Learning Progressions

A Learning Progression: A carefully sequenced set of building blocks consisting of subskills and bodies of enabling knowledge that, it is believed, students must master on route to mastering a more remote curricular aim.

Popham, 2008

Progressions for the Common Core State Standards in Mathematics (draft)
©The Common Core Standards Writing Team
20 May 2011

K. Counting and Cardinality; K-5, Operations and Algebraic Thinking

Counting and Cardinality and Operations and Algebraic Thinking are about understanding and using numbers. Counting and Cardinality involves understanding number names, the basic operations—addition and subtraction—and the basic number sense relationships they model, and consequently the kinds of problems they can be used to solve as well as their mathematical properties and relationships.

Although many of the standards organized under the OA heading involve whole numbers, the importance of the Progression is much more general because it describes concepts, properties, and representations that extend to other number systems, as measured, and to algebra. For example, if the focus of the unit is on integers, and the focus of the unit in the algebraic system is on integers, then the focus of the number system as a whole is on integers. In this example of additive reasoning, it doesn't matter whether x and y are whole numbers, fractions, decimals, or even variables. Likewise, a property such as distributive laws for all the number systems that students will study in K-12, including complex numbers.

The generality of the concepts involved in Operations and Algebraic Thinking means that students’ work in this area should be designed to help them understand arithmetic beyond whole numbers (i.e., the N and N0 Progressions) and extend and employ operations and equations in later grades (i.e., the U.S. Progressions). Addition and subtraction are the first operations studied. In
Constructing a Landscape

Big Ideas

Strategies

Mathematical Models
Big Ideas, Strategies, Models

Working in your group:

• Which are you familiar with? Which does your group need to know more about?

• How might you group these or place these in a “landscape” to show the mathematical connections?

• Do any build on each other?
Activity: “Big Idea”

Step 1: Critical Content

- Identify big ideas, strategies, and models related to (x and ÷) / (+ and -)
- Discuss with your face partner which critical content area(s) focus(es) on (x and ÷) / (+ and -)
Activity: “Big Idea”

Step 2:
• Quickly skim and scan cluster headings and standards related to multiplication and division or addition and subtraction.
BREAK TIME
Formative Assessment Process

1. Elicit Evidence
2. Interpret Evidence
3. Act on Evidence
4. Clarify Intended Learning
5. Questioning/Discussion
Eliciting Evidence of Learning

We have to know where we are **before** deciding where we need to go.
Mentally solve each problem. Record your answer using an **area model**.

- $2 \times 5$
- $2 \times 10$
- $2 \times 14$
- $20 \times 14$
Populate your Landscape
Groups can be regrouped

Doubling and halving

Doubling

Partial Products

Distributive Property

Using 10x's

Place Value Matters

Standard Algorithm

Area Model/Open Array
How can we use the area model as a strategy to show understanding of division?

140 ÷ 14

28 ÷ 14

168 ÷ 14

154 ÷ 14
Populate your Landscape
Connection of x and ÷

Doubling and halving

Doubling

Groups can be regrouped

Partial Products

Distributive Property

Using 10x’s

Partial Quotients

Place Value Matters

Unitizing

Grouping

Standard Algorithm

Area Model/Open Array
Norms for Watching Teaching Videos

• Teaching is multi-faceted.
  – *The video doesn’t show everything.*

• Teaching is incredibly hard work!
  – *Assume positive intent.*

• No lesson is ever perfect.
  – *Focus on what you can use to improve your classroom instruction.*
  – Focus on the student’s and teacher’s interaction with the Mathematics.
LUNCH
11:30 to 12:30

• See you promptly at 12:30.
• Remember to sign in when you return.
Connection of x and ÷

Place Value Matters

Counting 3 times

Skip Counting

Groups can be regrouped

Doubling and halving

Doubling

Partial Products

Repeated Addition

Number Line

Distributive Property

Partial Quotients

Using 10x’s

Commutative Property

Place Value Matters

Unitizing

Grouping

Reasoning and Error

Dealing

Social Knowledge

Arrays/area models

Standard Algorithm
Explore ways in which FA attributes and practices move learning forward.

- Participants will construct a progression of learning.
- Participants will discuss FA practices observed.

How to embed formative assessment practices into daily math instruction.

Context of Learning: x and ÷ + and -
Observing with Purpose
One day, in the teacher’s lounge, there was a conversation about planning a 5th grade science field trip. One hundred fifty-six 5th graders had signed up to attend and there were 13 chaperones to help. The teachers were trying to decide how many students should be in each group if they made 13 equal groups.

Considering strategies used by early 5th grade students, with your table group, what steps would you use to solve this task?

Solve and record your 5th grade strategies.
Task

There are two lots, each 50 yards by 100 yards. In one case, $\frac{3}{4}$ of the lot is to be playground and $\frac{2}{5}$ of that amount is to be blacktopped for the middle school students. In the second case, $\frac{2}{5}$ of the lot is to be playground and $\frac{3}{4}$ of that will be blacktopped. Explore and compare the two areas.

Adapted from: Fosnot/Dolk. Young Mathematicians at Work.
Populate your Landscape
Connection of x and ÷

Doubling and halving
Doubling
Groups can be regrouped
Partial Products
Commutative Property
Distributive Property
Partial Quotients
Using 10x’s
Place Value Matters
Unitizing
Grouping

Area Model/Open Array

Standard Algorithm
Balcony View

① Quiet Reflection: (3 min.)
Looking at your note taker, what formative assessment practices did you notice in these video clips?

② Discuss in your table groups.
Reviewing Task Considerations

✓ Does the task expose students’ current levels of understanding in relation to the mathematics learning target?

✓ Is it problematic for students?

✓ Does the cognitive demand enable students opportunities to expose the depth of their knowledge?

✓ Does the task have multiple entry and exit points?

✓ Is the context and the mathematics of the task relevant to students?

(Van de Walle et. all, p. 19, 2014)
TIPS for Class Discussions
Engaging all learners and keeping cognitive demand high

• Practice “wait time”
• Clarify students ideas in a variety of ways
• Emphasize reasoning
• Encourage student –student dialogue.

(Van de Walle et all, 2014)
Data Collection – Kidwatching and the landscape

NAME: Sharon V.

<table>
<thead>
<tr>
<th>Name</th>
<th>Not Yet</th>
<th>On Target</th>
<th>Above and Beyond</th>
<th>Comments</th>
</tr>
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<tbody>
<tr>
<td>Connie</td>
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<td>Jeanine</td>
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<td>Marti</td>
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<td>Bridget</td>
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<td>Robin</td>
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<td>Terikia</td>
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**Mathematical Practices**

- Makes sense of problems and perseveres: ✓
- Models with mathematics: ✓
- Uses appropriate tools: ✓

**Big Idea**

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<thead>
<tr>
<th>Date Observed</th>
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**Student Observations on 4th Grade Landscape**

<table>
<thead>
<tr>
<th>Comutative Property</th>
<th>Date Observed</th>
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Considering Formative Assessment and the Landscape

- Clarify intended learning
- Elicit evidence.
- Interpret evidence
- Act on evidence
Partner Conversation:
(4 min.)

1. What are the NVACS critical areas for my grade level?

2. How does the landscape support the critical areas?

3. What are some of the connections between the NVACS standards and the mathematical landscape?
Considering the Critical Areas and the Mathematical Landscape

1. Developing fluency with addition and subtraction of fractions with unlike denominators, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions).

2. Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations.

3. Developing understanding of volume.
<table>
<thead>
<tr>
<th>NVACS Standards</th>
<th>Language from the standards</th>
<th>Big Ideas from the landscape</th>
<th>Models and strategies to use with students</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.NBT.1</td>
<td>*10 times</td>
<td>*Distributive Property of Multiplication *place value patterns</td>
<td>*t-chart or ratio table *Array Model *partial products</td>
</tr>
<tr>
<td>5.NBT.2</td>
<td>*multiplying a number by the powers of 10</td>
<td>*Distributive Property of Multiplication *place value patterns</td>
<td>*T-Chart *array Model *partial Products *using ten-times</td>
</tr>
<tr>
<td>5.NBT.3a</td>
<td>*decimals *base ten numerals *power of 10</td>
<td>*Distributive Property of Multiplication *place value patterns</td>
<td>*T-Chart *partial quotients *using ten-times</td>
</tr>
<tr>
<td>5.NBT.5</td>
<td>*multiply multi digit numbers using the standard algorithm</td>
<td>*relationship between rows and columns *place value patterns</td>
<td>*T-Chart *array Model *partial Products *using ten-times</td>
</tr>
<tr>
<td>5.NBT.6</td>
<td>*find whole-number quotients of whole numbers</td>
<td>*relationship between multiplication and division *place value patterns</td>
<td>*open array *partial quotients</td>
</tr>
</tbody>
</table>
The Landscape’s Big Idea

- 5.NBT.1 “10 times”
- 5.NF.4b “multiplying”
- 5.MD.1 “convert”
- 5.NBT.2 “powers of 10”
- 5.MD.5a “volume”
- 5.MD.2 “operations”
- 5.OA.1 & 2 “expressions”
- 5.NBT.3a “decimals”
- 5.NF.7 “division”
- 5.OA.3 “patterns”
- 5.NBT.5 “algorithm”
- 5.NF.6 “multiplication”
- 5.NBT.7 “mult. and div.”
- 5.NBT.6 “quotients”

- Review the standards.
- Where are the connections to multiplication and division.
Explore ways in which FA attributes and practices move learning forward.

Plan next steps in embedding learning into practice.

PLCs will begin planning instruction, within today’s context, embedding formative assessment practices in plans.

- Participants will construct a progression of learning.
- Participants will discuss FA practices observed.

Context of Learning: x and ÷ + and -

Develop a conceptual understanding of the Formative Assessment Process and its benefits.

Compose and share a definition or quote capturing the essence of what formative assessment is.

How to embed formative assessment practices into daily math instruction.
Fall PD Opportunities

- District-Wide PLTs (16 hours, 1 credit)
  - Continue your learning from today!
  - Four follow-up sessions: 9/25, 12/4, 2/26, 5/21
  - Work in grade level teams
  - Content tied to “Pacing Timeline” to provide “just in time” connected learning opportunities to prepare for the next quarter of content

- Computational Strategies/Algorithms (8 hours, ½ credit)

- Core Connections: K-5 (8 hours, ½ credit) Same as offered last year. Join us if you missed it or have changed grade levels!

- eSuite Basic (no credit): 8/19 or 9/9

- Intensification Lab: During winter break

- Planning in Math (16 hours, 1 credit)

- Using Classroom Discussion to Promote Problem Solving and Solution Strategies in Mathematics

- Formative Assessment Webinars – (October – November)

Check Solutionwhere frequently for updates and additional opportunities!
Resources to Support

- WCSD Pacing Timeline
- WCSD Curriculum Documents
- Instructional Practice Guides

District Performance Plan Goal 1, Objective 4: The percentage of site administrators, instructional coaches, teachers, and instructional ESPs who report familiarity with WCSD Core NACS Materials and curriculum tools adopted and/or endorsed by WCSD will reach 50% by October 2014 and 75% by May 2015.
Break and Moving to Verticals
2:45-3:30 Vertical Teams with your School

Each school will turn in an exit ticket that addresses the guiding question. Individual copies of the questions have been provided for you to refer to during the discussion.

Guiding Question:

• How does the Mathematical landscape and formative assessment fit with my current teaching practices and my knowledge about nurturing and developing young mathematicians?

• Presenters: List schools and assigned room numbers here!
References


• Fosnot, Catherine, and Dolk, Maarten. (2001). **Young Mathematicians at Work. Constructing Multiplication and Division.** Heinemann, NH.

• Fosnot, Catherine, and Dolk, Maarten. (2001). **Young Mathematicians at Work. Constructing Addition and Subtraction.** Heinemann, NH.


• Gorski, Paul. (2013). **Reaching and Teaching Students in Poverty.** Teachers College Press, New York, NY

