Learning Progressions

Learning Progression Model

- Block of Knowledge
- Subskill
- Curricular Aim

Success Criteria
- FA Checks

3 2 4 9
(Lessons)

Learning Progression: A carefully sequenced set of building blocks consisting of subskills and bodies of enabling knowledge that, it is believed, students must master on route to mastering a more remote curricular aim.

Popham, 2008

Progressions for the Common Core State Standards in Mathematics (draft)

©The Common Core Standards Writing Team
20 May 2011

K, Counting and Cardinality; K-5, Operations and Algebraic Thinking

Counting and Cardinality and Operations and Algebraic Thinking are about understanding and using numbers. Counting and Cardinality includes Operations and Algebraic Thinking as well as Number and Operations in Base Ten. It begins with early counting and adding how many in one group of objects. Addition, subtraction, and increasingly complex counting patterns, such as skip counting, are foundational for every beginning. This Progression includes important ideas that are relevant to the domain. These ideas need to be taught or practiced so that students can form a strong foundation for future learning.

Although many of the standards presented under the OA Progression involve whole numbers, the importance of the Progression is much more general because it introduces concepts, properties, and representations that extend to other number systems, to measures, and to algebra. For example, if the base of the system is a digit, and the base of the number system is an integer, then the base of the number system is a base ten system for use in the same sense: 10 is a digit. Likewise, a property such as distributivity for all the number systems that students will study in K-12, including complex numbers. The generality of the concepts involved in Operations and Algebraic Thinking means that students’ work in this area should be designed to help them understand patterns beyond whole numbers (the N and OA Progressions) and understand and apply operations and properties at later grades (the K-12 Progressions).
Constructing a Landscape

Big Ideas

Strategies

Mathematical Models
Big Ideas, Strategies, Models

Working in your group:

• Which are you familiar with? Which does your group need to know more about?

• How might you group these or place these in a “landscape” to show the mathematical connections?

• Do any build on each other?
Activity: “Big Idea”

Step 1: Critical Content

- Identify big ideas, strategies, and models related to (x and ÷) / (+ and -)
- Discuss with your face partner which critical content area(s) focus(es) on (x and ÷) / (+ and -)
Activity: “Big Idea”

Step 2:
• Quickly skim and scan cluster headings and standards related to multiplication and division or addition and subtraction.

Packet Page 13
BREAK TIME
Formative Assessment Process

1. Clarify Intended Learning
2. Elicit Evidence
3. Interpret Evidence
4. Act on Evidence
5. Questioning/Discussion
Eliciting Evidence of Learning

We have to know where we are before deciding where we need to go.
In preparation for a big family gathering, I have purchased a turkey. According to my cookbook, the recommended cooking time for my turkey is 15 minutes per pound. If my turkey weighs 24 pounds, how many hours must my turkey cook?

Considering strategies used by early third grade students, with your table group, how might this task be solved? Record your strategy on chart paper.

Learning Target: Students will develop a conceptual understanding of the meaning of multiplication. (GRADE 3)
Task Activity – Third Grade

Gallery Walk: Making Connections to your Classroom

• As you observe solutions offered by other groups, consider what you might expect to observe with your students.

• Based on experience, what typical steps and missteps do you anticipate?

• Think about what Math Practices you experienced in this task.
The turkey is 24 pounds.

15 min. per pound.

1 jump is 15 min. and 1 pound cooking.

$15 \times 24 = 360$

---

**Skip Counting**

We split 15 into 10 and 5.

10 20 30 40 50 60
70 80 90 100 110 120
130 140

$10 \times 24 = 240$

6 hours and 20 min.

$40 \times 20 = 60$

---

**Ratio table**

We counted by 3s. So is pounds total. We counted 15 jumps total. The numbers were 1, 2, 3, and 15. In all hours and was the number 15.

$15 \times 24 = 360$

---

**Distributive property**

$30 + 30 = 60$

$60 + 30 = 90$

$90 + 30 = 120$

$120 + 30 = 150$

$150 + 30 = 180$

$180 + 30 = 210$

$210 + 30 = 240$

$240 + 30 = 270$

$270 + 30 = 300$

$300 + 30 = 330$

$330 + 30 = 360$

$360 \times 12 = 360$

We know that $15 + 15 = 30$ and $24 + 12 = 36$

So we took a shortcut and got 360. 6 hours.
Norms for Watching Teaching Videos

• Teaching is multi-faceted.
  – *The video doesn’t show everything.*

• Teaching is incredibly hard work!
  – *Assume positive intent.*

• No lesson is ever perfect.
  – *Focus on what you can use to improve your classroom instruction.*
  – Focus on the student’s and teacher’s interaction with the Mathematics.
The Landscape of Learning

• Big Ideas- “the central, organizing ideas of mathematics—principles that define mathematical order” (Schifter and Fosnot 1993, 35).

• Strategies- Use of organized patterns to develop mathematical understanding and reasoning.

• Models- Representation of thought that eventually serves as a tool for thought.

Populating Your Landscape

Proportional Reasoning

Ratio Table

Doubling

Doubling and halving

Groups can be regrouped

Skip Counting

Repeated Addition

Unitizing

Grouping

Distributive Property

Decomposition, Structures

Number Line
Balcony View

1. Quiet Reflection: (3 min.)
Looking at your note taker, what formative assessment practices did you notice in these video clips?

2. Discuss in your table groups.
Reviewing Task Considerations

✓ Does the task expose students’ current levels of understanding in relation to the mathematics learning target?

✓ Is it problematic for students?

✓ Does the cognitive demand enable students opportunities to expose the depth of their knowledge?

✓ Does the task have multiple entry and exit points?

✓ Is the context and the mathematics of the task relevant to students?

(Van de Walle et. all, p. 19, 2014)
LUNCH
11:30 to 12:30

- See you promptly at 12:30.
- Remember to sign in when you return.
Explore ways in which FA attributes and practices move learning forward.

- Participants will construct a progression of learning.
- Participants will discuss FA practices observed.

Context of Learning: x and ÷, + and -

How to embed formative assessment practices into daily math instruction.
Populating Your Landscape

- Proportional Reasoning
  - Ratio Table
  - Doubling
  - Doubling and halving
  - Groups can be regrouped
- Skip Counting
  - Repeated Addition
  - Number Line
- Distributive Property
  - Unitizing
  - Grouping

Elicit Evidence
Observing with Purpose
Task

• Task 1: There is a soda machine that has only one type of soda. The machine holds 156 cans of soda. How many six packs of soda would it take to fill the machine?

• Task 2: A drink machine holds 156 cans of soda. The machine has 6 columns, each containing a different type of drink. How many cans of each flavor?
Reflection

Before continuing with this observation, discuss with your table group:

• What did you notice?
• Where are the students on the landscape?
• What feedback would you provide?
Reflection

Before continuing with this observation, discuss with your table group:
• What did you notice?
• What did the teacher do?
• What are other strategies that you may have used?
Populating Your Landscape

• Add or revise your landscape to include new ideas discussed from watching these videos.
Populating Your Landscape

- Using Arrays As Models
- Decompositional Structures
- Distributive Property
- Commutative Property
- Relationship between partitive and quotative
- Using 10x’s
- Connection of x and ÷
- Grouping
- Unitizing
- Partial Products
- Number line
- Repeated Addition
- Groups can be regrouped
- Doubling and halving
- Doubling
- Skip Counting
- T-Chart
- Proportional Reasoning
Balcony View

1. Quiet Reflection: (3 min.)
   Looking at your note taker, what formative assessment practices did you notice in these video clips?

2. Discuss in your table groups.
TIPS for Class Discussions
Engaging all learners and keeping cognitive demand high

• Practice “wait time”
• Clarify students ideas in a variety of ways
• Emphasize reasoning
• Encourage student – student dialogue.

Want to learn more:
See the list of Professional Development Offerings!
Data Collection – Kidwatching and the landscape

NAME: Sharon V.

<table>
<thead>
<tr>
<th>DIVISION OF FRACTIONS</th>
<th>NOT THERE YET</th>
<th>ON TARGET</th>
<th>ABOVE AND BEYOND</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uses measurement interpretation</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uses partitive interpretation</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Set models</td>
<td>✓</td>
<td></td>
<td>Used two color counters to show $\frac{1}{2} \times 3$ =</td>
<td></td>
</tr>
<tr>
<td>Answers that are not whole numbers</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Understands standard algorithm</td>
<td>✓</td>
<td></td>
<td>Showing greater reasonableness</td>
<td></td>
</tr>
</tbody>
</table>

MATHMATICAL PRACTICES

| Makes sense of problems and perseveres | ✓ | Stated problem in own words |
| Models with mathematics | ✓ | Reluctant to use abstract models |
| Uses appropriate tools | ✓ | | |

Student Observations on 4th Grade Landscape

<table>
<thead>
<tr>
<th>Big Idea</th>
<th>Date Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commutative Property</td>
<td></td>
</tr>
<tr>
<td>Distributive Property</td>
<td></td>
</tr>
<tr>
<td>Relationship between rows and columns</td>
<td></td>
</tr>
<tr>
<td>Reason why answers are the same</td>
<td></td>
</tr>
<tr>
<td>Repeated addition can be regrouped</td>
<td></td>
</tr>
<tr>
<td>The relationship between the operations</td>
<td></td>
</tr>
</tbody>
</table>
Considering Formative Assessment and the Landscape

- Clarify intended learning
- Elicit evidence
- Interpret evidence
- Act on evidence
Review and Connections

Partner Conversation:
(4 min.)

1. What are the NVACS critical areas for my grade level?

2. How does the landscape support the critical areas?

3. What are some connections between the NVACS standards and the Mathematical landscape?
Considering the Critical Areas and the Mathematical Landscape

1. Developing understanding of multiplication and division and strategies for multiplication and division within 100.

2. Developing understanding of fractions, especially unit fractions (fractions with numerator 1).

3. Developing understanding of the structure of rectangular arrays and of area.

4. Describing and analyzing two-dimensional shapes.
## Considering the NVACS Standards

<table>
<thead>
<tr>
<th>NVACS Standards Pg. 23 – 26</th>
<th>Language from the standards</th>
<th>Big Ideas from the landscape</th>
<th>Models and strategies to use with students</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.OA.A.1</td>
<td>*number of objects is ____ groups.</td>
<td>*Grouping *Groups can be regrouped</td>
<td>*Skip counting *Repeated addition *Number Line</td>
</tr>
<tr>
<td>3.OA.A.2</td>
<td>*Objects are partitioned equally</td>
<td>*Connection of x and ÷</td>
<td>*Repeated Addition *Dealing *Number Line</td>
</tr>
<tr>
<td>3.OA.A.3</td>
<td>*equal groups *arrays *equation</td>
<td>Commutative Property</td>
<td>*Number line *Array</td>
</tr>
<tr>
<td>3.OA.A.4</td>
<td>*unknown number *multiplication of division</td>
<td>*Commutative Property</td>
<td>*Number line *Skip counting *Open array</td>
</tr>
<tr>
<td>3.OA.B.5.</td>
<td>*Commutative Property *Distributive Property *Properties of operations</td>
<td>*Commutative Property *Distributive Property</td>
<td>*Number line *Skip counting *Open array</td>
</tr>
</tbody>
</table>
The Landscape’s Big Idea

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.OA.A.1</td>
<td>“groups”</td>
</tr>
<tr>
<td>3.NBT.3</td>
<td>“mult. of 10”</td>
</tr>
<tr>
<td>3.MD.1</td>
<td>“intervals”</td>
</tr>
<tr>
<td>3.OA.A.2</td>
<td>“part. equally”</td>
</tr>
<tr>
<td>3.NF.1</td>
<td>“partitioned”</td>
</tr>
<tr>
<td>3.MD.4</td>
<td>“scale”</td>
</tr>
<tr>
<td>3.OA.B.6</td>
<td>“relationship”</td>
</tr>
<tr>
<td>3.OA.A.3</td>
<td>“equal groups”</td>
</tr>
<tr>
<td>3.NF.2a</td>
<td>“interval”</td>
</tr>
<tr>
<td>3.OA.C.7</td>
<td>“relationship”</td>
</tr>
<tr>
<td>3.OA.A.4</td>
<td>“Mult. or div.”</td>
</tr>
<tr>
<td>3.NF.3c</td>
<td>“equivalent”</td>
</tr>
<tr>
<td>3.OA.D.8</td>
<td>“auto. facts”</td>
</tr>
<tr>
<td>3.OA.A.5</td>
<td>“Commutative”</td>
</tr>
</tbody>
</table>

- Review the standards.
- Where are the connections to multiplication and division.
Explore ways in which FA attributes and practices move learning forward.

Plan next steps in embedding learning into practice.

Vertical Teams will begin planning instruction, within today’s context, embedding formative assessment practices in plans.

Participants will construct a progression of learning.

Participants will discuss FA practices observed.

Compose and share a definition or quote capturing the essence of what formative assessment is.

Context of Learning: $x$ and $\div$ + and -

Develop a conceptual understanding of the Formative Assessment Process and its benefits.

How to embed formative assessment practices into daily math instruction.
Fall PD Opportunities

✓ District-Wide PLTs (16 hours, 1 credit)
  ✓ Continue your learning from today!
  ✓ Four follow-up sessions: 9/25, 12/4, 2/26, 5/21
  ✓ Work in grade level teams
  ✓ Content tied to “Pacing Timeline” to provide “just in time” connected learning opportunities to prepare for the next quarter of content

✓ Computational Strategies/Algorithms (8 hours, ½ credit)

✓ Core Connections: K-5 (8 hours, ½ credit) Same as offered last year. Join us if you missed it or have changed grade levels!

✓ eSuite Basic (no credit): 8/19 or 9/9

✓ Intensification Lab: During winter break

✓ Planning in Math (16 hours, 1 credit)

✓ Using Classroom Discussion to Promote Problem Solving and Solution Strategies in Mathematics

✓ Formative Assessment Webinars – (October – November)

Check Solutionwhere frequently for updates and additional opportunities!
Resources to Support

- WCSD Pacing Timeline

- WCSD Curriculum Documents

- Instructional Practice Guides

District Performance Plan Goal 1, Objective 4: The percentage of site administrators, instructional coaches, teachers, and instructional ESPs who report familiarity with WCSD Core NACS Materials and curriculum tools adopted and/or endorsed by WCSD will reach 50% by October 2014 and 75% by May 2015.
Break and Moving to Verticals
2:45-3:30 Vertical Teams with your School

Each school will turn in an exit ticket that addresses the guiding question. Individual copies of the questions have been provided for you to refer to during the discussion.

Guiding Question:

• How does the Mathematical landscape and formative assessment fit with my current teaching practices and my knowledge about nurturing and developing young mathematicians?

• Presenters: List schools and assigned room numbers here!
References


