

Basic Math Facts: A Sequence of Learning

Students should be computing fluently with whole numbers. *Computational fluency* refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate flexibility in the computational methods they choose, understand and can explain these methods, and produce accurate answers efficiently. The computational methods that a student uses should be based on mathematical ideas that the student understands well. Fluency with whole-number computation depends, in large part, on fluency with basic number combinations—the single-digit addition and multiplication pairs and their counterparts for subtraction and division. Fluency with the basic number combinations develops from well-understood meanings for the four operations and from a focus on thinking strategies (Thornton 1990; Isaacs and Carroll 1999).

Principles and Standards for School Mathematics,
National Council of Teachers of Mathematics, 2000

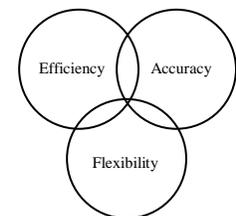
Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

Principles to Actions,
National Council of Teachers of Mathematics, 2014

Basic math facts are defined as computations involving the four basic math operations: addition, subtraction, multiplication, and division; using the single-digit numbers, 0 – 9. These basic facts are often referred to in current literature as “basic number combinations”. This phraseology is intended to focus less emphasis on discrete factual pieces of information and more on the relationships that exist and are critical to retention and recall of this information. The term “math facts” will be used here for the sake of clarity and consistency but, in fact, emphasis will be on constructing understanding of number and operation and on the development of strategies, an approach much more in line with “basic number combinations.”

The basic math facts form the foundation of successful subsequent computational fluency in two regards. First, they are the component operations used for multi-digit computation. Second, and equally important, they provide one of the foundations for strategies used for successful flexible, efficient, accurate and appropriate computation with multi-digit numbers.

Computational Fluency



Thus, both the actual retention and the activities leading to retention form an important step in the development of the sense of number and operation expected of all students. Fact acquisition is so important that the National Council of Teachers of Mathematics has identified the development of math facts in elementary school as focal points in grades 1 – 4 (*Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence*, National Council of Teachers of Mathematics, 2006)

The acquisition of basic math facts should occur in the following three phases:

- Phase I: Constructing meaning and counting strategies
- Phase II: Reasoning strategies
- Phase III: Working toward quick recall (occasionally referred to as *mastery*- producing answers efficiently and accurately)

These three phases should not be perceived as distinct, discreet units. There is much overlap and students often work simultaneously within these phases. Additionally, it is important to point out here that these three phases are critical for each of the four basic operations. However, addition is often connected to “related” subtraction facts and multiplication to “related” division facts. (Baroody, 2006 p. 22; Baroody, Bajwa, and Eiland, 2009; Carpenter et al. 1999; NCTM, Principles to Actions 2014)

Phase I: Constructing Meaning and Counting Strategies

Sense of number, the relationships between numbers, and understanding of operation, form the foundations upon which fact acquisition can occur. In their earliest work with quantity and size, children develop important sense about numbers – one-to-one correspondence, conservation of quantity, number pattern, counting, numeral, and relative size. These understandings are strengthened as children experience problems related to quantity and size. At the earliest stages of number development, children encounter many real-life

problems involving combining, separating, part/whole relationships, equal groups, and comparing sets. All of these experiences serve as the foundation for constructing meaning of both number and the basic operations.

As students' number sense grows, they experience the need to find strategies for solving problems related to numbers. These early strategies involving counting (from 1, counting on, counting back) further support students' sense of number.

This early development of number is critical for many reasons:

1. It provides the student with a deep conceptual understanding of both number (especially related to oneness) and operation.
2. It provides the student with a strategy. It is impossible to consider acquiring multiple strategies and flexibility with strategies until a single strategy is in place.
3. All other relationships and strategy development is directly dependent on these understandings.

Phase II: Reasoning Strategies

When focusing on the acquisition of basic math facts Phases I and II are by far the most critical. Phase II involves the student in seeking efficient strategies for the solving of basic number combinations. In so doing, students acquire the ability to look at numbers and operations in many equivalent ways. "Number relationships provide the foundation for strategies that help students remember basic facts." (Van De Walle, 2007, 2013 p. 171)

This phase focuses the student on continually using known number facts to solve those facts not yet retained. This phase encourages students to identify a wide array of relationships both between strategies and between the operations. Within Phase II, students develop the properties of operations (identity, commutative, associative, and distributive) for the purpose of relating many of the facts. Students discover many compositions of numbers and their usefulness in solving problems.

Through these patterns, relationships, and compositions, students continue to strengthen their senses of number and operation and develop strategies that can be applied as they solve problems involving larger whole numbers as well as fractions, decimals, other number systems, and symbolic generalizations (algebra).

Phase III: Working Toward Quick Recall

Quick recall of math facts is usually defined as the ability to solve a basic number computation in 3 seconds or less (without resorting to inefficient methods like counting). This is somewhat different than "instant recall" in that it does not preclude a student's ability to use a known fact to quickly derive an unknown one. Phase III is simply about increasing a student's quick recall. This involves:

- increasing the speed in which the student selects and applies a strategy for solving the problem and
- using highly organized and planned practice for the purpose of devoting facts to memory.

Phase III may include practice (and/or drill) with specific groups of facts through fact triangle cards, games, paper/pencil practice, and the use of software specifically designed for this purpose.

Successfully addressing Phases I and II will significantly decrease the need for prolonged work in Phase III. Often this phase focuses on particularly stubborn facts for which fewer relationships exist. Once students acquire the ability to quickly recall the facts, Phase III focuses on maintenance of the facts. Even better than the above-mentioned activities is the continual expectation of using mental computation for the purpose of reinforcing the basic number combinations.

Key Beliefs Related to the Acquisition of Math Facts

1. All children can learn the basic math facts with understanding. “All children are able to master the basic facts – including children with learning disabilities. All children can construct efficient mental tools that will help them.” (Van De Walle, 2007)
2. All three phases are critical to subsequent retention and recall of the basic math facts as well as the development of mathematical fluency in general. Students who skip any stage risk developing surface understanding of number and operation and/or fail to develop flexibility with number, which is necessary for successful problem solving. “Instruction must help students through these phases without rushing to know their facts from memory.” (Van de Walle, 2013 p. 171).
3. Phase III should not be an emphasis until the student has developed a deep understanding of number and the operation involved and has acquired a wide array of strategies for use in solving single-digit computation. “Premature drill, however, will certainly be ineffective, waste valuable time, and for many students contribute to a strong distaste for and a faulty view of learning mathematics.” (Van de Walle, 2007). “Drill in the absence of accomplishing these phases has repeatedly been demonstrated as ineffective” (Van de Walle, 2014 p. 184).
4. Acquisition of basic facts should be a component of the mathematics program not a pre-cursor to it. “There is no reason that a child who has not yet mastered all basic facts should be excluded from real mathematical experiences.” (Van de Walle, 2007) We should not restrict any student’s access to “the good stuff” of mathematically challenging concepts and problems. Mathematics is not solely about computation. Mathematics is about reasoning and using patterns and making sense of things. (Van de Walle, 2013, p.188)
5. We should never reach a point of believing that the student “just can’t retain the facts.” A student’s failure to retain math facts suggests the need for more focus on early concepts of number and operation and subsequently more experiences with representations of the strategies and relationships of Phase II.
6. “This approach (use of the phases) supports students, over time, in knowing, understanding, and being able to use their knowledge of number combinations meaningfully in new situations. (NCTM, Principles to Actions, 2014).
7. Software for the purpose of drill and practice must be designed and used to meet the specific needs of the student and adjust to present practice in a method that is both effective and efficient.

Students can learn to compute accurately and efficiently through regular experience with meaningful procedures. They benefit from instruction that blends procedural fluency and conceptual understanding (Ginsburg, Klein, and Starkey 1998; Hiebert 1999). This is true for all students, including those with special educational needs. Many children with learning disabilities can learn when they receive high-quality, conceptually oriented instruction. Special instructional interventions for those who need them often focus narrowly on skills instead of offering balanced and comprehensive instruction that uses the child's abilities to offset weaknesses and provides better long-term results (Baroody 1996). As students encounter problem situations in which computations are more cumbersome or tedious, they should be encouraged to use calculators to aid in problem solving. In this way, even students who are slow to gain fluency with computation skills will not be deprived of worthwhile opportunities to solve complex mathematics problems and to develop and deepen their understanding of other aspects of number.

Principles and Standards for School Mathematics,
National Council of Teachers of Mathematics, 2000

Grade-Level Timeline for Math Fact Development

		K	Grade 1				Grade 2				Grade 3				Grade 4				Grade 5
			Q.1	Q.2	Q.3	Q.4													
Addition & Related Subtraction Facts	Phase I Constructing Meaning	X	X	X	X	X													
	Phase II Reasoning Strategies		X	X	X	X	X	X	X	X									
	Phase III Working Toward Recall						X	X	X	X	X	X	X	X	X	X	X	X	X
Multiplication & Related Division Facts	Phase I Constructing Meaning									X	X	X	X	X					
	Phase II Reasoning Strategies									X	X	X	X	X	X	X	X	X	
	Phase III Working Toward Recall														X	X	X	X	X

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