Learning Progressions

Block of Knowledge
Success Criteria
FA Checks

Block of Knowledge
Success Criteria
FA Checks

Subskill
Success Criteria
FA Checks

Curricular Aim

Learning Progression: A carefully sequenced set of building blocks consisting of subskills and bodies of enabling knowledge that, it is believed, students must master en route to mastering a more remote curricular aim.

Popham, 2008

Progressions for the Common Core State Standards in Mathematics (draft)
©The Common Core Standards Writing Team
20 May 2011

K, Counting and Cardinality; K–5, Operations and Algebraic Thinking

Counting and Cardinality and Operations and Algebraic Thinking are about understanding and using numbers. Counting and Cardinality involves: Operations and Algebraic Thinking as well as Number and Operations in Base Ten. It begins with early counting and adding how many in one group of objects. Addition, subtraction, and number sentences. It is a whole concept. At every beginning, this Progression involves important ideas that are well beyond the “count there” there ideas used to be taught. In maps that are interesting and engaging for young children.

The Progressions of standard and Algebraic Thinking focus with the basic operations—the bonds of quantities, relationships of operations, and consequently the kinds of problems they can be used to solve as well as their mathematical properties and relationships. Although once the same standards are in each of these standards, the importance of the Progression is much more general because it describes concepts and properties, and reas-

ons that extend to other number systems, so measures, and to objects. For example, if the range of the two is a triangle, and the range of the two is a triangle, then the range of the two is a triangle. If the example of addition reasoning, it doesn’t matter whether $y$ is white numbers, numbers, decimals, or even variables. Likewise, a property such as distributivity for all the number systems that students will study in K–12 including complex numbers.

The generality of the concepts involved in Operations and Algebraic Thinking means that students’ work in this area should be designed to help them understand arithmetic beyond whole numbers (see the K and 1 Progression) and understand and apply properties and operations (such as in base ten) (see the Progressing Progressions). Addition and subtraction are the first operations studied. In
Constructing a Landscape

Big Ideas

Strategies

Mathematical Models
Skip Counting
Doubling
Doubling and halving
Associative Property
Social Knowledge
Arrays / area models
Distributive Property
Commutative Property
Number Line
Repeated Addition
Big Ideas, Strategies, Models

Working in your group:

• Which are you familiar with? Which does your group need to know more about?

• How might you group these or place these in a “landscape” to show the mathematical connections?

• Do any build on each other?
Activity: “Big Idea”

Step 1: Critical Content

- Identify big ideas, strategies, and models related to (x and ÷) / (+ and -)
- Discuss with your face partner which critical content area(s) focus(es) on (x and ÷) / (+ and -)
Activity: “Big Idea”

Step 2:
• Quickly skim and scan cluster headings and standards related to multiplication and division or addition and subtraction.
BREAK TIME
Formative Assessment Process

- Clarify Intended Learning
- Elicit Evidence
- Interpret Evidence
- Act on Evidence
- Questioning/Discussion
Eliciting Evidence of Learning

We have to know where we are **before** deciding where we need to go.
In preparation for the beginning of the school year, we were organizing our supplies and we found a large container of paperclips that need to be inventoried.

Considering strategies used by kindergarten students, with your table group, how might this task be solved? Record your strategy on chart paper.
Task Activity – Kindergarten

**Gallery Walk: Making Connections to your Classroom**

- As you observe solutions offered by other groups, consider what you might expect to observe with your students?

- Based on experience, what typical steps and missteps do you anticipate?
Based on observations that you made during the gallery walk and your work with this task; relook at your sort and make connections between the big ideas, strategies, and models.
Populate the Landscape

- Strategies
  - Big Ideas
  - Model

- Model with Groups
  - Uses 1-9 sequence when counting
  - Models quantities with tallies
  - Skip counting

- One-to-one correspondence
- One-to-one tagging
  - Synchrony: one word for every object

- Need for Organization & Keeping Track
- Cardinality
- Counting
- Modeling of action
- Modeling of situation
Norms for Watching Teaching Videos

• Teaching is multi-faceted.
  – The video doesn’t show everything.

• Teaching is incredibly hard work!
  – Assume positive intent.

• No lesson is ever perfect.
  – Focus on what you can use to improve your classroom instruction.
  – Focus on the student’s and teacher’s interaction with the Mathematics.
Observing Students: Discussion

• What do you think Jodi’s mathematical goal is in choosing this particular task?

• Given the investigation that Jodi developed, what mathematical ideas and strategies do you expect to see as the children set to work?
Observing Students: Preview Video 2 and 3
While watching the videos…

• Think about some of the strategies from the landscape you see Jodi’s students using.

• Find one example of how Jodi is using formative assessment to move her students along the landscape.
Observing Students: Discussion Video 2 and 3

- Think about some of the strategies from the landscape you see Jodi’s students using.

- Find one example of how Jodi is using formative assessment to move her students along the landscape.
Observing Students: Discussion Video 2 and 3

• Think about some of the strategies from the landscape you see Jodi’s students using.

• Find one example of how Jodi is using formative assessment to move her students along the landscape.
Populating Your Landscape

• Consider what big ideas, strategies or models we can now add to our landscape as a result of our learning.
Counting
Need for Organization & Keeping Track
Model with symbols
Cardinality
Modeling of action
Modeling of situation
Using 10 structure
Making 10s
Model with Groups
Uses 1-9 sequence when counting
Place determines value
Unitizing
Synchrony: one word for every object
One-to-one tagging
One-to-one correspondence
Models quantities with tallies
Skip counting
Counting on
Systematic production of arrangements
Strategies
Big Ideas
Model
Populate the Landscape
Balcony View

1. Quiet Reflection: (3 min.)
Looking at your note taker, what formative assessment practices did you notice in these video clips?

2. Discuss in your table groups.
Reviewing Task Considerations

✓ Does the task expose students’ current levels of understanding in relation to the mathematics learning target?

✓ Is it problematic for students?

✓ Does the cognitive demand enable students opportunities to expose the depth of their knowledge?

✓ Does the task have multiple entry and exit points?

✓ Is the context and the mathematics of the task relevant to students?

(Van de Walle et. all, p. 19, 2014)
LUNCH
11:30 to 12:30

• See you promptly at 12:30.
• Remember to sign in when you return.
Explore ways in which FA attributes and practices move learning forward.

- Participants will construct a progression of learning.
- Participants will discuss FA practices observed.

Context of Learning: x and ÷ + and -

How to embed formative assessment practices into daily math instruction.
Bringing it Back to Your Classroom

• Discuss with your table partners how this would look in your classroom at the beginning of the year.
  • How would you differentiate the task for students who are unable to count to 10? To 5?
  • What would these students count and how would you facilitate their learning?
  • How would you use your data from Kindergarten Portfolio Math Task 1 to structure this in your classroom?
  • How could the children organize their data?
• How would you adjust this task for mid year? End of year?
  • When would students be responsible for adding labels to the items being inventoried?
Observing with Purpose
How Many Do You See?
TIPS for Class Discussions

Engaging all learners and keeping cognitive demand high

• Helping Individual Students Clarify and Share Their Own Thoughts.

• Helping Students Orient to the Thinking of Others.

• Helping Students Deepen Their Own Reasoning.

• Helping Students Engage with the Reasoning of Others.
“The landmarks in this journey are not necessarily sequential. Many paths can be taken toward this horizon. Some landmarks are, of course, precursors to others.” ~Cathy Fosnot
Populate the Landscape

- Strategies
  - Big Ideas
  - Model

- Model with symbols
- Model withGroups
- Using 10 structure
- Uses 1-9 sequence when counting
- Need for Organization & Keeping Track
- Cardinality
- Counting
- Subitizing

- Using known facts
- Using doubles or near doubles
- Making 10s
- Combining that make 10
- Unitizing

- Systems production of arrangements
  - Counting on
  - Skip counting
  - Models quantities with tallies
  - One-to-one correspondence
  - One-to-one tagging
  - Synchrony: one word for every object

- Modeling of action
- Modeling of situation
Data Collection – Kidwatching and the landscape

<table>
<thead>
<tr>
<th>Name: Sharon V.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NAME:</strong> Sharon V.</td>
</tr>
<tr>
<td><strong>DIVISION OF FRACTIONS</strong></td>
</tr>
<tr>
<td>Uses measurement interpretation</td>
</tr>
<tr>
<td>Uses partitive interpretation</td>
</tr>
<tr>
<td>Set models</td>
</tr>
<tr>
<td>Answers that are not whole numbers</td>
</tr>
<tr>
<td>Understands standard algorithm</td>
</tr>
<tr>
<td><strong>MATHEMATICAL PRACTICES</strong></td>
</tr>
<tr>
<td>Makes sense of problems and perseveres</td>
</tr>
<tr>
<td>Models with mathematics</td>
</tr>
<tr>
<td>Uses appropriate tools</td>
</tr>
</tbody>
</table>

---

**Model Landscape**
- Commutative Property
- Associative Property
- Distributive Property
- Using tens times
- Using five times
- Partial products
- Using familiar facts
- Repeated addition can be regrouped
- Repeated Addition
- Skip Counting
- Unbinding
- The relationship between partition and quotient division
- Using out or counting at groups and counting the groups
- Try to make equal sized groups through trial and error

**Student Observations on 4th Grade Landscape**

<table>
<thead>
<tr>
<th>Big Idea</th>
<th>Date Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commutative Property</td>
<td></td>
</tr>
<tr>
<td>Distributive Property</td>
<td></td>
</tr>
<tr>
<td>Relationship between rows and columns</td>
<td></td>
</tr>
<tr>
<td>These values patterns that occur when using the 10</td>
<td></td>
</tr>
<tr>
<td>Repeated addition can be regrouped</td>
<td></td>
</tr>
<tr>
<td>The relationship between partition and quotient division</td>
<td></td>
</tr>
</tbody>
</table>
Balcony View

① Quiet Reflection: (3 min.)
Looking at your note taker, what formative assessment practices did you notice in these video clips?

② Discuss in your table groups.
Considering Formative Assessment and the Landscape

✔ Clarify intended learning

✔ Elicit evidence.

✔ Interpret evidence

✔ Act on evidence
Review and Connections

Partner Conversation:
(4 min.)

1. What are the NVACS critical areas for my grade level?

2. How does the landscape support the critical areas?

3. What are some of the connections between the NVACS standards and the Mathematical landscape?
Considering the Critical Areas and the Mathematical Landscape

Students use **numbers**, including **written numerals**, to represent **quantities** and to solve **quantitative problems**, such as **counting objects** in a set; **counting out** a given number of objects in a set; **comparing sets** or numerals; and modeling simple **joining** and **separating** situations with sets of objects, or eventually with **equations** such as \( 5 + 2 = 7 \) and \( 7 - 2 = 5 \).
## Considering the NVACS Standards

<table>
<thead>
<tr>
<th>NVACS Standards</th>
<th>Language from the standards</th>
<th>Big Ideas from the landscape</th>
<th>Strategies and Models to use with students</th>
</tr>
</thead>
</table>
| K.CC.4a          | • number name  
|                  | • standard order  
|                  | • pairing         | • Need for Organization and Keeping Track  
|                  |                  | • One-to-one correspondence            | • Actions  
|                  |                  |                                  | • Situation  
|                  |                  |                                  | • Synchrony  
|                  |                  |                                  | • 1-1 tagging  
| K.OA.3           | • decompose  
|                  | • equal         | • Commutative                     | • Pictures  
|                  | • pairs         | • Associative                     | • Model with Symbols  
|                  | • drawings      | • Equivalence                     | • Splitting  
| K.NBT.1          | • compose       | • Place Determines Value          | • Additive Structuring  
|                  | • Decompose     | • Unitizing                       | • 5 Structure  
|                  | • Tens and ones | • Grouping                        | • 10 Structure  
|                  | • further ones  |                                  | • Landmarks  
|                  |                  |                                  | • Counting  
|                  |                  |                                  | • Splitting  

**Notes:**
- **K.CC.4a:** Number names, standard order, and pairing.
- **K.OA.3:** Decomposition into equal pairs, with additional visual aids like decomposing drawings.
- **K.NBT.1:** Composition and decomposition of tens and ones, further ones, with emphasis on place value and unitizing.
Review the standards. Where are the connections to addition and subtraction?
Explore ways in which FA attributes and practices move learning forward.

Plan next steps in embedding learning into practice.

Vertical teams will begin planning instruction, within today’s context, embedding formative assessment practices in plans.

Compose and share a definition or quote capturing the essence of what formative assessment is.

- Participants will construct a progression of learning.
- Participants will discuss FA practices observed.

Context of Learning:
\[ x \quad \div \quad + \quad and \quad - \]
Fall PD Opportunities

✓ District-Wide PLTs (16 hours, 1 credit)
  ✓ Continue your learning from today!
  ✓ Four follow-up sessions: 9/25, 12/4, 2/26, 5/21
  ✓ Work in grade level teams
  ✓ Content tied to “Pacing Timeline” to provide “just in time” connected learning opportunities to prepare for the next quarter of content
✓ Computational Strategies/Algorithms (8 hours, ½ credit)
✓ Core Connections: K-5 (8 hours, ½ credit) Same as offered last year. Join us if you missed it or have changed grade levels!
✓ eSuite Basic (no credit): 8/19 or 9/9
✓ Intensification Lab: During winter break
✓ Planning in Math (16 hours, 1 credit)
✓ Using Classroom Discussion to Promote Problem Solving and Solution Strategies in Mathematics
✓ Formative Assessment Webinars – (October – November)

Check Solutionwhere frequently for updates and additional opportunities!
Resources to Support

- **WCSD Pacing Timeline**

- **WCSD Curriculum Documents**

- **Instructional Practice Guides**

District Performance Plan Goal 1, Objective 4: The percentage of site administrators, instructional coaches, teachers, and instructional ESPs who report familiarity with WCSD Core NACS Materials and curriculum tools adopted and/or endorsed by WCSD will reach 50% by October 2014 and 75% by May 2015.
Break and Moving to Verticals
2:45-3:30 Vertical Teams with your School

Each school will turn in an exit ticket that addresses the guiding question. Individual copies of the questions have been provided for you to refer to during the discussion.

Guiding Question:

- How does the Mathematical landscape and formative assessment fit with my current teaching practices and my knowledge about nurturing and developing young mathematicians?

- **Presenters: List schools and assigned room numbers here!**