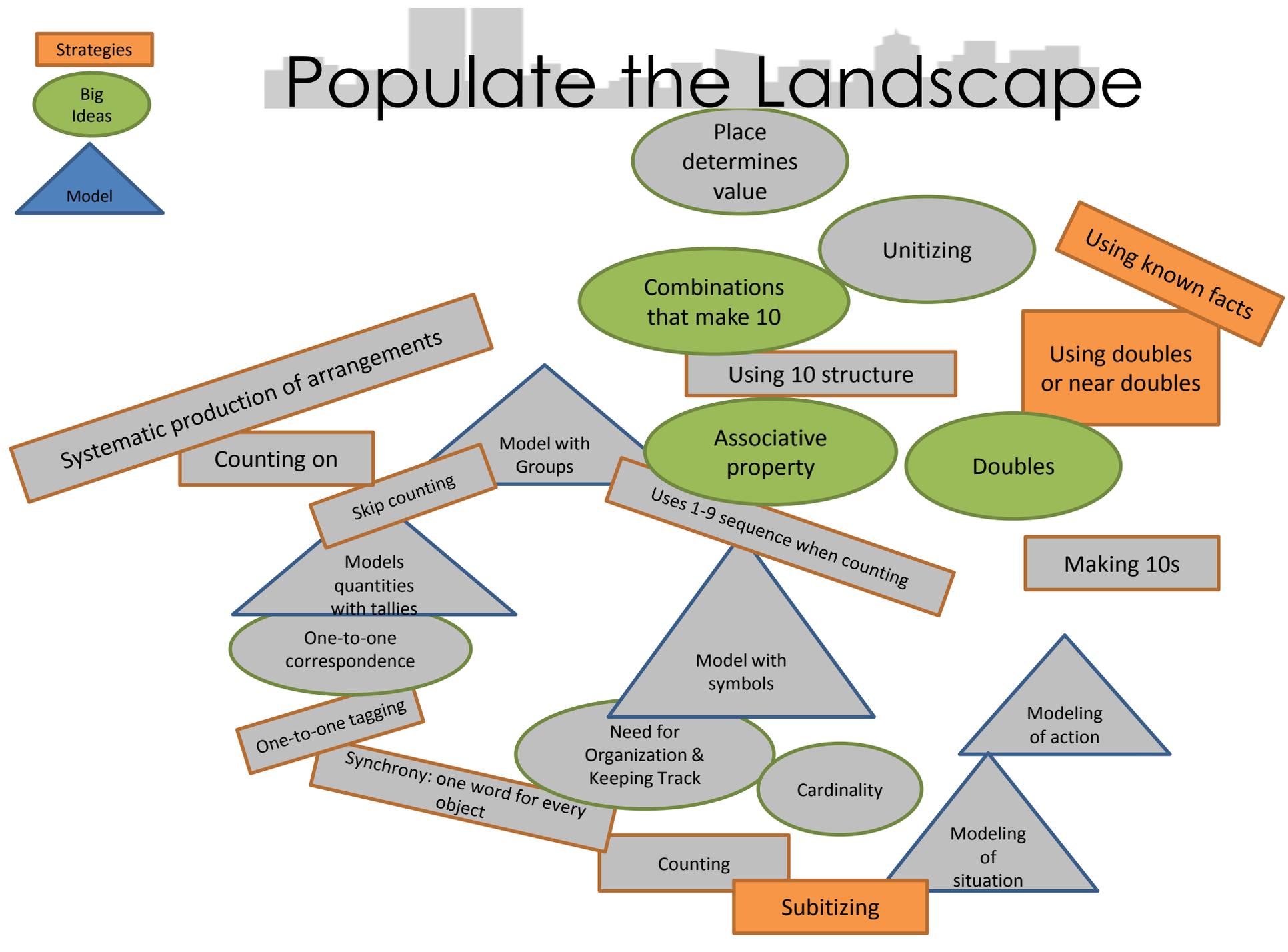


# Populate the Landscape

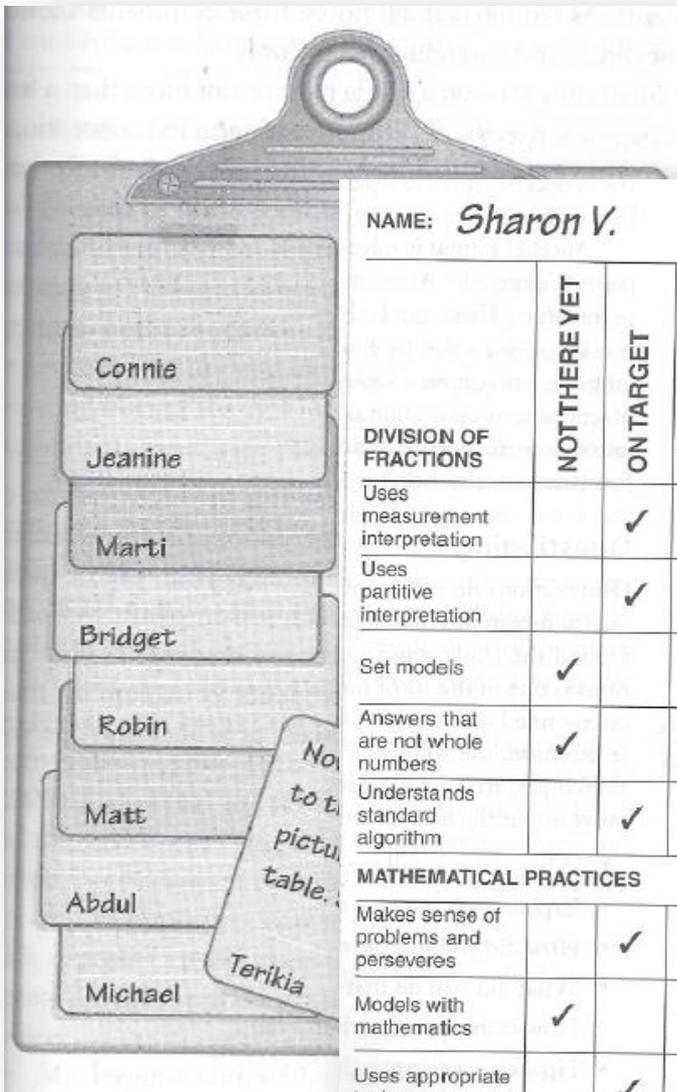
Strategies

Big Ideas

Model

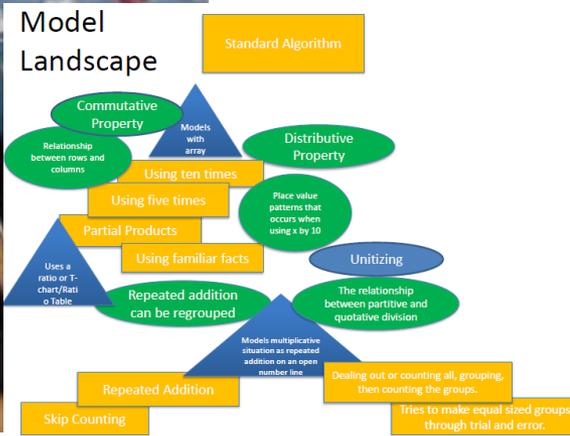


# Data Collection – Kidwatching and the landscape



NAME: *Sharon V.*

	NOT THERE YET	ON TARGET	ABOVE AND BEYOND	COMMENTS
<b>DIVISION OF FRACTIONS</b>				
Uses measurement interpretation		✓		
Uses partitive interpretation		✓		
Set models	✓			Used two color counters to show $1\frac{1}{4} \div 3 =$
Answers that are not whole numbers	✓			
Understands standard algorithm		✓		Showing greater reasonableness
<b>MATHEMATICAL PRACTICES</b>				
Makes sense of problems and perseveres		✓		Stated problem in own words
Models with mathematics	✓			Reluctant to use abstract models
Uses appropriate tools		✓		

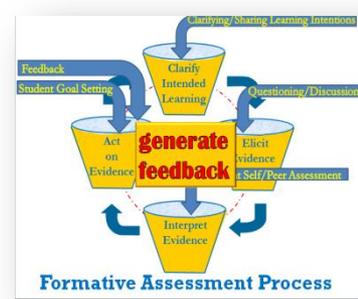


## Student Observations on 4<sup>th</sup> Grade Landscape

Student \_\_\_\_\_ School Year \_\_\_\_\_

Big Idea	Date Observed			
Commutative Property				
Distributive Property				
Relationship between rows and columns				
Place value patterns that occurs when using $\times$ by 10				
Repeated addition can be regrouped				
The relationship between partitive and quotative division				

# Balcony View



1

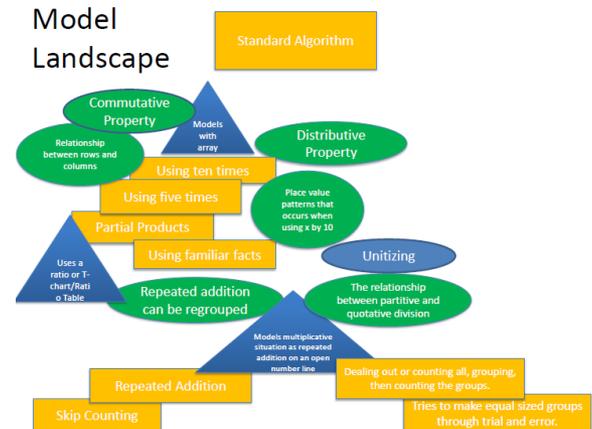
Quiet Reflection: (3 min.)  
Looking at your note taker, what formative assessment practices did you notice in these video clips?

2

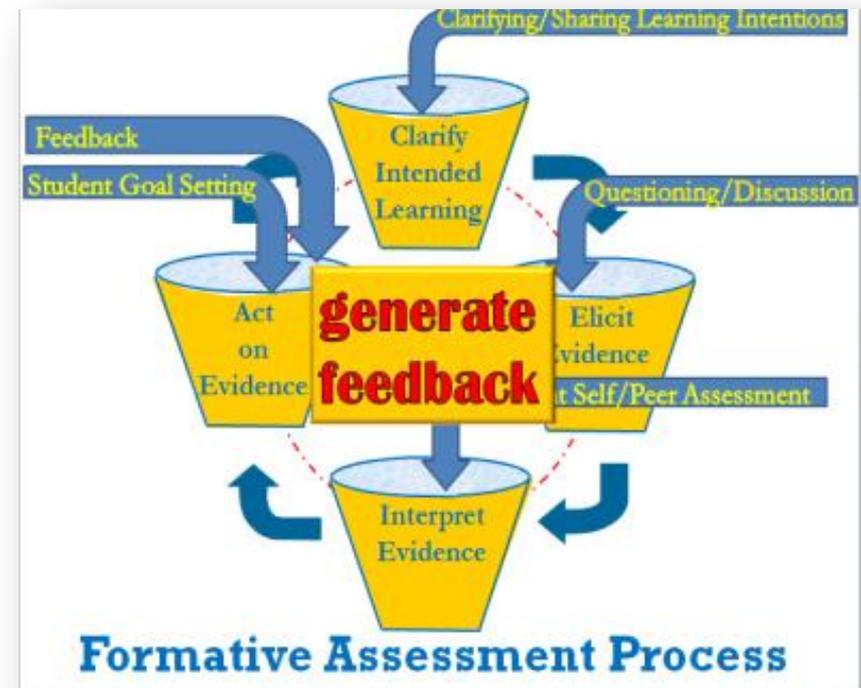
Discuss in your table groups.



# Considering Formative Assessment and the Landscape



- ✓ **Clarify intended learning**
- ✓ **Elicit evidence.**
- ✓ **Interpret evidence**
- ✓ **Act on evidence**





# Review and Connections

Partner Conversation:

(4 min.)

1. What are the NVACS critical areas for my grade level?
2. How does the landscape support the critical areas?
3. What are some of the connections between the NVACS standards and the Mathematical landscape?

# Considering the Critical Areas and the Mathematical Landscape

Students use **numbers**, including **written numerals**, to represent **quantities** and to solve **quantitative problems**, such as **counting objects** in a set; **counting out** a given number of objects in a set; **comparing sets** or numerals; and modeling simple **joining** and **separating** situations with sets of objects, or eventually with **equations** such as  $5 + 2 = 7$  and  $7 - 2 = 5$ .

# Considering the NVACS Standards

NVACS Standards Pg. 9 – 12	Language from the standards	Big Ideas from the landscape	Strategies and Models to use with students
K.CC.4a	<ul style="list-style-type: none"><li>• number name</li><li>• standard order</li><li>• pairing</li></ul>	<ul style="list-style-type: none"><li>• Need for Organization and Keeping Track</li><li>• One-to-one correspondence</li></ul>	<ul style="list-style-type: none"><li>• Actions</li><li>• Situation</li><li>• Synchrony</li><li>• 1-1 tagging</li></ul>
K.OA.3	<ul style="list-style-type: none"><li>• decompose</li><li>• equal</li><li>• pairs</li><li>• drawings</li></ul>	<ul style="list-style-type: none"><li>• Commutative</li><li>• Associative</li><li>• Equivalence</li></ul>	<ul style="list-style-type: none"><li>• Pictures</li><li>• Model with Symbols</li><li>• Splitting</li></ul>
K.NBT.1	<ul style="list-style-type: none"><li>• compose</li><li>• Decompose</li><li>• Tens and ones</li><li>• further ones</li></ul>	<ul style="list-style-type: none"><li>• Place Determines Value</li><li>• Unitizing</li><li>• Grouping</li></ul>	<ul style="list-style-type: none"><li>• Additive Structuring</li><li>• 5 Structure</li><li>• 10 Structure</li><li>• Landmarks</li><li>• Counting</li><li>• Splitting</li></ul>





Develop a conceptual understanding of the Formative Assessment Process and its benefits.



Compose and share a definition or quote capturing the essence of what formative assessment is.



Explore ways in which FA attributes and practices move learning forward.



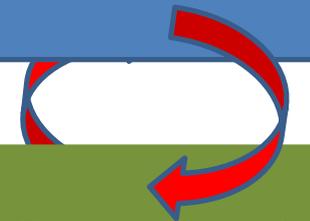
-Participants will construct a progression of learning.  
-Participants will discuss FA practices observed.



Plan next steps in embedding learning into practice.



Vertical teams will begin planning instruction, within today's context, embedding formative assessment practices in plans.



Context of Learning:  
x and ÷  
+ and -

How to embed formative assessment practices into daily math instruction.

# Fall PD Opportunities

- ✓ District-Wide PLTs (16 hours, 1 credit)
  - ✓ Continue your learning from today!
  - ✓ Four follow-up sessions: 9/25, 12/4, 2/26, 5/21
  - ✓ Work in grade level teams
  - ✓ Content tied to “Pacing Timeline” to provide “just in time” connected learning opportunities to prepare for the next quarter of content
- ✓ Computational Strategies/Algorithms (8 hours, 1/2 credit)
- ✓ Core Connections: K-5 (8 hours, 1/2 credit) Same as offered last year. Join us if you missed it or have changed grade levels!
- ✓ eSuite Basic (no credit): 8/19 or 9/9
- ✓ Intensification Lab: During winter break
- ✓ Planning in Math (16 hours, 1 credit)
- ✓ Using Classroom Discussion to Promote Problem Solving and Solution Strategies in Mathematics
- ✓ Formative Assessment Webinars – (October – November)

**Check Solutionwhere frequently for updates and additional opportunities!**

# Resources to Support

Quarters 3 & 4	Unit 6 Whole-Number Operations & Number Stories	Unit 7 Patterns & Rules	Unit 8 Fractions	Unit 9 Measurement
	Number of lessons: 12 over 13 days. D/E: 3 CCSS Focus Domains: OA - NBT - MD	Number of lessons: 10 over 13 days. D/E: 2 CCSS Focus Domains: NBT - MD	Number of lessons: 8 D/E: 0 CCSS Focus Domains: OA - G - MD	Number of lessons: 4 D/E: 0 (see 4th Q) CCSS Focus Domains: OA - NBT - MD - G
	Expanded Unit Topic: Whole-Number Operations & Number Stories Additional Instructional Days: 3 Total Days: "19"	Expanded Unit Topic: Parts & Wholes (2.MD.6, 2.G.2, 2.G.3) Additional Instructional Days: 2 Total Days: "15"	Expanded Unit Topic: Parts & Wholes (2.MD.6, 2.G.2, 2.G.3) Additional Instructional Days: 2 Total Days: "10"	Total Days: "4"
Quarter 3: 48 total instructional days				

WCSD K-6 Mathematics Curriculum and Instruction Grade 2

- WCSD Pacing Timeline

► Grade 4 Unit 1: Naming and Constructing Geometric Figures

Big Conceptual Idea: [Geometry](#) (pp. 2-5, 14-15)

This unit focuses on naming and constructing geometric figures. See column notes about specific geometric terminology and shapes to consider. At the conclusion of this unit, students should be able to draw figures including circles aligned with 4.G.1 (yet establishing foundations for angles 4.MD.9, 2), begin to classify two-dimensional figures (see 4.G.2) and some students may be able to recognize that the diameter of a circle is a line of symmetry (4.G.3). Focus on having students use mathematics terminology accurately and precisely, Mathematical Practice 6.

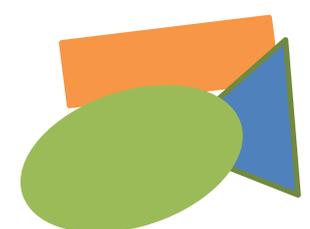
Beginning with this geometry unit supports children engaging in mathematics through the Standards of Mathematical Practices through exploring and using tools, justifying reasoning, and attending to precision which includes precise language. The terminology in this unit is vital to establishing mathematically correct explanations which will build throughout the year. For example, children will explore fractions on a number line. The equal iterations within the number line are all equivalent line segments. Line segments are also used when determining distance in both linear measurements and fractional understandings ( $1/4, 1/4, 1/4$  is the distance of all three fourths and named  $3/4$ ths). Understanding the difference between a line and line segments is foundation when working with such distance measures.

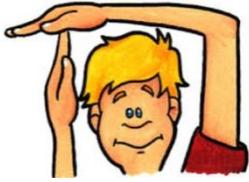
Unit 1 Naming and Constructing Geometric Figures
Number of lessons: 9 over 28 days. D/E: 2 CCSS Focus Domains: G
Total Days: 12

- WCSD Curriculum Documents

CORE INSTRUCTIONAL PRACTICE GUIDE		
CORE ACTION 3: Provide all students with opportunities to exhibit mathematical practices in connection with the content of the lesson.		
INDICATORS	ILLUSTRATIVE STUDENT BEHAVIOR*	EVIDENCE OBSERVED OR GATHERED**
1. The teacher uses strategies to keep all students participating with challenging questions.	Even after receiving a pair of feedback, student persists in asking a challenging question.	1 2 3 4
2. The teacher establishes a classroom culture in which students expect free thinking.	Students volunteer with a relevant question, hypothesis, or prediction for the teacher or another student to explain their thinking, and comment on their own responses.	1 2 3 4
3. The teacher establishes communication protocols that have been shared with all students.	Students talk about and ask questions, clarify or rephrase free time, to obtain clarity or rephrase free time, to obtain clarity or rephrase free time, to obtain clarity or rephrase free time.	1 2 3 4
4. The teacher connects students' informal language to precise mathematical language expressions in their papers.	Students use precise mathematical language in their explanations and discussions.	1 2 3 4
5. The teacher has established a classroom culture in which students expect and use appropriate tools when solving a problem.	Students use appropriate tools strategically when solving a problem.	1 2 3 4
6. The teacher asks students to explain and justify their work and provide feedback that helps students refine their work.	Students explain their thinking, especially when explaining their justifications.	1 2 3 4

- Instructional Practice Guides





# Break and Moving to Verticals



# ***2:45-3:30 Vertical Teams with your School***

Each school will turn in an exit ticket that addresses the guiding question. Individual copies of the questions have been provided for you to refer to during the discussion.

## **Guiding Question:**

- How does the Mathematical landscape and formative assessment fit with my current teaching practices and my knowledge about nurturing and developing young mathematicians?
- **Presenters: List schools and assigned room numbers here!**