Learning Progressions

Learning Progression Model

Block of Knowledge → Block of Knowledge → Subskill → Curricular Aim

FA Checks
FA Checks
FA Checks
FA Checks

3 2 4 9
(Lessons)

Learning Progression: A carefully sequenced set of building blocks consisting of subskills and bodies of enabling knowledge that, it is believed, students must master en route to mastering a more remote curricular aim.

Popham, 2008

Progressions for the Common Core State Standards in Mathematics (draft)

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20 May 2011

K, Counting and Cardinality; K-5, Operations and Algebraic Thinking

Counting and Cardinality and Operations and Algebraic Thinking are about understanding and using numbers. Counting and Cardinality includes Operations and Algebraic Thinking as well as Number and Operations in Base Ten. It begins with early counting and adding how many in one group of objects. Addition, subtraction, multiplication, and division is taught over the K-5 grades. As the learner begins, the Progression includes important ideas that are usually taught in isolation, there ideas need to be taught in ways that are interesting and engaging in young children.

The Progression is of professional and Algebraic Thinking starts with the basic operations—the kinds of quantities relationships they model, and consequently the kinds of problems they can be used to solve as well as their mathematical properties and relationships. Although most of the standards organized under the OA teaching involve whole numbers, the importance of the Progression is much more general because it describes concepts properties, and representations that extend to other number systems and to objects. For example, if the mass of the sun is a constant, and the mass of the sun system is a whole in the same, it is a kilogram. In this kind of algebraic reasoning, it doesn’t matter whether y and z are whole numbers, fractions, decimals, or even matrices. Likewise, a property such as distributivity for all the number systems that students will study in K-12, including complex numbers.

The generality of the concepts intended in Operations and Algebraic Thinking means that students’ work in this area should be designed to help them extend arithmetic beyond whole numbers use the K- and H- Progressions and understand and apply operations and properties at later grades beyond the K- Progression. Addition and subtraction are the first operations studied. In
Constructing a Landscape

Big Ideas

Strategies

Mathematical Models
Skip Counting
Doubling
Doubling and halving
Associative Property
Social Knowledge
Arrays/area models
Distributive Property
Commutative Property
Repeated Addition
Number Line
Big Ideas, Strategies, Models

Working in your group:

• Which are you familiar with? Which does your group need to know more about?

• How might you group these or place these in a “landscape” to show the mathematical connections?

• Do any build on each other?
Activity: “Big Idea”

Step 1: Critical Content

- Identify big ideas, strategies, and models related to \((x \text{ and } ÷) / (+ \text{ and } -)\)
- Discuss with your face partner which critical content area(s) focus(es) on \((x \text{ and } ÷) / (+ \text{ and } -)\)
Activity: “Big Idea”

Step 2:

- Quickly skim and scan cluster headings and standards related to multiplication and division or addition and subtraction.
BREAK TIME
Interpret Evidence
Clarify Intended Learning
Act on Evidence
Elicit Evidence

Formative Assessment Process
Eliciting Evidence of Learning

We have to know where we are before deciding where we need to go.
Class Inventory

- In preparation for the beginning of the school year, we were organizing our supplies and we found a large container of paperclips that need to be inventoried.

- Consider how first graders at the beginning of the year problem solve, then count the paperclips as a group as if you were a first grader.

- As a group, use chart paper to record how this task might be solved using their strategies.
Task Activity – Gallery Walk

First Grade: Making Connections to your Classroom

- Consider strategies and big ideas that this task may expose.
- As you observe solutions offered by other groups, consider what you might expect to observe with your students?
- Based on experience, what typical steps and missteps do you anticipate?
Populate your Landscape

What mathematical ideas did we discuss during the inventory task?

Choose ideas from the landscape sort that is in the middle of your table.
Strategies

Big Ideas

Model

Populate the Landscape

Model with Groups

Uses 1-9 sequence when counting

Model with symbols to represent amounts

Need for Organization & Keeping Track

Cardinality

Modeling of action

Modeling of situation

Skip counting

Models quantities with tallies

One-to-one correspondence

One-to-one tagging

Synchrony: one word for every object

Counting
Norms for Watching Teaching Videos

Teaching is multi-faceted.

- *The video doesn’t show everything.*

Teaching is incredibly hard work!

- *Assume positive intent.*

No lesson is ever perfect.

- *Focus on what you can use to improve your classroom instruction.*
- *Focus on the student’s and teacher’s interaction with the Mathematics.*
Think about Jodi’s mathematical goal in choosing this particular investigation.

What mathematical big ideas, strategies and models do you expect to see as the children set to work?
Clip 1 Discussion

What do you think Jodi’s mathematical goal is in choosing this particular investigation?

What mathematical big ideas, strategies and models do you expect to see as the children set to work?
As you watch the next clip, look for student’s understanding or use of the big ideas, strategies and models.

Find one example of how Jodi is scaffolding her students to move along the landscape.
Clip 2 Discussion

- What big ideas, strategies and models did you observe?
- How did Jodi scaffold her students to move along the landscape?
Notice Jodi’s strategic use of questioning.

Look for the big ideas, strategies and models you observe.
What did you notice about Jodi’s strategic use of questioning?
What big ideas, strategies and models did you observe?
Look for ways that class discussion supports learning for all students.
Clip 4 Discussion

How did the class discussion support learning for all students?
Populating Your Landscape

- After watching these videos, what big ideas, strategies or models would you like to add to your landscape?
- How do these elements connect?
Populate the Landscape

- Strategies
- Big Ideas
- Model

1. Need for Organization & Keeping Track
2. Model with symbols
3. Place determines value
4. Unitizing
5. Using 10 structure
6. Making 10s
7. Modeling of action
8. Modeling of situation

- Counting
- Systematic production of arrangements
- Counting on
  - Skip counting
  - Models quantities with tallies
  - One-to-one correspondence
  - One-to-one tagging
  - Synchrony: one word for every object

- Model with Groups
- Use 1-9 sequence when counting

- Need for Organization & Keeping Track
- Cardinality
- Populating the Landscape
Balcony View

Quiet Reflection: (3 min.)
Looking at your note taker, what formative assessment practices did you notice in these video clips?

Discuss in your table groups.
LUNCH
11:30 to 12:30

• See you promptly at 12:30.
• Remember to sign in when you return.
Explore ways in which FA attributes and practices move learning forward.

- Participants will construct a progression of learning.
- Participants will discuss FA practices observed.

How to embed formative assessment practices into daily math instruction.

Context of Learning: x and ÷ + and -
Eliciting Evidence of Learning
We have to know where we are before deciding where we need to go.

What techniques or strategies do you currently use in your classroom to elicit evidence of learning?
Taking this Back to Your Classroom

Discuss with your table partners how you could do a similar investigation in your classroom at the beginning, middle, and end of the school year.
Observing with Purpose
While watching this video again focus on Haley (the boy in the yellow shirt) and his counting strategy.
Did focusing your observation change what you observed?

What do Haleys’ actions tell us about his thoughts?
Clip 6 Discussion

What do Haley’s actions tell us about his thoughts now?

What big idea is Haley and Alexander demonstrating in this clip?
Chip Change

▶ Follow the directions in your handout to play the game.

▶ Play the game in a pair. The first pair plays while the second pair observes.

▶ The goal is to change the color of the chips so that at the end all ten chips have the other color facing up.

▶ Now the second pair plays and the first pair observes.

*from *Games for Early Number Sense* by Catherine Twomey Fosnot and Antonia Cameron
Taking this Back to Your Classroom

How could you use this game in your classroom?

Do you have a similar activity that you already use in your classroom? If so share with your group.
TIPS for Class Discussions
Engaging all learners and keeping cognitive demand high

- Practice “wait time”
- Clarify students ideas in a variety of ways
- Emphasize reasoning
- Encourage student–student dialogue.

Want to learn more? See professional development opportunities.

(Van de Walle et al., 2014)
Classroom Discussion Opportunities

- Help Individual Students Clarify and Share their own Thinking
- Helping Students Orient to the Thinking of Others
- Helping Students Deepen Their Own Reasoning
- Helping Students Engage with the Reasoning of Others
After watching these videos, what big ideas, strategies or models would you like to add to your landscape?

How do these elements connect?
Data Collection – Kidwatching and the landscape

NAME: Sharon V.

DIVISION OF FRACTIONS

<table>
<thead>
<tr>
<th>NOT YET</th>
<th>ON TARGET</th>
<th>ABOVE AND BEYOND</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uses measurement interpretation</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uses partitive interpretation</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model [specific model]</td>
<td>✓</td>
<td>Used two-color counters to show $\frac{1}{2} + \frac{1}{3} = $</td>
<td></td>
</tr>
<tr>
<td>Answers that are not whole numbers</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Understands standard algorithm</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

MATHEMATICAL PRACTICES

<table>
<thead>
<tr>
<th>NOT YET</th>
<th>ON TARGET</th>
<th>ABOVE AND BEYOND</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Makes sense of problems and perseveres</td>
<td>✓</td>
<td></td>
<td>Stated problem in own words</td>
</tr>
<tr>
<td>Models with mathematics</td>
<td>✓</td>
<td></td>
<td>Reluctant to use abstract models</td>
</tr>
<tr>
<td>Uses appropriate tools</td>
<td>✓</td>
<td></td>
<td></td>
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</tbody>
</table>
Balcony View

1

Quiet Reflection: (3 min.)
Looking at your note taker, what formative assessment practices did you notice while engaging in the videos and game?

2

Discuss in your table groups.
Considering Formative Assessment and the Landscape

- Clarify intended learning
- Elicit evidence.
- Interpret evidence
- Act on evidence
Partner Conversation:  
(4 min.)

1. What are the NVACS critical areas for my grade level?

2. How does the landscape support the critical areas?

3. What are some of the connections between the NVACS standards and the Mathematical landscape?
Considering the Critical Areas and the Mathematical Landscape

At the first grade level, addition and subtraction is found in two critical areas. The first is **developing understanding** of addition, subtraction, and **strategies for addition and subtraction** within 20. The second critical area focuses on **developing understanding of whole number relationships** and place value, including grouping in tens and ones.
## Considering the NVACS Standards

<table>
<thead>
<tr>
<th>NVACS Standards</th>
<th>Language from the standards</th>
<th>Big Ideas from the landscape</th>
<th>Models to use with students</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.NBT.2</td>
<td>Two-digit numbers Tens and ones</td>
<td>Unitizing, Place-determines value</td>
<td>*groups *ten-frame *Open array</td>
</tr>
<tr>
<td>1.NBT.2c</td>
<td>Tens and ones</td>
<td>Unitizing, place-determines value</td>
<td>*groups *number grid *ten frame</td>
</tr>
<tr>
<td>1.NBT.4</td>
<td>Multiples of 10 Strategies based on place value Properties of operation Tens and ones</td>
<td>Place value patterns that occur when making and adding groups of 10, Commutativity, Associativity</td>
<td>*Number line *ten-frame *Math rack</td>
</tr>
<tr>
<td>1.NBT.6</td>
<td>Multiples of 10 Concrete models Strategies Properties of operation</td>
<td>Place-determines value,</td>
<td>*Number line *ten-frame *Math rack</td>
</tr>
</tbody>
</table>
The landscape provides a different lens to recognize the big mathematical ideas in the standards.

The mathematical ideas have not changed, but a better and deeper understanding will help us to know how to teach and differentiate instruction.
The Landscape’s Big Idea

1.OA.2 “Solving word problem with three whole numbers”

1.NBT.6 “Concrete models to subtract to 90”

1.OA.1 “Start, change or results unknown”

1.MD.4 “Represent and interpret data”

1.NBT.4 “Concrete models to add to 100”

1.NBT.5 “Ten more, ten less”

1.OA.3 “Commutative and associative properties”

1.OA.6 “Composition and decomposition”

1.OA.5 “Adding and subtracting within 20”

1.OA.4 “Unknown - Addend”

1.NBT.3 “Comparison of two-digit numbers”

1.NBT.2 “Tens and ones”

1.OA.1 “Represent and solve”

1.OA.3 “Represent and solve”

1.OA.2 “Solving word problem with three whole numbers”

1.NBT.6 “Concrete models to subtract to 90”

1.OA.1 “Start, change or results unknown”

1.MD.4 “Represent and interpret data”

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1.OA.6 “Composition and decomposition”

1.OA.5 “Adding and subtracting within 20”

1.OA.4 “Unknown - Addend”

1.NBT.3 “Comparison of two-digit numbers”

1.NBT.2 “Tens and ones”

Review the standards. Where are the connections to place value, unitizing and addition and subtraction.
Explore ways in which FA attributes and practices move learning forward.

Plan next steps in embedding learning into practice.

- Participants will construct a progression of learning.
- Participants will discuss FA practices observed.

PLCs will begin planning instruction, within today’s context, embedding formative assessment practices in plans.

Compose and share a definition or quote capturing the essence of what formative assessment is.

Context of Learning: x and ÷ + and -

Develop a conceptual understanding of the Formative Assessment Process and its benefits.

How to embed formative assessment practices into daily math instruction.
Resources to Support

• WCSD Pacing Timeline

<table>
<thead>
<tr>
<th>Unit</th>
<th>Whole-Number Operations in Number Stories</th>
<th>Patterns &amp; Rules</th>
<th>Fractions</th>
<th>Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of Lessons: 12 over 13 Days</td>
<td>Number of Lessons: 10 over 12 Days</td>
<td>Number of Lessons: 6 over 8 Days</td>
<td>Number of Lessons: 6, 5, 4, 8, 2, 2, 1</td>
</tr>
</tbody>
</table>

Expanding Unit
Topic: NBT & Operations
Number Stories:
Additional Instructional Days: 8
Total Days: 13
Quarter 3: 48 total instructional days

• WCSD Curriculum Documents

• Instructional Practice Guides

District Performance Plan Goal 1, Objective 4: The percentage of site administrators, instructional coaches, teachers, and instructional ESPs who report familiarity with WCSD Core NACS Materials and curriculum tools adopted and/or endorsed by WCSD will reach 50% by October 2014 and 75% by May 2015.
Fall PD Opportunities

✓ District-Wide PLTs (16 hours, 1 credit)
  ✓ Continue your learning from today!
  ✓ Four follow-up sessions: 9/25, 12/4, 2/26, 5/21
  ✓ Work in grade level teams
  ✓ Content tied to “Pacing Timeline” to provide “just in time” connected learning opportunities to prepare for the next quarter of content

✓ Computational Strategies/Algorithms (8 hours, ½ credit)

✓ Core Connections: K-5 (8 hours, ½ credit) Same as offered last year. Join us if you missed it or have changed grade levels!

✓ eSuite Basic (no credit): 8/19 or 9/9

✓ Intensification Lab: During winter break

✓ Planning in Math (16 hours, 1 credit)

✓ Using Classroom Discussion to Promote Problem Solving and Solution Strategies in Mathematics

✓ Formative Assessment Webinars – (October – November)

Check Solutionwhere frequently for updates and additional opportunities!
Break and Moving to Verticals
2:45-3:30 Vertical Teams with your School

Each school will turn in an exit ticket that addresses the guiding question. Individual copies of the questions have been provided for you to refer to during the discussion.

Guiding Question:

• How does the Mathematical landscape and formative assessment fit with my current teaching practices and my knowledge about nurturing and developing young mathematicians?
References


• Fosnot, Catherine, and Dolk, Maarten. (2001). *Young Mathematicians at Work. Constructing Multiplication and Division*. Heinemann, NH.

• Fosnot, Catherine, and Dolk, Maarten. (2001). *Young Mathematicians at Work. Constructing Addition and Subtraction*. Heinemann, NH.


