

10.1 Operations with Matrices ⁺ 10.1 w.s.

Dimension of a matrix = row x column

$$A = \begin{bmatrix} 1 & 7 \\ 4 & 2 \\ 8 & 3 \end{bmatrix} \quad \begin{array}{l} 3 \text{ rows} \\ 2 \text{ columns} \end{array} \quad \text{dimension} = 3 \times 2$$

$$B = [1 \ 4 \ 8 \ -7] \quad \begin{array}{l} 1 \text{ row} \\ 4 \text{ columns} \end{array} \quad \text{dimension} = 1 \times 4$$

ex] Identify the element of each matrix

$$A = \begin{bmatrix} 4 & -2 \\ -3 & 7 \end{bmatrix} \quad \text{Find } a_{11} = 4$$

First row
First column

$$b) a_{12} = -2$$

First row
Second column

$$\text{ex] } A = \begin{bmatrix} -7 & 3 \\ 1 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 0 & -2 \\ 4 & 7 \end{bmatrix}$$

$$a) A + B = \begin{bmatrix} -7+0 & 3+(-2) \\ 1+4 & 8+7 \end{bmatrix} = \begin{bmatrix} -7 & 1 \\ 5 & 15 \end{bmatrix}$$

$$b) B - A = \begin{bmatrix} 0 - (-7) & -2 - 3 \\ 4 - 1 & 7 - 8 \end{bmatrix} = \begin{bmatrix} 7 & -5 \\ 3 & -1 \end{bmatrix}$$

$$c) 3B = \begin{bmatrix} 3(0) & 3(-2) \\ 3(4) & 3(7) \end{bmatrix} = \begin{bmatrix} 0 & -6 \\ 12 & 21 \end{bmatrix}$$

10.2 Matrix Multiplication

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} aw+by & ax+bz \\ cw+dy & cx+dz \end{bmatrix}$$

ex) $\begin{bmatrix} -2 & 1 \\ -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -1 & -5 \\ 0 & 4 \end{bmatrix}$

$$\begin{bmatrix} -2(-1)+1(0) & -2(-5)+1(4) \\ -1(-1)+0(0) & -1(-5)+0(4) \end{bmatrix} = \begin{bmatrix} 2 & 14 \\ 1 & 5 \end{bmatrix}$$

b) $\begin{bmatrix} -1 & -5 \\ 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 \\ -1 & 0 \end{bmatrix}$

$$\begin{bmatrix} -1(-2)+-5(-1) & -1(1)+-5(0) \\ 0(-2)+4(-1) & 0(1)+4(0) \end{bmatrix} = \begin{bmatrix} 7 & -1 \\ -4 & 0 \end{bmatrix}$$

↑ notice
not the
same

Different Dimensions multiplying

$(2 \times 2)(2 \times 2)$

must
be the
same

2×2

$(2 \times 3)(3 \times 1)$

2×1

$(3 \times 2)(3 \times 1)$

can't do

ex) $[.5 \ .3 \ .2]$

$$\begin{bmatrix} 90 & 80 \\ 95 & 70 \\ 75 & 85 \end{bmatrix}$$

$(1 \times 3)(3 \times 2)$

1×2

$$[.5(90)+.3(95)+.2(75) \quad .5(80)+.3(70)+.2(85)]$$

$$[88.5 \ 78]$$

10.4 Inverse and Determinants ⁺ 10.4 w.s.

Determinant $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\det = ad - bc$

Inverse $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

ex] $A = \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}$

a) Find det

$$2(0) - 1(3) = -3$$

b) Find Inverse

$$\frac{1}{-3} \begin{bmatrix} 0 & -1 \\ -3 & 2 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 0 & 1/3 \\ 1 & -2/3 \end{bmatrix}$$

ex] Does the inverse of the matrix exist?

$$B = \begin{bmatrix} 6 & 8 \\ 3 & 4 \end{bmatrix}$$

Find det

$$6(4) - 3(8) = 0$$

Since the det equals 0, then no inverse.

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(Need more)

10.5 Solve Matrix Equations

$$\text{ex)} \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \end{bmatrix}$$

↑
Find inverse

$$\det = 2(2) - (-1)(0) = 4$$

$$\begin{bmatrix} 1/2 & 1/4 \\ 0 & 1/2 \end{bmatrix} = \text{Inverse}$$

$$\begin{bmatrix} 1/2 & 1/4 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \end{bmatrix} \begin{bmatrix} 1/2 & 1/4 \\ 0 & 1/2 \end{bmatrix}$$

I (1x2) (2x2) 1x2

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2(\frac{1}{2}) + 8(\frac{1}{4}) \\ 2(0) + 8(\frac{1}{2}) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\text{ex)} \begin{cases} 3x - 5y = 7 \\ 2x + y = 4 \end{cases}$$

write matrix
equation

$$\begin{bmatrix} 3 & -5 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$$